Chapter 3: Formal Logic

Background

Formal logic is a branch of logic that studies the logical structure of arguments, identifying the conditions under which a conclusion of an argument can be said to follow logically from the premises of the argument even if the premises are false. The study of formal logic concerns the form (structure) of an argument rather than its content, the latter being a concern of informal logicians.

About Chapter 3

This chapter concludes the unit theme of reasoning about reasoning by examining the logical structures of arguments. It also examines what it means to be logical. The key question that this chapter addresses is: When is a conclusion logically entailed in the premises? The logical entailment of a conclusion means that, according to the logical structure or form of an argument, if the premises are accepted as true then the conclusion must be true as well. This chapter includes the development of techniques for testing logical entailment in arguments, as well as descriptions of various logical structures that give rise to logical entailment. Faulty logical structures (formal logic fallacies) are also examined.

This chapter is the most theoretical of the three in this unit and therefore may be the most challenging for students. Completing the activities in Teaching Plan 1 will more than satisfy curriculum expectations; depending on your comfort level with this material and on the interests and abilities of the students, you may decide to stop here. If, however, you wish to delve more deeply into formal logic, then Teaching Plan 2 will enable you to do so.

Features

Not applicable.

BIG IDEAS

- Formal logic is the study of the logical form of an argument, not its content. (SE pp. 66-67)
- Venn diagrams can be used to test the validity (logical entailment) of a categorical syllogism, a three-line argument that categorizes things as having, or not having, certain properties. (SE pp. 68-71)
- The logical connectives used in propositional logic are introduced: "not," "and," "or" and "if…then…" (SE pp. 72-76)
- The meanings of logical connectives can be used to establish logically valid forms of argumentation (e.g., *modus ponens*, *modus tollens*, *disjunctive syllogism*). (SE pp. 72-77)
- Certain logical forms are fallacious. These include: fallacy of *affirming the consequent*, fallacy of *denying the antecedent*, and fallacy of *affirming the disjunct.* (SE pp. 74-76)
- *Rules of inference* and *rules of replacement* (valid logical structures) can be used to determine whether a particular argument is valid. (SE pp. 80-82)
- Any proposition logically follows from a contradiction. (SE p. 83)

Teaching Plan 1 (SE pp. 66-85)

Activity Description

Students are guided through all of Chapter 3. They read the chapter individually, answering and discussing the section questions in pairs or groups as they go.

Learning Goal

Students are introduced to formal logic, categorical syllogisms, and Venn diagrams. Students are also introduced to propositional logic, logical connectives, and valid and invalid forms of argumentation.

Assessment Opportunities for Chapter Questions

The table below summarizes assessment opportunities for selected chapter questions, including questions in the Chapter Review, which are relevant to this teaching plan.

Assessment Type	Assessment Tool	Feature Questions	Section Questions	Chapter Review Questions
Assessment for Learning	Text answers		a)-e), SE p. 71 1, 3, 4, 5, SE pp. 78-79 a)-d), SE p. 82	
Assessment as Learning	Text answers		2, SE p. 78	
Assessment of Learning	Text answers			1–9, SE pp. 84-85

Timing

225 minutes
(three 75-minute classes)

Learning Skills Focus

- Independent Work (students complete some chapter questions individually)
- Collaboration (students complete answers to chapter questions in pairs or small groups)

Resources Needed

Make copies of this Blackline Master:

- BLM 3.1 Categorical Syllogisms

Possible Assessment of Learning Task

Students may complete the Chapter Review questions (SE pp. 84-85) as a take-home or open-book test

Assessment (For/As Learning)

As teachers move through each chapter, opportunities will be highlighted to provide assessment for/as learning in preparation for assessment of learning at the end of each chapter.

Task/ Project	Achievement Chart Category	Type of Assessment	Assessment Tool	Peer/Self/ Teacher Assessment	Learning Skill	Student Textbook Page(s)	Blackline Master
Section questions	Knowledge; Thinking; Communication; Application	As	Text answers; discussion	Self; peer	Collaboration	71	
Section questions	Knowledge; Thinking	As	Text answers; discussion	Peer	Collaboration	77-78	
Section questions	Knowledge; Thinking; Communication; Application	As	Text answers; discussion	Peer	Collaboration	82	

Prior Learning Needed

Students should be familiar with the following key terms/concepts from Chapter 1: *argument*, *conclusion*, *contradiction*, *deductive argument*, *formal logic*, *invalid deductive argument*, *law of the excluded middle*, *law of non-contradiction*, *logical entailment*, *premise*, *proposition*, *syllogism*, and *valid deductive argument*.

Teaching/Learning Strategies

1. To whet students' appetite for formal logic, show a clip from the film *Monty Python and the Holy Grail*, in which faulty logic is used to determine that a woman is a witch. (The British comedy troupe Monty Python often injected philosophical humour into their films and comedy skits.) A Web search for *Monty Python witch* will give you several YouTube versions of the scene. After students watch the witch skit, have a brief class discussion as to what was faulty about the reasoning in the skit.

   The argument in the witch skit is utterly absurd and, for that reason, is difficult to reconstruct for analysis. Below is a reconstruction for reference. You may, after showing the skit, place the following on the board and ask students to identify flaws in the various sub-arguments that make up the whole argument. (Note: students may have difficulty extracting the arguments from the dialogue, so it helps to present the arguments on the board.)

   P1: If she weighs the same as a duck, then she floats.

   P2: She weighs the same as a duck.

   C: Therefore: She floats.

P1: Wood floats.

P2: She floats. [This premise is established from the above argument.]

C: Therefore: She is made of wood.

P1: We burn witches.

OR: Witches belong to the category of things we burn.

P2: We also burn wood.

OR: Wood belongs to the category of things we burn.

C: Therefore: Witches are made of wood.

P1: Witches are made of wood.

P2: She is made of wood. [This premise is established from one of the above arguments.]

C: Therefore: She is a witch.

After students have had an opportunity to discuss and find flaws with the arguments, you may wish, during whole-class discussion, to use some of the points noted below, in response to each of the sub-arguments:

P1: If she weighs the same as a duck, then she floats.

P2: She weighs the same as a duck.

C: Therefore: She floats.

This is a valid argument in that the conclusion is logically entailed in the premises. The valid inference in this case is known as *modus ponens* (see SE p. 73). But the argument is not sound (SE p. 68), meaning, in this case, that it is based on faulty premises. This argument provides a good opportunity to introduce the distinction between soundness and validity.

P1: Wood floats.

P2: She floats. [This premise is established from the above argument.]

C: Therefore: She is made of wood.

This is not a valid argument: the conclusion is not logically entailed in the premises. While all wood does indeed float, not all floating things are made of wood and this is what the argument assumes. In technical language, this argument commits the fallacy of the undistributed middle (see SE p. 30).

P1: We burn witches.

OR: Witches belong to the category of things we burn.

P2: We also burn wood.

OR: Wood belongs to the category of things we burn.

C: Therefore: Witches are made of wood.

We can reconstruct this argument as a *categorical syllogism* (SE p. 68): All witches burn; all wood burns; therefore, all witches are made of wood. Apart from the absurd and faulty first premise, we burn witches(!), this argument also commits the fallacy of the undistributed middle in assuming that, from the premise that all wood burns, all burning things are made of wood.

P1: Witches are made of wood.

P2: She is made of wood. [This premise is established from one of the above arguments.]

C: Therefore: She is a witch.

Both premises are faulty in their own right and they were established from faulty arguments. The argument also commits the fallacy of the undistributed middle in assuming that all things made of wood are witches.

Again, the above arguments can be used to point out the following ideas: deduction, logical entailment, soundness, and validity.

2. Now have students read SE pp. 68–71 and complete the questions on SE p. 71 in pairs. When students are finished, ask different pairs of students to place their Venn diagrams on the board for whole-class discussion.

DI If extra practise is needed, have students complete BLM 3.1 in pairs, again having different pairs of students placing their diagrams on the board for whole-class discussion.

3. Students should now read and study SE pp. 72-77. As students finish their reading, arrange them into groups of four to answer, collaboratively, the questions on SE pp. 78-79; each group should try to reach consensus regarding their answers. After students have had an opportunity to do this, resolve any differences of opinion by referring to the answers provided below. The discussion of these questions will probably take the entire class.

4. Students should now read SE pp. 80-81 and, in pairs, answer the questions on SE p. 82. After students are finished, ask different pairs to place their answers on the board for whole-class discussion.

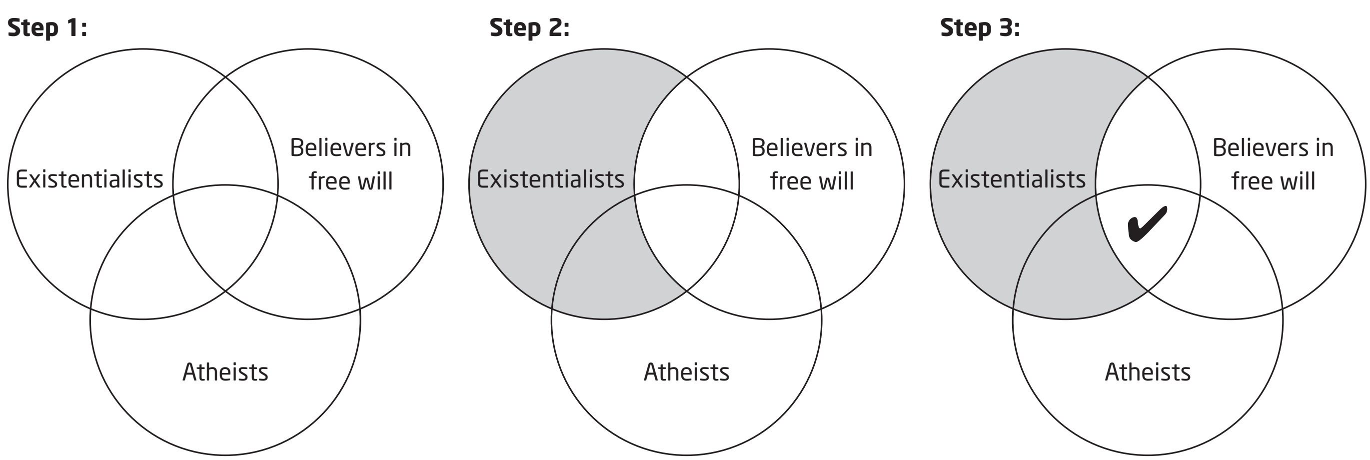
5. The Chapter Review could be used as an assessment *of* learning by having students complete and submit their answers, individually or in groups, to the questions on a separate sheet of paper (treating this as an open-book test or take-home assignment).

Text Answers

Page 71: Section questions

The solutions below follow the steps described on SE p. 69. To complement the technical step-by-step process, ask students to also determine the validity of these arguments by reasoning about them informally.

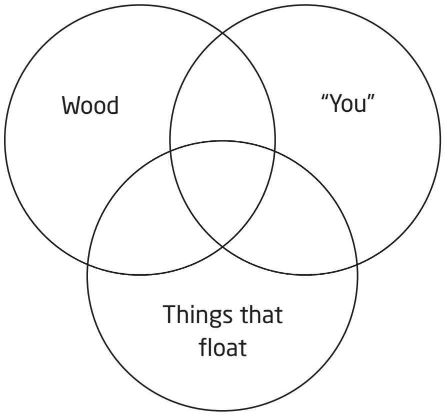
a)



Step 4 (conclusion): Valid.

b)

Step 1:



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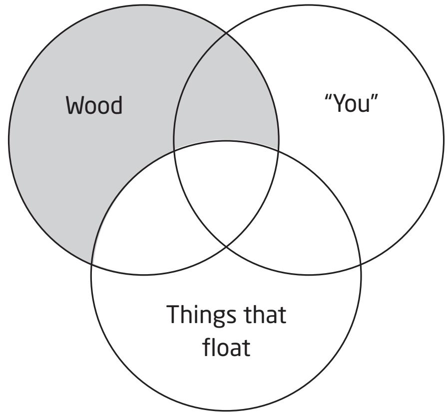

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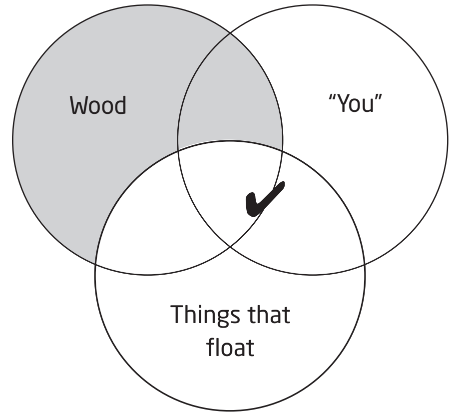

Step 4 (conclusion): Invalid.

c)

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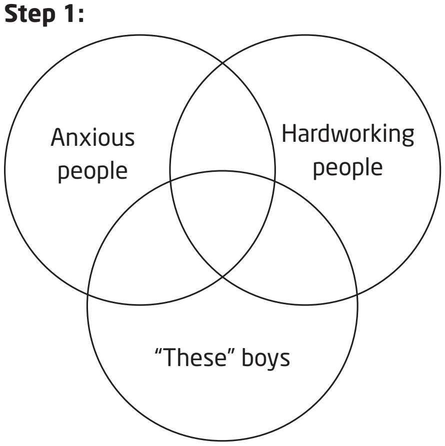

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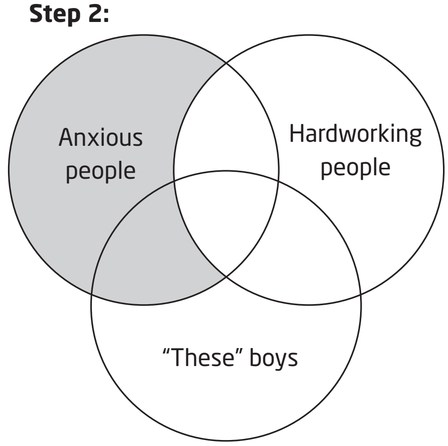

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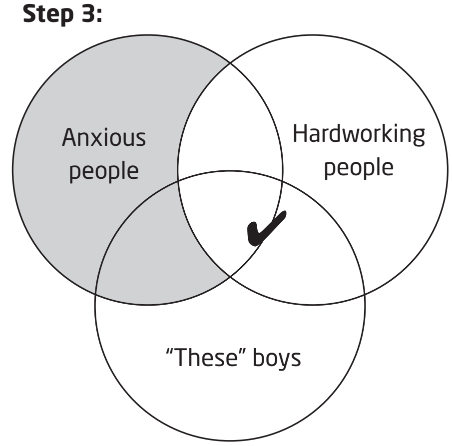
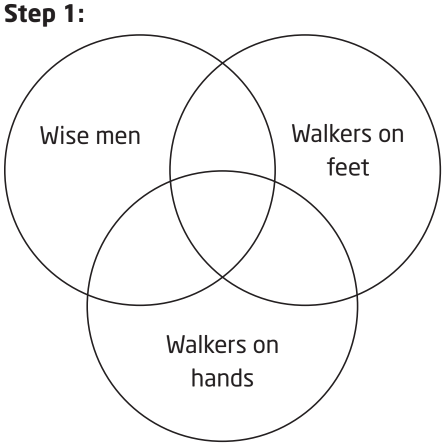


Step 4 (conclusion): Valid.

d)

Step 1:



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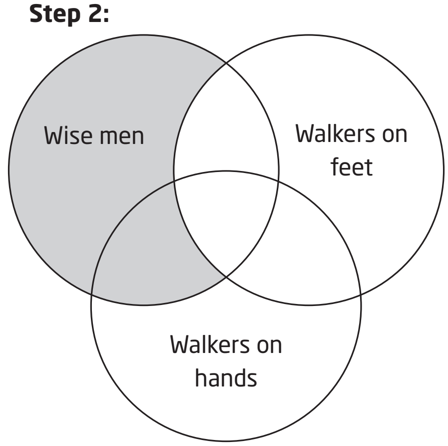

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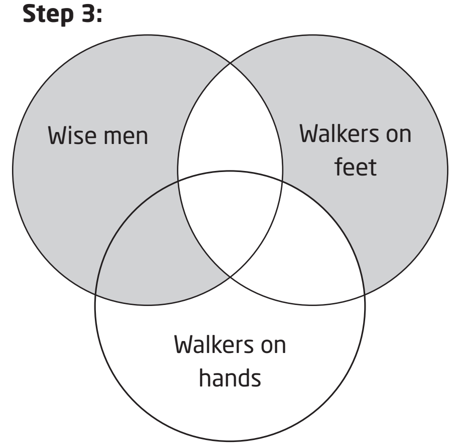
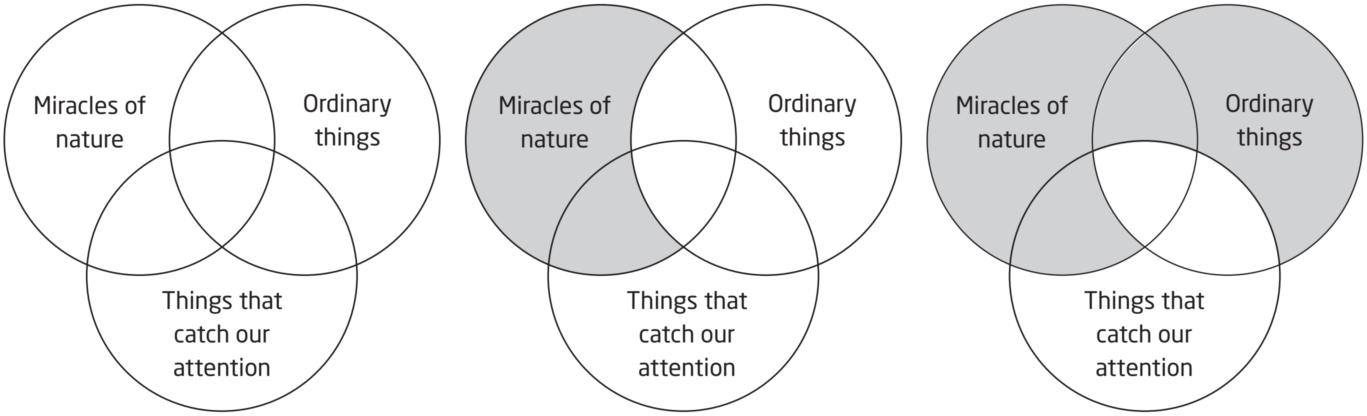


Step 4 (conclusion): Invalid.

e)

Step 1: **Step 2:** **Step 3:**



Step 4 (conclusion): Invalid.

Pages 78-79: Section questions

1. a) Invalid: Fallacy of affirming the consequent.

   b) Valid: *Modus tollens*.

   c) Invalid: Fallacy of denying the antecedent.

   d) Valid: *Modus ponens*.

2. The rule is: "If a card has a vowel on one side, then it has an even number on the other side." Using this rule and *modus ponens*, we should turn over any card that has a vowel on one side to see if it has an even number on the other. Thus, we should turn over the third card. Using this rule and *modus tollens*, we should turn over any card that does not have an even number on one side (i.e., an odd number) to make sure it does not have a vowel on the other side. Thus, we should turn over the fourth card.

3. The two statements are logically equivalent. For example, if it is true that "If Devon is going to the party, then Aidan is also going to the party," then, by *modus tollens*, it follows that if Aidan is not going to the party then neither is Devon.

4. a) is equivalent to ii)

   b) is equivalent to iv)

   c) is equivalent to v)

   d) is equivalent to i)

   e) is equivalent to iii)

5. a) Valid.

   b) Valid.

   c) Invalid: suppose p is false and q is true, then the premise is true and the conclusion is false.

   d) Invalid: suppose p is false and q is true, then the premise is still true (it has not been contradicted) but the conclusion is false.

   e) Valid.

Page 82: Section questions

1. a) Valid. Using *H* to stand for the proposition "Humans have a soul," and *S* to stand for the proposition "Humans are self-aware," the argument can be symbolized as follows:

$$H \supset S$$
$$\therefore \ {\sim}H \vee S$$

The statement $H \vee \sim H$ is always true (by the law of the excluded middle). We can replace *H* with *S* since, according to the premise in the argument, if *H* is true then so is *S*. Thus, $H \vee \sim H$ becomes $S \vee \sim H$ which is the same as the conclusion but written in reverse.

Note that the above illustrates, in part, a rule of replacement called *material implication*. This rule asserts that statements of the form $H \supset S$ can be replaced with statements of the form $\sim H \vee S$ and vice versa.

b) Valid. The argument can be symbolized as follows:

$$D \supset (E \vee F)$$
$$\sim E \wedge \sim F$$
$$\therefore \ {\sim}D$$

We can replace the second premise, $\sim E \wedge \sim F$, with $\sim (E \vee F)$ using De Morgan's Rule (see SE p. 80). With this replacement, we can use *modus tollens* to establish ~*D*:

$$D \supset (E \vee F)$$
$$\sim (E \vee F)$$
$$\therefore \ {\sim}D$$

c) Valid. Using *T* to stand for the proposition "A Heffalump is in the trap," *S* for the proposition "It is a special day," and *H* for the proposition "We should say 'Ho ho!'," the argument can be symbolized as follows:

$$T \supset S$$
$$(\sim T \vee S) \supset H$$
$$\therefore H$$

Using the rule of material implication (described in relation to a), above), we can replace $T \supset S$ with $\sim T \vee S$. With this replacement, we can use *modus ponens* to establish *H*:

$$\sim T \vee S$$
$$(\sim T \vee S) \supset H$$
$$\therefore H$$

d) Valid. Using *M* to stand for the proposition "The Mosaic account of the cosmogony is strictly correct," *S* for "The sun was not created until the fourth day," *A* for "The sun could not have been the cause of alternation of day and night for the first three days," and *D* for "The word 'day' is used in scripture in a different sense from that which is commonly accepted now," the argument can be symbolized as follows:

$$M \supset S$$
$$S \supset A$$
$$D \vee \sim A$$
$$\therefore \sim M \vee D$$

From $M \supset S$ and $S \supset A$ we can use the rule *hypothetical syllogism* to get $M \supset A$. The statement $D \vee \sim A$ can be rewritten as $\sim A \vee D$ which can be replaced with $A \supset D$ (see the note in question a) above). We have now established that $M \supset A$ and $A \supset D$ are both logically true. Combining these, we get $M \supset D$ (again, hypothetical syllogism). And $M \supset D$ can be rewritten as $\sim M \vee D$ (again, see question a) above). Note: We could have symbolized things differently; for example, *A* could stand for "The sun was the cause of alternation of day and night for the first three days." This would mean that instead of *A* above we would have ~*A* and instead of ~*A* we would have *A*, but the argument would still be valid as shown above.

Pages 84-85: Chapter Review

1. A sound argument is a valid argument in which the premises are accepted as true; a valid argument is an argument in which the conclusion is logically entailed in the premises but the premises are not necessarily accepted as true.

2. a) False. A valid argument may or may not be sound.

b) False. Part of the argument can be rewritten as follows:

P1: If the sun is shining, then the game is on.

P2: The game is on.

C: The sun is shining.

This argument commits the fallacy of affirming the consequent.

c) True. The two statements are logically equivalent. (See also question 3, SE p. 78.)

d) False. If the argument went: "He can have his dessert *if* he eats his mashed potatoes," then it can be symbolized as $m \supset d$. But the argument uses the phrase *only if* and this makes a big difference. Consider the statement "There is fire only if there is oxygen present." This is actually true and, thankfully, it does not mean "If oxygen is present, then there is fire," otherwise we would be in big trouble! It does mean, however, that if there is a fire, then there must be oxygen present, which can be symbolized as $F \supset O$. Similarly, "He can have his dessert only if he eats his mashed potatoes" is symbolized as $d \supset o$, which is saying that if he got his dessert, then he had his mashed potatoes.

e) True. The proposition *p* is either true or false (law of the excluded middle). If *p* is true, then the statement $\sim p \supset p$ is also true, making the entire disjunction, $(p \supset \sim p) \vee (p \supset \sim p)$, true as well. If *p* is false, then the statement $p \supset \sim p$ is true, making $(p \supset \sim p) \vee (\sim p \supset p)$ true as well.

f) False. $\sim(p \wedge \sim p)$ is *always* true because $p \wedge \sim p$ is *always* false.

3. a) Valid. If we use *p* to represent "Patrick is going to the party" and *q* to represent "Qiana is going to the party, then the premise establishes that Patrick is going and Qiana is not, which entails the conclusion that it is not the case that both Patrick and Qiana are going.

b) Valid. The premise is a contradiction: "Patrick is both going and not going to the party." From a contradiction, any proposition can be proven to be true (see SE p. 83).

c) Invalid. The premise, "Patrick is or is not going to the party," is always true but it does not logically entail the proposition *q*, "Qiana is going to the party."

d) Invalid. The premise is given as true, but the conclusion, which is a contradiction, is always false.

e) Valid. Since the conclusion is always true, there will never be an instance in which the premise is true and the conclusion is false.

4. To answer these questions, use the strategy of substituting "Patrick is going to the party" for proposition *p* and "Qiana is going to the party" for proposition *q*.

a) Equivalent.

b) Equivalent.

c) Equivalent.

d) Equivalent.

5. Many answers are possible, but each one must have the following structure:

P1: $p \supset q$

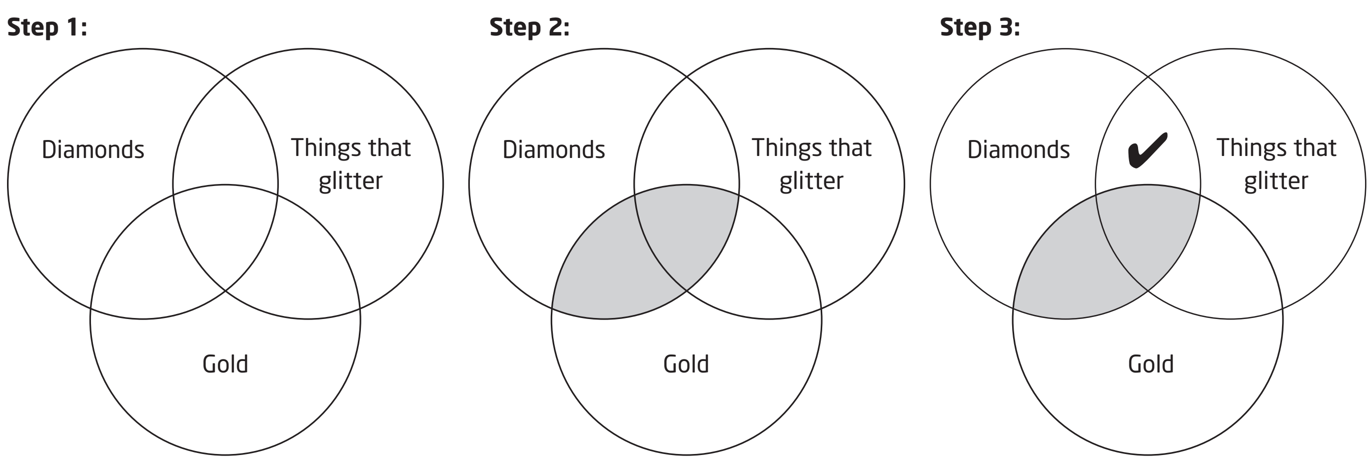
P2: q

C: p

6. Many answers are possible. For example, students could create Venn diagrams showing how their answer to part a) is a valid categorical syllogism and their answer to part b) is an invalid categorical syllogism.

7. It is not possible to draw a conclusion from two particular statements. A little experimentation can demonstrate this; for example, use Venn diagrams for arguments consisting of particular statements.

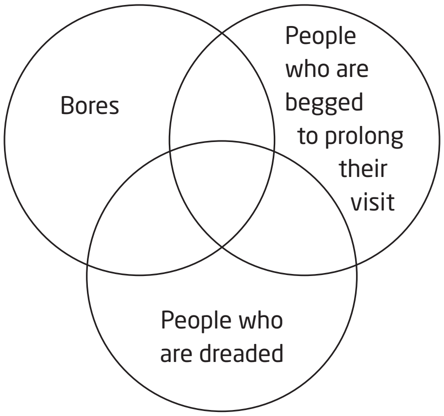
8. a)



Step 4 (conclusion): Valid.

b)

Step 1:



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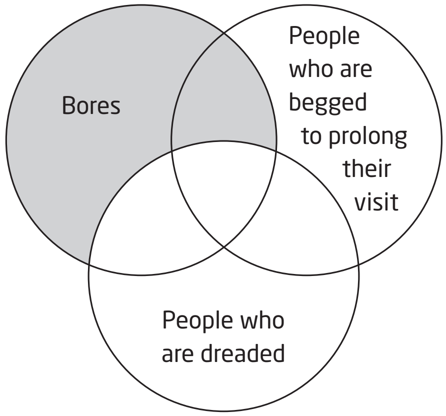

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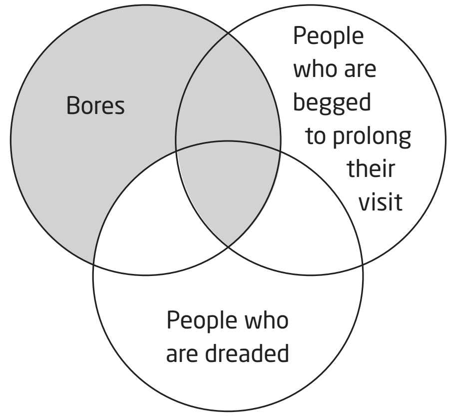

Step 4 (conclusion): Invalid.

c)

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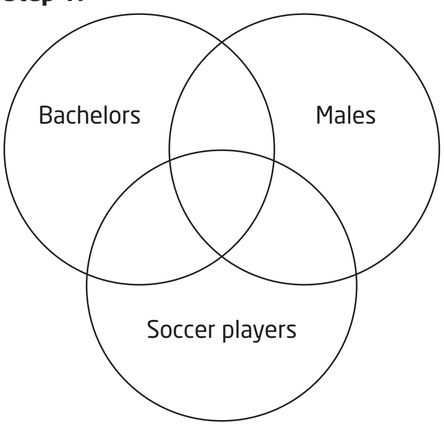

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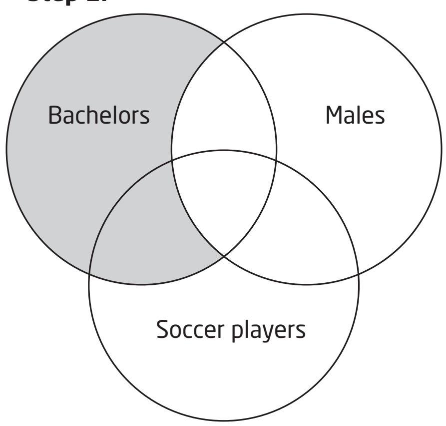

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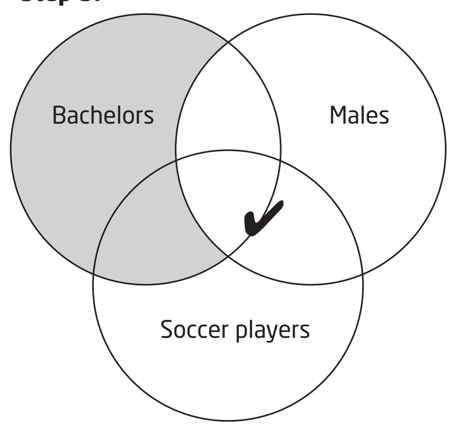

Step 4 (conclusion): Invalid.

9. a) Valid. The argument can be symbolized as:

$$
\begin{array}{r}
(N \vee J) \supset P \\
(P \vee Q) \supset R \\
Q \vee N \\
\sim Q \\
\hline
\therefore R
\end{array}
$$

The argument can be developed, using rules of inference, as follows:

$Q \vee N$
$\sim Q$
$\therefore N$ (disjunctive syllogism)

N
$\therefore N \vee J$ (addition)

$(N \vee J) \supset P$
$N \vee J$ (from above)
$\therefore P$ (*modus ponens*)
P
$\therefore P \vee Q$ (addition)

$(P \vee Q) \supset R$
$P \vee Q$ (from above)
$\therefore R$ (*modus ponens*)

b) Valid. Using A to represent "Ashwini had completed her homework regularly," the argument can be symbolized as:

$\sim A \supset F$
$F \supset \sim E$
E
$\therefore A$

The argument can be developed, using rules of inference, as follows:

$\sim F \supset \sim E$
E
$\therefore \sim F$ (*modus tollens*)

$\sim A \supset F$
$\sim F$
$\therefore \sim\sim A$ (*modus tollens*)

$\sim\sim A$
$\therefore A$ (double negation)

Learning Goal

Students will consolidate and extend their understanding of formal logic. To accomplish this, they will discover more rules of inference and replacement, develop their ability to translate arguments into symbolic form and, through further practise, become more adept at using the rules to prove that various arguments are valid. These exercises are technical and theoretical and should be done as enrichment only, not as part of the core curriculum.

Timing

225 minutes
(three 75-minute classes)

Learning Skills Focus

- Independent Work (students will work on exercises independently)
- Collaboration (students will work with others to compare answers and arrive at a consensus)

Teaching Plan 2 (SE pp. 66-85)

Activity Description

Students will complete an exercise in which they will discover new rules of inference and replacement. They will then be given a summary of established rules of inference and replacement from Irving Copi's *Introduction to Logic*, checking that these rules make sense. After practising translating statements into symbolic form, students will then be able to prove that various arguments are valid, using Copi's list.

Assessment Opportunities for Chapter Questions

The table below summarizes assessment opportunities for selected chapter questions, which are relevant to this teaching plan.

Assessment Type	Assessment Tool	Feature Questions	Section Questions
Assessment for Learning	BLM 3.2		
Assessment for Learning	BLM 3.4		
Assessment as Learning	BLM 3.5		

Resources Needed

Make copies of these Blackline Masters:

- BLM 3.2 Formal Logic: Valid vs. Invalid Arguments
- BLM 3.3 Rules of Inference and Replacement
- BLM 3.4 Translating Statements into Symbols
- BLM 3.5 Applying Formal Logic

Possible Assessment of Learning Task

See Teaching Plan 1.

Assessment (For/As Learning)

As teachers move through each chapter, opportunities will be highlighted to provide assessment for/as learning in preparation for assessment of learning at the end of each chapter.

Task/ Project	Achievement Chart Category	Type of Assessment	Assessment Tool	Peer/Self/ Teacher Assessment	Learning Skill	Student Textbook Page(s)	Blackline Master
Valid vs. invalid arguments exercise	Knowledge; Thinking;	For	Answer key	Self; peer	Collaboration		BLM 3.2
Converting statements into sym-bolic form exercise	Knowledge; Thinking; Communication; Application	For	Answer key	Self; peer	Independent work; col-laboration		BLM 3.4
Applying formal logic exercise	Knowledge; Thinking; Communication; Application	For	Answer key	Self; peer	Independent work; col-laboration		BLM 3.5

Prior Learning Needed

In preparation for this activity, students will need to be familiar with the concept of validity (SE p. 30), understand the meanings of logical symbols ($\sim$, $\vee$, $\wedge$, $\supset$) (SE p. 76), and have some familiarity with expressing arguments in symbolic form and recognizing them as valid or invalid (SE pp. 80-81). Students should have read and completed the questions in Chapter 3 (SE pp. 66-83). See Teaching Plan 1.

Teaching/Learning Strategies

1. This activity begins with developing the rules of inference and replacement, rules that will be used to determine if an argument is valid or not. Students can better appreciate formal logic rules of inference and replacement after they have had a chance to discover these rules for themselves. To that end, hand out BLM 3.2 and have students work in pairs or groups of four to determine which of the arguments are valid and which are invalid. Students should be encouraged to do this by reflecting on the meanings of the different logical connectives ("not," "and," "or," "if…then…") and testing some concrete scenarios. After students have completed this exercise, take up their answers using the answer key provided.

   Point out that a *rule of inference* allows one to make a conclusion from known premises (e.g., from the two premises $p \supset q$ and $p \supset q$ we can make the conclusion $p \supset q$ according to the rule of inference called *hypothetical syllogism*. A *rule of replacement* allows one to replace one statement with another (e.g., the rule of replacement called *material implication*, which states $p \supset q \Leftrightarrow \sim p \vee q$, allows one to replace $p \supset q$ with $\sim p \vee q$ and vice versa; the symbol $\Leftrightarrow$ indicates that the replacement works both ways.)

   Hand out BLM 3.3, a summary of rules of inference and rules of replacement. Students will have discovered some of these rules while completing BLM 3.2. Here are two discussion points regarding some of the rules:

   - Material Implication (MI): $p \supset q \Leftrightarrow \sim p \vee q$. Students may ask why this rule is valid; here is an explanation: First, we know that $p \vee \sim p$ is always true by the law of the excluded middle. If p is true, then because of $p \supset q$, q is true as well (*modus ponen*). So we can rewrite $p \vee \sim p$ as $q \vee \sim p$ and rearrange it as $\sim p \vee q$.
   - De Morgan's Rules (De M): $\sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$ and $\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$. To help students understand these rules, substitute actual propositions for p and q (e.g., p stands for "Patrick is going to the party" and q stands for "Qiana is going to the party"). The rules act like mathematical operators whereby you "multiply" by $\sim$ and change the $\wedge$ symbol to $\vee$.

2. Now that these rules have been established, students can use them to determine if an argument is valid. But before doing this, students will need practise with converting statements into symbolic form (BLM 3.4).

   Before assigning BLM 3.4, you may decide to explain how to symbolize "only if." Or, assign BLM 3.4 and explain how to symbolize "only if" while taking up question 9. To explain, ask students to consider the following statement: "The streets are wet (S) *if* it is raining (R)." This statement is symbolized as $R \supset S$, for clearly the statement is claiming that if it is raining, then the streets are wet. But suppose the statement now reads: "The streets are wet (S) *only if* it is raining (R)." This statement tells us that the only circumstance in which the streets are wet is that it is raining. In other words, if the streets are wet, then it must be raining! *It does not tell us that if it is raining then the streets are wet.* Therefore, the statement is symbolized as $S \supset R$.

Students may complete BLM 3.4 in pairs, possibly placing answers on the board. Take up their answers using the answer key provided.

3. After checking and discussing the answers to BLM 3.4, and before assigning BLM 3.5, it may help to walk students through the following examples:

Example 1: Using the rules of inference and/or replacement, prove that the following argument is valid: "Either Patrick is going to Toronto or David is going to Toronto. If Andrew goes to Toronto, then David is not going. But Patrick is not going to Toronto. Therefore, Andrew is not going."

The argument can be symbolized as follows. Note how each rule of inference or replacement must be referenced (e.g., *DS* means *disjunctive syllogism*). Also, the proof uses only premises or statements proved using rules of inference and/or replacement. The abbreviation QED stands for the Latin phrase *quod erat demonstrandum,* meaning *which was to be demonstrated*. This phrase is commonly used to signify the end of a proof.

$$\begin{array}{r} p \vee d \\ a \supset \sim d \\ \sim p \\ \hline \sim a \end{array}$$

Proof:

$p \vee d$	(Premise)
$\sim p$	(Premise)
d	(DS)

d	(From the argument above)
$\sim\sim d$	(DN)

$a \supset \sim d$	(Premise)
$\sim\sim d$	(From the argument above)
$\sim a$	(MT)

QED

It may be tempting to symbolize "Either Patrick is going to Toronto or David is going to Toronto" as $(p \vee d) \wedge \sim (p \wedge d)$. This captures the meaning of *either/or*. However, using the rule of inference *simplification*, we can extract the statement $p \vee d$ and use it alone. So whenever an *either/or* statement appears in an argument, one only need translate it as a single disjunct of the form $p \vee q$. This will make translating arguments into symbolic form easier.

Example 2: Using the rules of inference and/or replacement, prove that the following argument is valid: "If the laws are good and their enforcement is strict, then crime will diminish. If strict enforcement of laws will make crime diminish, then our problem is a practical one. The laws are good. Therefore, our problem is a practical one."

The argument can be symbolized as follows:

$$\begin{array}{r}(l \wedge e) \supset c \\ (e \supset c) \supset p \\ l \\ \hline p\end{array}$$

Proof:

$$\begin{array}{rl}(l \wedge e) \supset c & \text{(Premise)} \\ \hline l \supset (e \supset c) & \text{(Exp.)}\end{array}$$

$$\begin{array}{rl}l \supset (e \supset c) & \text{(From the argument above)} \\ l & \text{(Premise)} \\ \hline e \supset c & \end{array}$$

$$\begin{array}{rl}(e \supset c) \supset p & \text{(Premise)} \\ e \supset c & \text{(From the argument above)} \\ \hline p & \text{(MP)}\end{array}$$

QED

Now assign BLM 3.5. Students may complete it individually or in pairs. Again, students could be asked to place their answers on the board for discussion. Take up their answers using the answer key provided.