Chapter 1 Quadratic Functions

Prerequisite Skills

1. a) 7   **b)** -2 **c)** 4   **d)** -15

2. a) -13 **b)** 11   **c)** $-\dfrac{10}{7}$ **d)** 2

3. a)

x	y
-2	-15
-1	-10
0	-5
1	0
2	5

b)
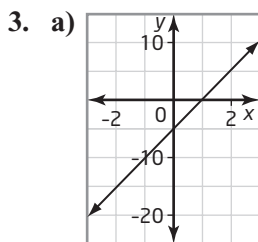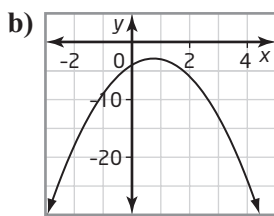
x	y
-2	-18
-1	-9
0	-4
1	-3
2	-6

4. a)

x	y	First Differences	Second Differences
-3	-11		
		3	
-2	-8		0
		3	
-1	-5		0
		3	
0	-2		0
		3	
1	1		0
		3	
2	4		0
		3	
3	7		

b)

x	y	First Differences	Second Differences
-3	9		
		-10	
-2	-1		4
		-6	
-1	-7		4
		-2	
0	-9		4
		2	
1	-7		4
		6	
2	-1		4
		10	
3	0		

5.

1.1 Identify Functions

Prerequisite Skills

1. a) not a function; e.g., the x-value 1 is mapped onto two y-values, 3 and -4

   b) function; e.g., the graph passes the vertical line test

   c) function; e.g., each x-value is mapped onto only one y-value

   **d)** not a function; e.g., for $x = 8$, there are two y-values, 2 and -2

2. a), b), d); explanations may vary

A

1. a) function

   b) not a function; e.g., for $x = 9$, there are two y-values, 3 and -3

   c) not a function; e.g., for some x-values, a vertical line intersects the graph at more than one point

2. a) function; e.g., each x-value is mapped onto only one y-value

   b) function; e.g., each x-value is mapped onto only one y-value

   c) not a function; e.g., some x-values, such as -2, are mapped onto more than one y-value, 4 and -4 in this case
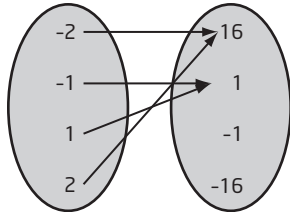
B

3. a) 1 **b)** 3 **c)** 36

4. a)

Rock's Height vs. Time
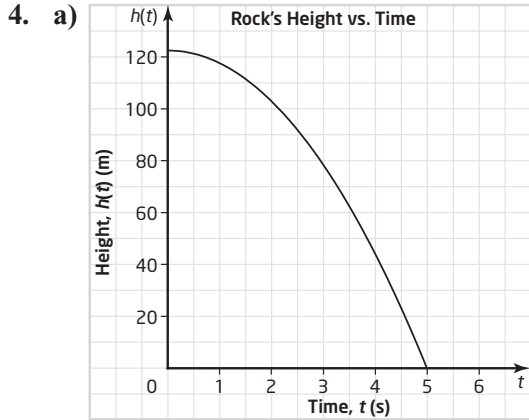
b) 122.5 m

c) 0 m; the rock hits the ground after 5 s

C

5. a) $\frac{1}{2}$ **b)** 8 **c)** 2

6. $h(x) = -7x + 5$

7. a)

$5x^2 - y = 4$

b)

$5x^2 - y^2 = 4$
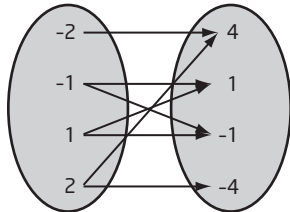
1.2 Domain and Range

Prerequisite Skills

1. a) Domain: $\{-1, 0, 1, 2\}$

Range: $\{2, 0, -2, -4\}$

b) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

c) Domain: $\{-2, 0, 2, 4\}$

Range: $\{-4, -3, -2, -1\}$

d) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

2. a) Domain: $\{x \in \mathbb{R}\}$

b) Range: $\{y \in \mathbb{R} \mid 0 \le y \le 20\}$

c) The domain is restricted by the speed the apple falls. The range is restricted by the initial height the apple falls.

3. a) Sketches may vary.

b) Domain: $\{t \in \mathbb{R} \mid 0 \le t \le 25\}$

Range: $\{d \in \mathbb{R} \mid 0 \le d \le 100\}$

A

1. a) Domain: $\{-4, -3, -2, -1, 0\}$

Range: $\{0, 1, 2\}$

b) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \ge 2\}$

c) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \le 0\}$

d) Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{f(x) \mid f(x) \in \mathbb{R}\}$

2. In words: The domain is the set of real numbers greater than or equal to 0. The range is the set of real numbers greater than or equal to 0.
As intervals:
Domain: $\{x \in \mathbb{R} \mid x \ge 0\}$
Range: $\{y \in \mathbb{R} \mid y \ge 0\}$

3. a) 0; 1   **b)** 0.5

c) The graph represents a function because each *x*-value is mapped onto only one *y*-value.

d) Domain: $\{x \mid x \in \mathbb{R}\}$
Range: {0, 0.5, 1}

B

4. a) Answers may vary. For example:

Length of Pen, *l* (m)	Width of Pen, *w* (m)
1	60
2	30
3	20
4	15
5	12
6	10
10	6
12	5
15	4
20	3
30	2
60	1

b) Diagrams may vary.

c) a curve

d) In words: The domain is the set of real numbers between 1 and 60. The range is the set of real numbers between 1 and 60.
As intervals:
Domain: $\{l \in \mathbb{R} \mid 1 \le l \le 60\}$
Range: $\{w \in \mathbb{R} \mid 1 \le w \le 60\}$

e) The domain is the set of real numbers between 0.5 and 60. The range is the set of real numbers between 0.5 and 60.
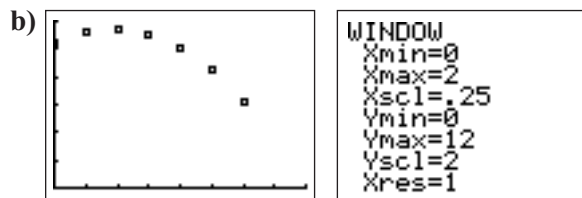
5. a)

Time, *t* (s)	Height, *h* (m)
0	34.3
1	58.8
2	73.5
3	78.4
4	73.5
5	58.8
6	34.3
7	0.0

b) 78.4 m   **c)** 7 s

d) Domain: $\{t \in \mathbb{R} \mid 0 \le t \le 7\}$
Range: $\{h \in \mathbb{R} \mid 0 \le h \le 78.4\}$

6. a) $3x^2 - 7$

b) Yes, it is a function. Each *x*-value maps onto only one y-value in the graph of the equation.

c) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid -7 \le y\}$

1.3 Analyse Quadratic Functions

Prerequisite Skills

1. a) 11.48 m

b)


```
WINDOW
 Xmin=0
 Xmax=2
 Xscl=.25
 Ymin=0
 Ymax=12
 Yscl=2
 Xres=1
```

c) Estimates may vary. e.g., 0 m

d) $h = -4.9t^2 + 4.6t + 10.4$

e) 0 m

f) Answers may vary. e.g., yes

2. a)

First Differences	Second Differences
12	
	0
12	
	0
12	
	0
12	
	0
12	
	0
12	

b)

First Differences	Second Differences
−10	
	4
−6	
	4
−2	
	4
2	
	4
6	
	4
10	

A

1. a) linear   **b)** quadratic

   c) linear   **d)** quadratic

   e) quadratic   **f)** linear

   g) linear   **h)** linear

2. a) linear   **b)** quadratic   **c)** neither

B

3. a)

Time, t (s)	Height, h (m)
0	0.6
1	34.9
2	59.4
3	74.1
4	79.0
5	74.1
6	59.4
7	34.9
8	0.6

   b) yes, $h(t)$ is quadratic

   c) Diagrams may vary.

d) Axis of symmetry: $t = 4$
Direction of opening: Downward
Vertex: (4, 79)
Domain: $\{t \in \mathbb{R} \mid 0 \le t \le 8\}$
Range: $\{h(t) \in \mathbb{R} \mid 0.6 \le h(t) \le 79\}$

4. a) The values of x have to be real numbers between 0 and 40. For x equals 40 or more, the width of the box will be a negative value, which is not possible.

   b) $6400 - 4x^2$ **c)** 15 cm

5. a) Yes   **b)** Yes   **c)** $600

C

6. 170

7. $24(s - 10)^2$

1.4 Stretches of Functions

Prerequisite Skills

1. a) The graph of $y = \frac{1}{4}x^2$ is a vertical compression of the graph of $y = x^2$ by a factor of $\frac{1}{4}$. Every point on the graph of $y = \frac{1}{4}x^2$ is 4 times closer to the x-axis as the corresponding point on the graph of $y = x^2$. Sketches may vary.

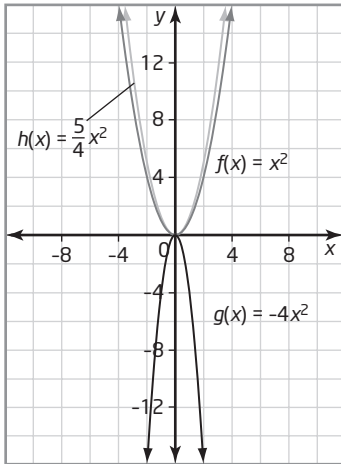   **b)** The graph of $y = -x^2$ is a reflection in the x-axis. Sketches may vary.

2. $a = \frac{2}{3}$

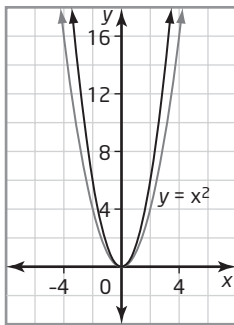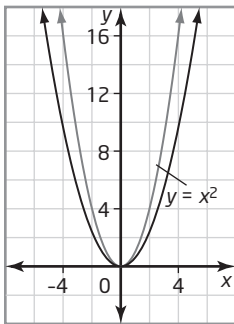**3.** Answers may vary. e.g., $(-1, -2)$, $(2, -4)$

A

1. a) B   **b)** C   **c)** A

2.

3. a)

b)

c)
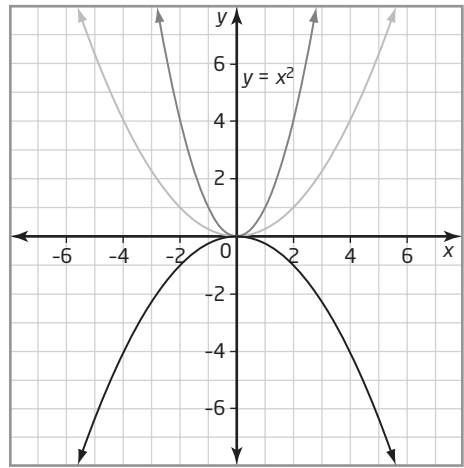
d)
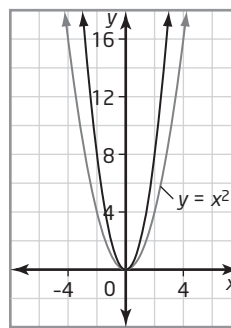
4. a) $f(x) = 0.9x^2$

     b) $p(x) = 20x^2$

     c) $q(x) = 7x^2$

5. $a = 0.5$

B

6. a) 0.71 m

     b) approximately 12 s

     c) Answers may vary. e.g., the graph of $y = 0.71x^2$ represents a vertical compression of the graph $y = x^2$ by a factor of 0.71.

7. e.g., $y = -\dfrac{2}{3}x^2$

C

8. b and c

9. a) Diagrams may vary.

   b) Answers may vary. For example, the graph of $y = \frac{1}{3}x^2$ is wider than the graph of $y = x^2$. As the values of x increases, the value of y decreases by $\frac{1}{3}$ for corresponding y values of $y = x^2$.

   c) Answers may vary. For example, the graph of $y = 3x^2$ is narrower than the graph of $y = x^2$. As the values of x increases, the value of y increases by 3 for corresponding y values of $y = x^2$.

1.5 Translations of Functions

Prerequisite Skills

1. a) $f(x) = x^2 - 2$

   i) This represents a translation of 2 units down.

   ii) The vertex is at $(0, -2)$.

   iii) Sketches may vary.

  b) $g(x) = (x - 4)^2$

   i) This represents a translation of 4 units to the right.

   ii) The vertex is at $(4, 0)$.

   iii) Sketches may vary.

  c) $h(x) = (x + 3)^2 + 1$

   i) This represents a translation of 3 units to the left and 1 unit up.

   ii) The vertex is at $(-3, 1)$.

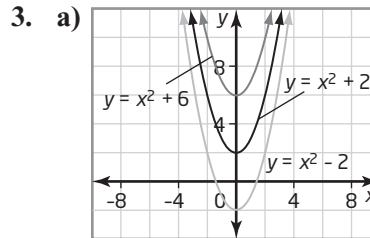   **iii)** Sketches may vary.

  d) $y = (x - 1)^2 + 5$

   i) This represents a translation of 1 unit to the right and 5 units up.

   ii) The vertex is at $(1, 5)$.

   iii) Sketches may vary.

2. Answers and diagrams may vary. e.g., The vertex must lie on the line of symmetry, so $h = 3$. Substitute this and the coordinates of one point into the function $f(x) = (x - h)^2 + k$ to solve for k. The coordinates of the vertex are (h, k), or $(3, -2)$.

3. a)

  b) $y = (x + 2)^2 + 6$, $y = (x + 2)^2 + 2$, $y = (x + 2)^2 - 2$

  c) $y = x^2 + 4$, $y = x^2$, $y = x^2 - 4$

A

1. a) C   **b)** A   **c)** D   **d)** B

2. a) $f(x) = (x + 10)^2$

  b) $g(x) = x^2 + 10$

  c) $s(x) = (x - 1)^2$

  d) $v(x) = x^2 - 1$

3. a) $(0, -6)$ **b)** $(5, 0)$ **c)** $(-3, -9)$

  d) $(1, 3)$; Sketches may vary.

4. a) $(0, 10)$ **b)** $(-8, 0)$

  c) $(12, -2)$ **d)** $(-6, 6)$

B

5. a) 1   **b)** $q(x) = (x - 1)^2 - 9$

6. a) -1 **b)** $q(x) = (x + 1)^2 - 9$

7. Answers may vary. E.g., the shapes of the two graphs are the same. The graph for the friend who jumps second is a shift of the graph for the friend who jumps first by 3 min to the right on the Time axis.

8. a) $f(x) = (x - 3)^2 - 9$

b) Sketches may vary.

c) $(3, -9)$ d) 0 and 6

9. a) $g(x) = \sqrt{x - 4} - 3$

b) Sketches may vary.

c) Domain: $\{x \in \mathbb{R} \mid x \geq 4\}$
Range: $\{g(x) \in \mathbb{R} \mid g(x) \geq -3\}$

C

10. $(x + 4)^2 + y^2 = 4$

11. a) $(-2, 2)$; graph the function

b) $f(x) = 2(x - 5)^2$

1.6 Sketch Graphs Using Transformations

Prerequisite Skills

1. a) Answers may vary. E.g., compared to the graph of $f(x) = x^2$, the graph of $f(x) = 2(x + 3)^2 - 4$ represents a vertical stretch by a factor of 2 and a translation of 3 units to the left and 4 units down.

b) $(-3, -4)$

c) $x = -3$

d) Sketches may vary.

e) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \geq -4\}$

2. The graph of $y = -2(x + 5)^2 + 2$ is a vertical stretch of the graph of $y = x^2$ by a factor of 2 followed by a reflection in the x-axis and a translation of 5 units to the left and 2 units up.

3. a)

$g(x) = (x - 3)^2 - 4$

$f(x) = -(x - 3)^2 + 4$

$h(x) = 2(x - 3)^2 - 8$
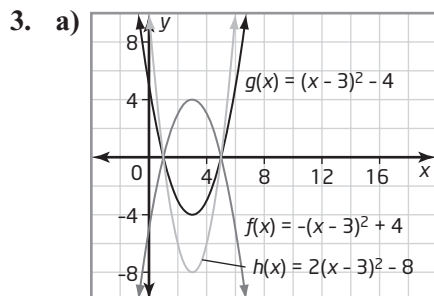
b) Answers may vary. e.g., The three graphs have the same x-intercepts and axis of symmetry. The graph of $f(x)$ is a reflection of the graph of $g(x)$ through the x-axis.

A

1. a) Vertex: $(0, 10)$
Axis of symmetry: $x = 0$
Direction of opening: downward
Range: $\{f(x) \in \mathbb{R} \mid f(x) \leq 10\}$

b) Vertex: $(-6, 0)$
Axis of symmetry: $x = -6$
Direction of opening: downward
Range: $\{g(x) \in \mathbb{R} \mid g(x) \leq 0\}$

c) Vertex: $(2, 12)$
Axis of symmetry: $x = 2$
Direction of opening: upward
Range: $\{h(x) \in \mathbb{R} \mid h(x) \geq 12\}$

d) Vertex: $(-4, -9)$
Axis of symmetry: $x = -4$
Direction of opening: upward
Range: $\{y \in \mathbb{R} \mid y \geq -9\}$

2. a) The graph of $f(x) = 4(x - 6)^2$ is a vertical stretch of the graph of $y = x^2$ by a factor of 4 followed by a translation of 6 units to the right. Sketches may vary.

b) The graph of $g(x) = -2x^2 - 2$ is a vertical stretch of the graph of $y = x^2$ by a factor of 2 followed by a reflection in the x-axis and a translation of 2 units down. Sketches may vary.

c) The graph of $h(x) = 3(x - 3)^2 + 7$ is a vertical stretch of the graph of $y = x^2$ by a factor of 3 followed by a translation of 3 units to the right and 7 units up. Sketches may vary.

d) The graph of $y = -5(x + 1)^2 + 5$ is a vertical stretch of the graph of $y = x^2$ by a factor of 5 followed by a reflection in the x-axis and a translation of 1 unit to the left and 5 units up. Sketches may vary.

3. a) $f(x) = 2(x - 1)^2 + 5$

b)
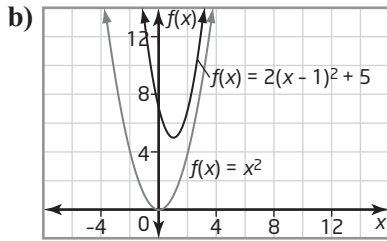
4. a) $y = 5(x - 6)^2 - 3$

b) $y = 0.5(x + 4)^2 - 1$

c) $y = -3(x + 2)^2 + 12$

B

5. a) Answers may vary. e.g., (2, 2) and (4, 8)

b) Answers may vary. e.g., Translate each point 8 units to the left and 12 units up. These points lie on the graph of $g(x) = 0.5(x + 8)^2 + 12$.

c) Sketches may vary.

Domain: $\{x \mid x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \geq 12\}$

6. a) $(-4, 7)$ **b)** $y = -2(x + 4)^2 + 7$

7. a) approximately 0.5 m

b) 31.1 m, when $t = 2.5$

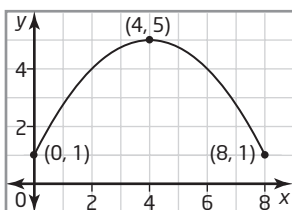**c)** $\{h(t) \in \mathbb{R} \mid 0 \leq h(t) \leq 31.1\}$

d) approximately 5 s

e) $\{t \in \mathbb{R} \mid 0 \leq t \leq 5\}$

8. a) $y = -(x + 2)^2 + 1$

b) $y = 0.5(x - 3)^2 - 0.5$

C

9. a) Sketches may vary. e.g.,

b) Yes. Explanations may vary, e.g., a function that represents the path of the shuttle is $h(x) = -0.25(x - 4)^2 + 5$, where x is the horizontal distance the shuttle is from the badminton player, in metres, and $h(x)$ is the height of the shuttle, in metres. Since $h(0.5) = 1.94$, the shuttle will be at a height of 1.94 m when it is directly above the net.

10. a) Answers may vary. For example, the axis of symmetry of the parabola is between $x = 7$ and $x = 3$, which is $x = 5$. This means the x-coordinate of the vertex is 5.

b) $f(x) = 4(x - 5)^2 - 12$

Chapter 1 Review

1. a) function   **b)** not a function

c) function   **d)** function

2. a) 0   **b)** -12

c) $-4\frac{13}{16}$ **d)** $-8\frac{8}{9}$

3. a) Answers may vary. e.g., $h(5)$ represents the height of the flare after 5 s, in metres.

b) approximately 437 m

4. The domain is the set of real numbers and the range is the set of real numbers less than or equal to -1.

5. Domain: $\{x \mid x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \geq -4\}$

6. Domain : $\{n \in \mathbb{I} \mid 0 < n \leq 24\}$
Range: $\{258, 266, 274, \ldots , 426, 434, 442\}$

7. a) not quadratic   **b)** quadratic

c) not quadratic

8. a) neither   **b)** linear

c) neither   **d)** quadratic

9. a)

Base Length of Figure (unit)	Area of Figure (units^2)
1	3
2	8
3	11
4	14
5	17

    b) neither

10. a) The graph of $y = 3x^2 - 2$ represents a vertical stretch of the graph of $y = x^2$ by a factor of 3 followed by a translation of 2 units down. Sketches may vary.

    b) The graph of $m(x) = -(x + 4)^2$ represents a reflection of the graph of $y = x^2$ in the x-axis followed by a translation of 4 units to the left. Sketches may vary.

    c) The graph of $r(t) = 2(t - 1)^2 - 3$ represents a vertical stretch of the graph of $y = x^2$ by a factor of 2 followed by a translation of 1 unit to the right and 3 units down. Sketches may vary.

11. a) $y = -2(x + 3)^2 - 2$

    b) $y = 0.8(x + 2)^2$

Chapter 1 Practice Exam

1. C   **2.** B   **3.** C   **4.** D   **5.** D

6. a) function   **b)** not a function

    c) not a function   **d)** function

7. a) Domain: $\{x \mid x \in \mathbb{R}\}$
      Range: $\{y \mid y \in \mathbb{R}\}$

    b) Domain: $\{x \mid x \in \mathbb{R}\}$
      Range: $\{y \in \mathbb{R} \mid y \le 2\}$

8. a) $S(3)$ represents the surface area of a cone with a radius of 3 cm and a slant height of 4.5 cm, in square centimetres.

    b) $S(3) = 22.5\pi$

9. a) $h(x) = (x - 3)^2$ **b)** $f(r) = r^2 - 2$

    c) $m(n) = -3(n + 5)^2$ **d)** $p(x) = ax^2 + 7$

10. a) $(0, -1)$ **b)** $(-1, 1)$ **c)** $(2, 9)$

11. a) $y = -x^2 + 3$

   **b)** $y = (x - 2)^2$

    c) $y = -x^2 + 4$

12. a) The graph of $y = -x^2 - 2$ represents a reflection of the graph of $y = x^2$ in the x-axis followed by a translation of 2 units down.

    b) The graph of $y = (x + 1)^2 - 2$ represents a translation of 1 unit to the left and 2 units down of the graph of $y = x^2$.

    c) The graph of $y = 3(x - 1)^2 + 1$ represents a vertical stretch of the graph of $y = x^2$ by a factor of 3 followed by a translation of 1 unit to the right and 1 unit up.

    d) The graph of $y = -0.5(x + 2)^2 - 3$ represents a vertical stretch of the graph of $y = x^2$ by a factor of 0.5 followed by a reflection in the x-axis and a translation of 2 units to the left and 3 units down.

13. a)
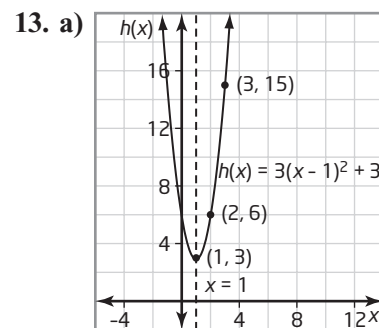
    b) Domain: $\{x \mid x \in \mathbb{R}\}$
      Range: $\{y \in \mathbb{R} \mid y \ge 3\}$

14. a) 1242   **b)** Answers may vary.

15. a) B   **b)** A   **c)** C

Chapter 2 Quadratic Functions

Prerequisite Skills

1. a)

x	$f(x)$ $= 3x + 9$	First Differences	Second Differences
-3	0		
		3	
-2	3		0
		3	
-1	6		0
		3	
0	9		0
		3	
1	12		0
		3	
2	15		0
		3	
3	18		

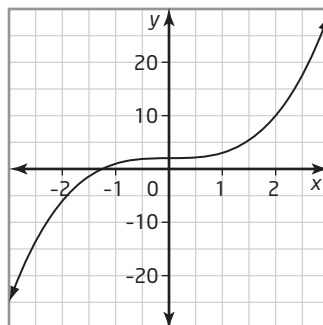The first differences are constant, so the function is linear.
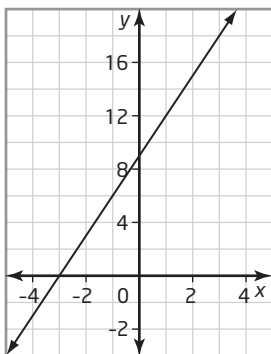
b)

x	$f(x)$ $= x^3 + 2$	First Differences	Second Differences
-3	-25		
		19	
-2	-6		-12
		7	
-1	1		-6
		1	
0	2		2
		3	
1	5		2
		5	
2	10		14
		19	
3	29		

The function is neither linear nor quadratic.

c)

x	$f(x)$ $= x^2 - x - 4$	First Differences	Second Differences
-3	8		
		-6	
-2	2		2
		-4	
-1	-2		2
		-2	
0	-4		2
		0	
1	-4		2
		2	
2	-2		2
		4	
3	2		

The function is quadratic.
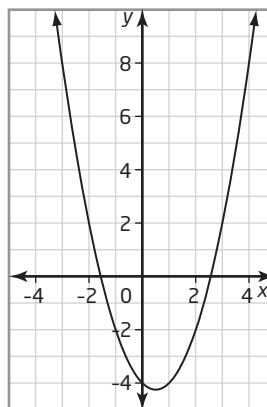
2. direction of opening: upward; vertex: $(-1, -4)$; equation of axis of symmetry: $x = -1$; x-intercepts: -3 and 1; y-intercept: -3

3. a)

b)
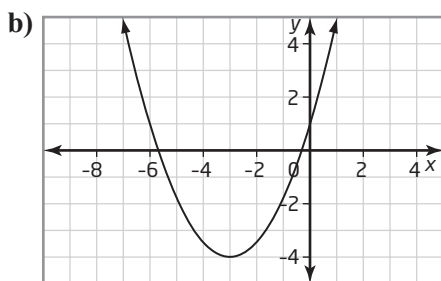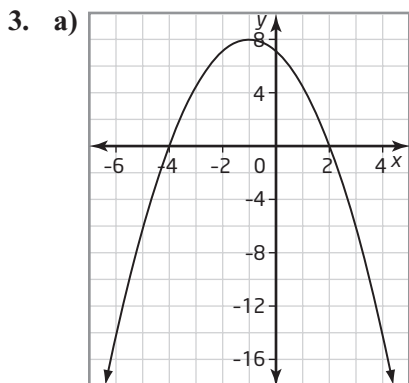
4. a) to d) $3(x - 3) = 3x - 9$

5. a) $2x + 10$ **b)** $-4a - 4$

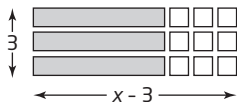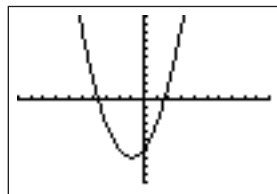 **c)** $\frac{1}{2}y^2 + 3y$ **d)** $-6k^2 - 12k + 12$

  e) $-\frac{15}{4}b^2 + 9b - \frac{9}{2}$ **f)** $h^2 - 5h + 6$

  g) $n^2 + 2n - 15$ **h)** $6s^2 - 8s$

6. a) $3(4y + 1)$ **b)** $2x(3x - 1)$

  c) $5(n^2 + n + 4)$ **d)** $-4(m - 6)$

  e) $(q - 6)(q + 1)$ **f)** $(a + 7)(a - 3)$

  g) $(a - 5)(a - 2)$ **h)** $(g - 7)(g + 6)$

Section 2.1

Prerequisite Skills

1. a) direction of opening: upward
   vertex: $(-1, -7)$
   axis of symmetry: $x = -1$
   x-intercepts: approximately 1.6
            and -3.6
   y-intercept: -6

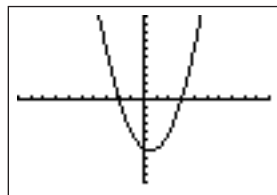**b)** direction of opening: downward
   vertex: $(-3, 18)$
   axis of symmetry: $x = -3$
   x-intercepts: approximately 1.2 and
            -7.2
   y-intercept: 9
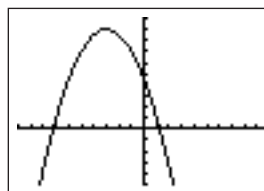
c) direction of opening: upward
   vertex: $(0.5, -6.25)$
   axis of symmetry: $x = 0.5$
   x-intercepts: -2 and 3
   y-intercept: -6

2. a), b) Sketches may vary.

A

1. a) $a = -1$, $b = 9$, $c = 20$; downward; maximum

b) $a = 3$, $b = -9$, $c = 6$; upward; minimum

2. a) direction of opening: upward
x-intercepts: 2 and 8;
vertex: $(5, -9)$;
minimum axis of symmetry: $x = 5$;
y-intercept: 16

b) direction of opening: downward
x-intercepts: 1 and 3;
vertex: $(2, 5)$;
maximum axis of symmetry: $x = 2$;
y-intercept: -16

c) direction of opening: upward
x-intercepts: none;
vertex: $(2, 3)$;
minimum axis of symmetry: $x = 2$;
y-intercept: 7

d) direction of opening: downward
x-intercepts: none;
vertex: $(4, -6)$;
maximum axis of symmetry: $x = 4$;
y-intercept: -38

3. $f(x) = -x^2 + 9x + 20$
Graphs may vary.
Positive approximately when
$-1.8 < x < 10.8$; negative approximately
when $x < -1.8$ and $x > 10.8$; increasing
when $x < 4.5$; decreasing when $x > 4.5$.

$y = 3x^2 - 9x + 6$
Graphs may vary.
Positive when $x < 1$ and $x > 2$; negative
when $1 < x < 2$; increasing when
$x > 1.5$; decreasing when $x < 1.5$.

$f(x) = (x - 2)(x - 8)$
Graphs may vary.
Positive when $x < 2$ and $x > 8$; negative
when $2 < x < 8$; increasing when $x > 5$;
decreasing when $x < 5$.

$y = -5(x - 1)(x - 3)$
Graphs may vary.
Positive when $1 < x < 3$; negative when
$x < 1$ and $x > 3$; increasing when $x < 2$;
decreasing when $x > 2$.

$f(x) = (x - 2)^2 + 3$
Graphs may vary.
Positive for all $x \in \mathbb{R}$; negative–never;
increasing when $x > 2$; decreasing
when $x < 2$.

$y = -2(x - 4)^2 - 6$
Graphs may vary.
Positive–never; negative for all
$x \in \mathbb{R}$; increasing when $x < 4$;
decreasing when $x > 4$.

B

4. a) approximately 1.25 m

b) approximately 4.25 m

c) approximately 2.25 s

5. a) A second x-intercept is approximately
-0.3 s. This value has no meaning since
time cannot be negative in this context.

b) vertex: $(1, 4.25)$
$y = -3(x - 1)^2 + 4.25$

c) y-intercept will increase;
the x-coordinate of the vertex stays the
same but the y-coordinate increases;
the maximum height achieved
increases; x-intercept increases,
hang time increases

6. a) Stage 4   Stage 5
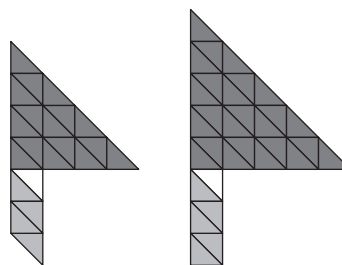
b) The number of white triangles remains
constant from one stage to the next.

c) The number of light grey triangles increases by one for each stage. The number of dark grey triangles changes by the square of the stage number.

d)

Stage	1	2	3	4	5
White Triangles	1	1	1	1	1
Light Grey Triangles	1	2	3	4	5
Dark Grey Triangles	1	4	8	16	25
Total Triangles	3	7	12	21	31

e) The relation between the number of white triangles and stage number is constant. The relation between the number of light grey triangles and stage number is linear. The relation between the number of dark grey triangles and stage number is quadratic.

7. a), b) Answers may vary.

C

8. a) $f(x) = 2(x + 3)^2 - 2$

b) Yes, there is only one function.

9. a) When expanded, the degree of the polynomial is three. It is a cubic function with two x-intercepts, r and s.
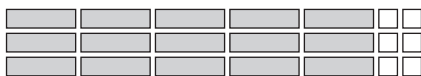
b) The answer for part a) is verified graphically.
The function is increasing when $x < -2$ and $x > 3.3$. The function is decreasing when $-2 < x < 3.3$.

Section 2.2

Prerequisite Skills

1. $3(5x - 2) = 15x - 6$

2. a) i)

b) ii)

c)
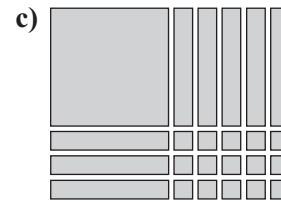
3. a) $x^2 + 9x + 20$ **b)** $y^2 - 8y + 7$
c) $4h^2 + 12h - 6$ **d)** $-n^2 + n + 6$

A

1. a) $3x + 15$ **b)** $6y + 24$
c) $2a^2 - 2a$ **d)** $b^2 + 11b + 28$
e) $4n^2 + 7n + 2$ **f)** $q^2 + 8q + 16$

2. a) $2x - 20$ **b)** $-3b - 21$
c) $32w - 4w^2$ **d)** $y^2 - 14y + 45$
e) $3n^2 + 3n - 6$ **f)** $a^2 - 8a + 16$
g) $9x^2 + 48x + 64$ **h)** $4q^2 - 9$

3. a) $y = x^2 - 5x - 6;\ -6$
b) $y = x^2 - 10x + 21;\ 21$
c) $y = 5x^2 - 20x - 25;\ -25$
d) $y = x^2 + 6x + 8;\ 8$
e) $y = x^2 - 8x + 10;\ 10$
f) $y = -3x^2 + 12x - 7;\ -7$

4. a) $3x^2 + 6x$
b) $x^2 + 9x + 20$
c) $4x^2 + 15x + 20$

5. a) $7x + 19$ **b)** $10y^2 + 16y$
c) $6k^2 + 34k + 36$ **d)** $5t + 14$

6. a) $8x - 42$ **b)** $8 - 10a$
c) $13y + 16$ **d)** $7n - 23$
e) $26b + 66$ **f)** $-2s^2 + 58$

B

7. a) *x*-intercept form; where $a = 14.7$, $r = 0$, and $s = 6$

  b) *t*-intercepts are 0 and 6; These represent the times when the rocket is on ground in seconds.

  c) $h = -14.7t^2 + 88.2t$

  d) vertex: (3, 132.3); maximum;

    It represents the maximum height the rocket will reach.

  e)
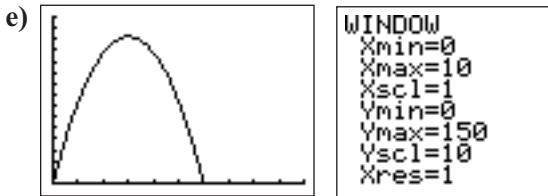
**8. a)** $40 + x$ represents the price of one sweatshirt, which is $40 plus the number of $1 increases or *x*. $150 - 2x$ represents the number of sweatshirts sold at a maximum of 150 less double the number of the $1 increases.

  b) The *y*-intercept is $6000. This represents the revenue when 150 sweatshirts are sold at a price of $40 each.

  c) $y = -2x^2 + 70x + 6000$; Use this equation to substitute values for *x*. R(*x*) and *y* should be the same for a given *x*.

  d) $47.50; $6612.50

9. a) $R(x) = (40 + x)(150 - 5x)$

  b) $35; $6125.00

  c) The company should increase their price, then the revenue will be greater.

C

10. a) 225 cm²

  b) base of 15 cm, height of 15 cm

Section 2.3

Prerequisite Skills

1. a), b), c)
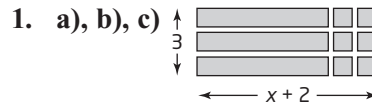
$3x + 6 = 3(x + 2)$

  d) The dimensions represent the factors of the expression $3x + 6$.

2. a) $2x + 8 = 2(x + 4)$

  b) $4y + 16 = 4(y + 14)$

3. a) $3(r + 4)$ **b)** $x(5x + 2)$

  c) $7q(q - 2)$ **d)** $2(n^2 + 3n - 16)$

  e) $13(x - 3)$ **f)** $(m - 3)(m - 4)$

4. a) $(y - 9)(y - 2)$ **b)** $(b + 7)(b - 3)$

  c) $(x - 6)(x - 4)$ **d)** $(q - 9)(q + 2)$

A

1., 2. a) $x^2 + 10x + 16 = (x + 8)(x + 2)$

  b) $y^2 + 10y + 9 = (y + 9)(y + 1)$

  c) $k^2 + 8k + 15 = (k + 3)(k + 5)$

  d) $b^2 - 3b - 54 = (b + 9)(b - 6)$

  e) $n^2 + 5n - 24 = (n + 8)(n - 3)$

  f) $a^2 - 9a + 36 = (a - 12)(a + 3)$

  g) $t^2 + t - 56 = (t + 8)(t - 7)$

  h) $m^2 - 13m + 42 = (m - 7)(m - 6)$

3., 4. a) $2x^2 + 9x + 9 = (2x + 3)(x + 3)$

  b) $4k^2 + 16k + 7 = (2k + 7)(2k + 1)$

  c) $6y^2 + 7y + 2 = (3y + 2)(2y + 1)$

  d) $3b^2 - 2b - 16 = (3b - 8)(b + 2)$

  e) $2a^2 - 15a + 25 = (2a - 5)(a - 5)$

  f) $3t^2 + t - 14 = (3t + 7)(t - 2)$

  g) $5m^2 - 6m - 8 = (5m + 4)(m - 2)$

  h) $8n^2 + 2n - 3 = (2n - 1)(4n + 3)$

  i) $7p^2 + 19p + 10 = (7p + 5)(p + 2)$

  j) $9q^2 - 15q + 6 = (3q - 2)(3q - 3)$

5. a) $t(x) = (x + 17)(x − 1)$; x-intercepts are $−17$ and 1

b) $y = (x + 15)(x + 3)$; x-intercepts are $−15$ and $−3$

c) $y = (x − 9)(x + 8)$; x-intercepts are 9 and $−8$

d) $y = (2x + 5)(x + 2)$; x-intercepts are $−\dfrac{5}{2}$ and $−2$

e) $y = (3x − 5)(x + 3)$; x-intercepts are $\dfrac{5}{3}$ and $−3$

f) $f(x) = (5x + 2)(x − 3)$; x-intercepts are $−\dfrac{2}{5}$ and 3

g) $g(x) = (7x + 4)(x − 2)$; x-intercepts are $−\dfrac{4}{7}$ and 2

h) $h(x) = 2(x + 6)(x + 1)$; x-intercepts are $−6$ and $−1$

B

6. a) $t(x) = x^2 + 16x − 17$
x-intercepts are $−17$ and 1
y-intercept: $−17$
axis of symmetry: $x = −8$
Sketches may vary.

b) $y = x^2 + 18x + 45$
x-intercepts are $−15$ and $−3$
y-intercept: 45
axis of symmetry: $x = −9$
Sketches may vary.

c) $y = x^2 − x − 72$
x-intercepts are 9 and $−8$
y-intercept: $−72$
axis of symmetry: $x = 0.5$
Sketches may vary.

d) $y = 2x^2 + 9x + 10$
x-intercepts are $−\dfrac{5}{2}$ and $−2$
y-intercept: 10
axis of symmetry: $x = −2.25$
Sketches may vary.

e) $y = 3x^2 + 4x − 15$
x-intercepts are $\dfrac{5}{3}$ and $−3$
y-intercept: $−15$
axis of symmetry: $x \doteq −0.67$
Sketches may vary.

f) $f(x) = 5x^2 − 13x + 6$
x-intercepts are $−\dfrac{2}{5}$ and 3
y-intercept: 6
axis of symmetry: $x = 1.3$
Sketches may vary.

g) $g(x) = 7x^2 − 10x − 8$
x-intercepts are $−\dfrac{4}{7}$ and 2
y-intercept: $−8$
axis of symmetry: $x \doteq 0.71$
Sketches may vary.

h) $h(x) = 4x^2 + 14x + 12$
x-intercepts are $−6$ and $−1$
y-intercept: 12
axis of symmetry: $x = −3.5$
Sketches may vary.

7., 8. a) $3x^2 + 15x + 18 = 3(x + 2)(x + 3)$

b) $5m^2 + 20m + 15 = 5(m + 3)(m + 1)$

c) $2q^2 − 2q − 12 = 2(q − 3)(q + 2)$

d) $8a^2 − 28a + 20 = 4(2a − 5)(a − 1)$

9. a) Polynomials may vary.

b) Diagrams may vary.

10. a) $b = +7$ and $c = +9$. There are no factors of $+9$ that will have a sum of $+7$.

b) You cannot create a perfect rectangle using the algebra tiles that models the polynomial.
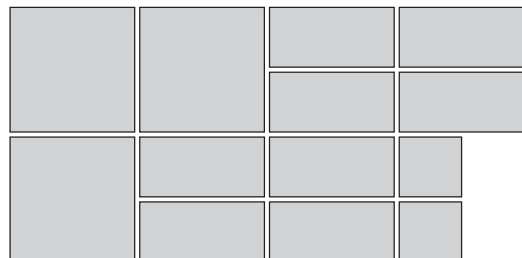
11. a), b) Expressions may vary.

　　c) Answers may vary.

C

12. a) $(x - 2)(x - 6)(x + 5)$

　　b) $(x + 1)(x - 2)(x + 2)$

　　c) $(x - 2)(2x + 1)$

　　d) $(x + 3)(x - 4)$

13. a) $\left(x + \dfrac{1}{2} \right)\left(x + \dfrac{1}{3} \right)$

　　b) $(x + 2)\left(x + \dfrac{1}{2} \right)$

　　c) $\left(x - \dfrac{2}{3} \right)\left(x + \dfrac{1}{3} \right)$

　　d) $\left(x - \dfrac{3}{5} \right)\left(x + \dfrac{3}{5} \right)$

Section 2.4

Prerequisite Skills

1.　a) 25　　**b)** 81^2　　**c)** 36　　**d)** 144

　　e) 169　　**f)** 49　　**g)** 121　　**h)** 64

2.　a) $x(4x + 7)$　　　　**b)** $9(r + 3)$

　　c) $2q(q - 9)$　　　　**d)** $3(n^2 + 4n - 2)$

　　e) $(m - 2)(m - 5)$　**f)** $(2 + 4x)(x - 1)$

3.　a) $(x - 7)(x + 1)$　**b)** $(y - 5)(y + 3)$

　　c) $(b + 10)(b - 4)$　**d)** $(q - 7)(q + 3)$

　　e) $(n + 8)(n - 4)$　**f)** $(p - 13)(p - 2)$

　　g) $(a + 9)(a - 6)$　**d)** $(m - 11)(q - 4)$

A

1.　a) $(y + 5)^2$　　　　**b)** $(x + 4)^2$

　　c) $(3q + 8)^2$　　　　**d)** $(p - 1)^2$

　　e) $(4z + 3)^2$　　　　**f)** $(2x + 9)^2$

　　g) $(5m - 7)^2$　　　　**h)** $(2k + 10)^2$

　　i) $(3b + 5)^2$　　　　**j)** $(6n + 2)^2$

2.　a) $(x + 3)(x - 3)$

　　b) $(3k + 10)(3k - 10)$

　　c) $(z + 15)(x - 15)$

　　d) $(2a + 13)(2a - 13)$

　　e) $(6m + 8)(6m - 8)$

　　f) $(9b + 12)(9b - 12)$

　　g) $(7q + 5)(7q - 5)$

　　h) $(11p + 1)(11p - 1)$

3.　a) $(b + 7)(b + 2)$

　　b) not factorable

　　c) $16(x + 1)^2$

　　d) not factorable

　　e) $(2k + 13)(2k - 13)$

　　f) $(4n + 7)(n + 7)$

　　g) $(5p + 1)(5p - 7)$

　　h) not factorable

4.　a) $7(2x^2 - x - 6)$

　　b) Yes, it can be factored further;
　　　　$7(2x + 3)(x - 2)$

　　c), d) $14x^2 + 21x - 28x + 42$
　　　　$= 7x(2x + 3) - 7(4x - 6)$
　　　　$= 7x(2x + 3) - 7(2)(2x + 3)$
　　　　$= (7x - 14)\,(2x + 3)$
　　　　$= 7(x - 2)\,(2x + 3)$

　　e) The answers are the same.

　　f) Answers may vary. Factoring first then
　　　　finding the binomials was easiest.

5.　a) $(9y - 10)^2$

　　b) $81y^2 - 90y - 90y + 100$
　　　　$= 9y(9y - 10) - 10(9y - 10)$
　　　　$= (9y - 10)^2$

　　c) The results are the same.

　　d) Answers may vary. Using the pattern
　　　　method was easy because you only had
　　　　to find the square root of a and b, and
　　　　then express the factors in this form
　　　　$(\sqrt{a} - \sqrt{b})^2$. This method saved
　　　　time.

6. a) b equals 0 and $c = -25$. Two factors of -25 whose sum is 0 are -5 and $+5$.

  i) The values -5 and $+5$ are the x-intercepts of the graph of $y = x^2 - 25$. They are opposite values.

  ii) The axis of symmetry can be found by finding the mean of the x-intercepts. $\dfrac{-5 + 5}{2} = 0$. The equation of the axis of symmetry is $y = 0$ or the y-axis.

  b) Sketches may vary.

7. a) b equals -3 and $c = 0$.

  i) You can determine the x-intercepts by letting $y = 0$.
  $$x^2 - 3x = 0$$
  $$x(x - 3) = 0$$
  $$x = 0 \qquad x - 3 = 0$$
  $$x = 3$$

  ii) Since the y-intercept $= 0$, its value is equal to one of the x-intercepts.

  b) Sketches may vary.

8. All polynomials can be factored by decomposition. The preferred method is described.

  a) Factor out 6 first, then factor the difference of squares.

  b) Factor out 3 first, then find two factors of c in the simplified trinomial whose sum is b.

  c) Factor out 2 first, then factor the perfect square trinomial.

  d) Factor out 10 first, then factor the perfect square trinomial.

  e) Factor out 14. Then find two factors of ac in the simplified trinomial whose sum is b.

  f) Factor the difference of squares.

  g) Find two factors of ac whose sum is b.

  h) Factor out 3. Then find two factors of c in the simplified trinomial whose sum is b.

  i) Factor out 7. Then factor the difference of squares.

  j) Factor out 2. Then find two factors of ac in the simplified trinomial whose sum is b.

  k) Factor out 2. Then factor the difference of squares.

  l) Factor out 3. Then find two factors of ac in the simplified trinomial whose sum is b.

9. a) $6(x - 4)(x + 4)$

  b) $9(q + 7)(q + 3)$

  c) $2(p - 2)^2$

  d) $10(4z + 7)^2$

  e) $14(b + 2)(b + 1)$

  f) $(a - 17)(a + 17)$

  g) $(2m - 19)(m + 1)$

  h) $3(k + 6)(k - 4)$

  i) $7(z - 10)(z + 10)$

  j) $(3n + 1)(3n + 2)$

  k) $2(4k - 5)(4k + 5)$

  l) $3(a + 5)(a + 1)$

10. not factorable; The tile model is not a rectangle.

11. a) $a \leq -1$ **b)** $a = 0$ **c)** $a \geq 1$

## Section 2.5

Prerequisite Skills

1. a) $(x - 9)(x + 9)$

  b) $(x - 5)(x + 3)$

  c) $(6x + 7)(x + 1)$

  d) $4(x - 2)^2$

  e) $3(x + 5)(x + 1)$

  f) $(5 - 2x)(5 + 2x)$

  g) $(7x + 1)(x + 2)$

  h) $(4x - 7)^2$

2. a)-h) Sketches may vary.
The factored form of the quadratic function was easiest to use for graphing because it provides the values of the x-intercepts, the equation of the axis of symmetry, and thus, the coordinates of the vertex, and the y-intercept. These key features can be used to make a sketch of the graph.

A

1. a) $x = 0, x = 2$ **b)** $y = 1, y = -4$

   c) $a = 7, a = -\dfrac{2}{3}$ **d)** $c = -\dfrac{3}{2}, c = \dfrac{1}{4}$

   e) $k = -2, k = -\dfrac{7}{6}$ **f)** $n = 3, n = -9$

2. a) $y = -19, y = -1$ **b)** $k = -11, k = 3$

   c) $p = 9, p = -4$ **d)** $a = -5, a = 2$

   e) $n = -\dfrac{11}{2}, n = \dfrac{11}{2}$

   f) $b = \dfrac{3}{5}, b = 2$ **g)** $h = \dfrac{5}{4}, h = -\dfrac{5}{2}$

   h) $m = -3, m = 3$

3. a) The zeros of the function are $t = 5$ and $t = -2$. $t = -2$ s must be rejected since negative time is not permissible in the given context.

   b)

WINDOW
Xmin=-5
Xmax=10
Xscl=1
Ymin=-5
Ymax=40
Yscl=5
Xres=1
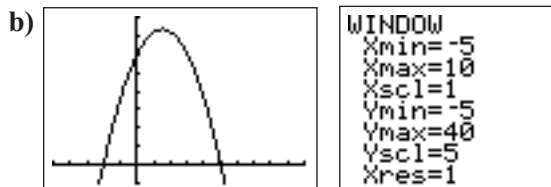
For $t < 0$ and $t > 5$, the graph has no meaning.

B

4. a) $y = a(x + 3)(x - 11)$

   b) Express the function in x-intercept form, $y = a(x - r)(x - s)$.

c) If a is any given value, then the function in part b) is the only possible function. If a was specific, for example, equal to 1, then there would be other possible functions.

5. a) $x = 6, x = -6$

   b) The x-intercepts of the graph are 6 and -6.

   c) $x = 6$

   d) The three methods produce the same result.

   e) Answers may vary. Solving by factoring and solving by rearranging and taking square roots are preferred because you can readily obtain precise values without having to use technology for this specific case.

6. a) $x^2 - 172$ or $x^2 - 289$

   b) 22 cm   **c)** 28 cm

7. Answers may vary.

   a) • The positive zero of the function will be less.

     • The positive zero of the function will be less.

     • The positive zero of the function will be greater.

   b) All scenarios should have one inadmissible zero.

8. a) $x = 0$ and $x = 24$

   The value 0 is the reference point from the left side of the lamp. The value 24 is the reference point from the right side of the lamp.

   b) 24 cm; The difference between the two zeros is 24.

   c) 12 cm; This is the perpendicular distance from the centre of the parabola to the vertex.

d) All negative *y*-values are admissible because the parabola modelling the shape of the headlamp is below the *x*-axis. The parabola opens upward.

C

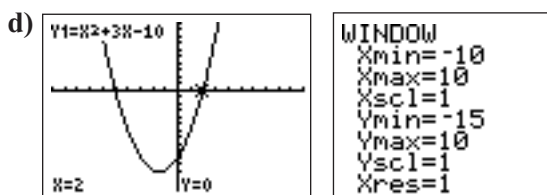**9.** Answers may vary. Quadratic equations in this form have one root, and it is given by $-\dfrac{b}{2a}$.

Chapter 2 Review

1. a) standard form

b) $a = 1$, $b = 3$, $c = -10$; upward

c) *y*-intercept: -10

d)

e) $x = 2$, $x = -5$; vertex $(1.5, -3.25)$

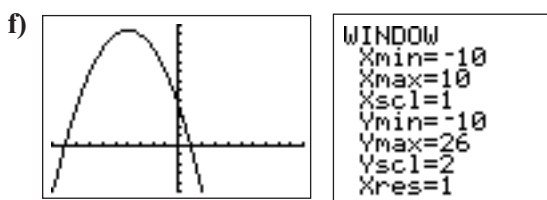**f)** Since *a* is positive, the vertex is a minimum.

2. a) *x*-intercept form; downward

b) $x = 1$, $x = -9$

c) vertex $(-4, 25)$; Since *a* is negative, the vertex is a maximum.

d) axis of symmetry: $x = -4$

e) *y*-intercept: 9

f)

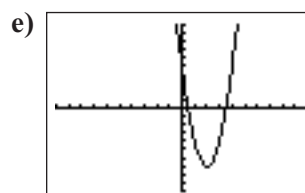**g) i)** positive when $-9 < x < 1$; negative when $x < -9$ and $x > 1$

ii) increasing for $x < -5$; decreasing for $x > -5$

3. a) vertex form; upward

b) vertex $(2, -7)$; Since *a* is positive, the vertex is a minimum.

c) axis of symmetry: $x = 2$

d) *y*-intercept: 5

e)

f) increasing for $x > 2$; decreasing for $x < 2$

4. a)

b) $a = 1$; This indicates that the number of dark grey squares grow quadratically but not multiplicatively. $b = 3$; This indicates that the number of light grey squares grows by a factor of 3. $c = 2$; This indicates that the number of white squares stays the same with each stage.

5. a) i)

ii)
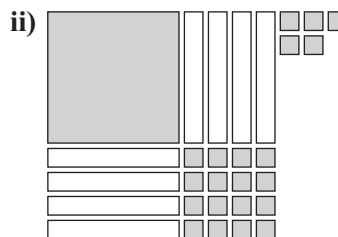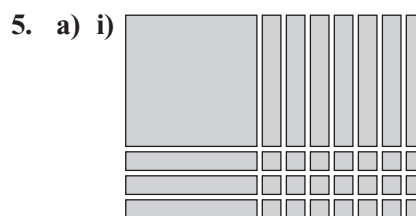
b) i) $x^2 + 10x + 21$ **ii)** $x^2 - 8x + 21$

c) i) $a = 1$, $b = 10$, $c = 21$

ii) $a = 1$, $b = -8$, $c = 21$

d) i) *y*-intercept: 21   **ii)** *y*-intercept: 21

6. a) $8x - 17$ **b)** $m - 6$
c) $4y^2 - 5y + 16$ **d)** $-19w^2 + 66w - 9$

7. a), b) Quadratic functions may vary.

8. a) $(x + 4)(x + 1)$ **b)** $(3x + 1)(x + 2)$

9. a) $(y + 9)(y + 3)$ **b)** $(k + 8)(k + 2)$

   c) not factorable; There are no two factors of -10 whose sum is 2.

   d) not factorable; There are no two factors of $5(2)$ or 10 whose sum is 9.

   e) $(2v + 5)(v - 5)$ **f)** $(3x + 1)(3x + 1)$

10. a) $y = 2(x + 6)(x + 1)$; $x = -6$, $x = -1$

   b) $y = 3(3x + 5)(x + 1)$; $x = -\dfrac{5}{3}$, $x = -1$

   c) $y = 4(x - 3)(x - 3)$; $x = 3$

   d) $y = 3(7x + 2)(x - 1)$; $x = -\dfrac{2}{7}$, $x = 1$

   e) $y = 2(17x + 1)(x - 1)$; $x = -\dfrac{1}{17}$, $x = 1$

   f) $y = 8(x - 7)(x - 1)$; $x = 7$, $x = 1$

11. a), b) Questions and models may vary.

12. a) $(x + 5)(x + 5)$ or $(x + 5)^2$

   b) The binomial factors of the trinomial are the same, so it is a perfect square trinomial. In a tile model, the dimensions are the same.

13. a), b) Answers may vary.

14. a) $2(x + 3)^2$
   b) $5(n - 2)(n + 2)$
   c) $7(g - 5)(g - 5)$
   d) $4(m - 8)(m - 8)$
   e) $3(3w - 7)(3w + 7)$
   f) $2(11v - 5)(11v + 5)$

15. a) $(p + 13)(p + 4)$; $p = -13$, $p = -4$

   b) $(4x + 17)(x - 1)$; $x = -\dfrac{17}{4}$, $x = 1$

   c) $(3r - 7)(r - 2)$; $r = \dfrac{7}{3}$, $r = 2$

   d) $(2q - 19)(q - 1)$; $q = \dfrac{19}{2}$, $q = 1$

   e) $(5n - 4)(n + 2)$; $n = \dfrac{4}{5}$, $n = -2$

   f) $(3m - 7)(m + 2)$; $m = \dfrac{7}{3}$, $m = -2$

16. a) $y = -4(t - 7)(t - 1)$

   b) 6 s; The hang time is the difference between the start of the path to the end which is the difference between the values of the x-intercepts.

Chapter 2 Practice Exam
1. A   **2.** D   **3.** C   **4.** D   **5.** B

6. a)
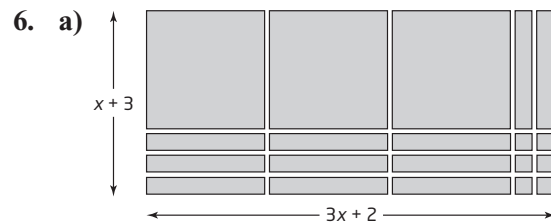
   b) $3x^2 + 11x + 6 = (x + 3)(3x + 2)$

7. a) $4x^2 - 16x - 20$; y-intercept: -20
   b) $-2(x^2 + 14x + 37)$; y-intercept: 37

8. a) $(2a + 11)(a - 1)$
   b) $(p + 6)(p + 3)$
   c) not factorable using integral factors; The expression looks like a difference of squares, but 24 is not a perfect square.
   d) not factorable; There are no two factors of -32 whose sum is 2.
   e) not factorable; There are no two factors of -7 whose sum is -10.
   f) $(h - 5)(h - 3)$

9. a) $2(4f - 7)(f - 1)$ **b)** $3(q - 5)(q + 5)$

10. a) $(x - 9)(x - 3)$; $x = 9$, $x = 3$

b) $(2x + 5)(x - 3)$; $x = -\dfrac{5}{2}$, $x = 3$

11. 59 m

12. a) x-intercept form; downward

b) $x = 6$, $x = -2$

c) vertex $(2, 32)$; Since a is negative, the vertex is a maximum.

d) axis of symmetry: $x = 2$

e) y-intercept: 24
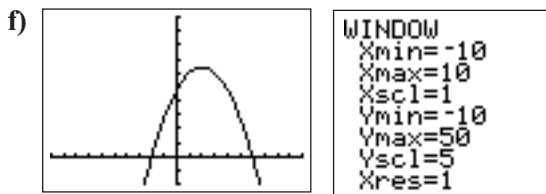
f)

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=50
Yscl=5
Xres=1
```

g) i) positive for $-2 < x < 6$; negative for $x < -2$ and $x > 6$

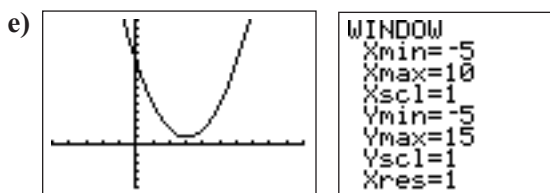**ii)** increasing when $x < -2$, decreasing when $x > -2$

13. a) vertex form; upward

b) vertex $(3, 1)$; Since a is positive, the vertex is a minimum.
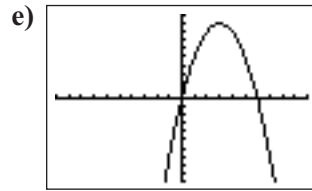
c) axis of symmetry: $x = 3$

d) y-intercept: 10

e)

```
WINDOW
Xmin=-5
Xmax=10
Xscl=1
Ymin=-5
Ymax=15
Yscl=1
Xres=1
```

f) increasing when $x > 3$, decreasing when $x < 3$

14. a) vertex $(3, 9)$ **b)** $-x^2 + 6x$

c) $-x(x + 6)$ **d)** $x = 0$, $x = -6$

e)

f) Yes, an 8 cm bulb will fit. The maximum height of a bulb that will fit is 9 cm.

g) For $x < 0$ and $x > 6$, the range values have no meaning.

15. a) $x = -6$, $x = 8$

b) axis of symmetry: $x = 1$; vertex $(1, 122\ 500)$; maximum

c) One $10 increase will provide a maximum revenue. The selling price should be $70.

16. a) The dark grey squares are growing at a rate of n^2, where n is the stage number. The white squares are growing at a rate of $2n$. The light grey squares stay the same as 1 with each stage.

b) • The dark grey squares grow as a quadratic function.

• The white squares grow as a linear function.

• The light grey squares represent a constant function.

See explanations in part a).

c) $T(n) = n^2 + 2n + 1$

d) $a = 1$ represents the number of dark grey squares of side length n. $b = 2$ represents the number white squares. $c = 1$ represent the constant number of light grey squares.

e) $T(n) = (n + 1)^2$; The factors $(n + 1)$ and $(n + 1)$ indicate that the models are square images with the same dimension.

Chapter 3 Represent Quadratic Functions

Prerequisite Skills

1. a) 12   **b)** 21   **c)** 24

    d) 18   **e)** 60   **f)** 36

2. a) $\dfrac{9}{10}$ **b)** $-\dfrac{1}{2}$ **c)** $\dfrac{1}{12}$

    d) $\dfrac{7}{16}$ **e)** $\dfrac{33}{35}$ **f)** -3

3. a) -6 **b)** $-\dfrac{4}{3}$ **c)** 1

    d) 11   **e)** 22   **f)** 28

4. a) 13   **b)** 4.58   **c)** -11.18

    d) 10   **e)** 9   **f)** -15

5. a) $4x$ **b)** $3b$ **c)** $4y^2$

    d) $2y$ **e)** $6e$ **f)** $7g^2h$

6. a) $3x(4x + 3)$ **b)** $4x(2x - 1)$

    c) $-5x(x + 4)$ **d)** $2x(4x + 3)$

    e) $4x(x - 7)$ **f)** $3y(5x + 3)$

7. a) $(x + 7)(x + 4)$

    b) $(x - 12)(x - 3)$

    c) $(x + 9)(x - 2)$

    d) $(x + 2)(x - 1)$

    e) $(x - 4)(x + 4)$

    f) $(x - 1)(x - 1)$

8. a) $(7x - 3)(x + 1)$

    b) $(3x + 5)(x - 2)$

    c) $(8x - 1)(x - 1)$

    d) $(5x + 5)(x + 1)$

    e) $(4x - 3)(2x + 1)$

    f) $3(x - 5)(x + 5)$

3.1 Identify Functions

Prerequisite Skills

1. a) $y = (x - 4)^2 - 7$

    b) $y = 2(x + 3)^2 + 2$

2. a) maximum: -1

    b) minimum: -8

3. a)
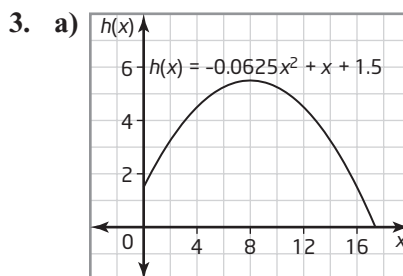
    b) 5.5 m   **c)** 8 m

4. $4.80; $768

A

1. a) $y = (x + 3)^2 - 4$

    b) $y = (x + 1)^2 + 4$

    c) $y = (x + 4)^2 - 18$

    d) $y = (x + 2)^2 - 8$

    e) $y = (x + 5)^2 - 10$

    f) $y = (x + 3)^2 - 14$

2. a) 4   **b)** 25   **c)** 1

    d) 30.25   **e)** 121   **f)** 56.25

3. a) $y = (x + 6)^2 - 18$

    b) $y = (x + 7)^2 - 42$

    c) $y = (x - 1)^2 - 7$

    d) $y = 2(x + 4)^2 - 16$

    e) $y = 4(x - 1)^2 - 4$

    f) $y = -3(x + 1)^2 - 6$

4. e.g., correct

5. a) 0   **b)** $-2\dfrac{1}{3}$

    c) -1 **d)** $-2\dfrac{9}{20}$

    e) 46   **f)** 1

6. e.g., correct

7. a) 16.8 m

    b) 25 m

    c) approximately 2.4 m

    d) 1.8 m

8. a) Sketches may vary.

    b) 18 m   **c)** 23 m   **d)** 2 s

B

9. a) $(45 - 2x)$; $(30 + 3x)$

    b) $R(x) = (45 - 2.5x)(30 + 3x)$

    c) \$35
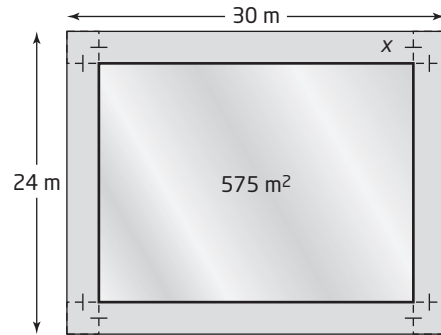
    d) \$1470

C

10. $(-2, -14)$

11. a) $2(x + 2b)^2 + 10 - 8b^2$

     b) $(-2b, 10 - 8b^2)$

3.2 The Quadratic Formula

Prerequisite Skills

1. a) -2, $\dfrac{1}{3}$ **b)** Yes

2. $x \doteq 2.54$ or $x \doteq -2.10$

3. a) 6.35 s   **b)** 4.02 s   **c)** 49.70 m

4. a) Diagrams may vary. e.g.,

    b) $4x^2 - 108x + 145 = 0$

    c) 1.4 m

A

1. a) $x = \dfrac{-4 + \sqrt{48}}{2}$ or $x = \dfrac{-4 - \sqrt{48}}{2}$

    b) $x = \dfrac{6 + \sqrt{56}}{2}$ or $x = \dfrac{6 - \sqrt{56}}{2}$

    c) $x = \dfrac{3 + \sqrt{5}}{2}$ or $x = \dfrac{3 - \sqrt{5}}{2}$

    d) $x = \dfrac{-9 + \sqrt{57}}{2}$ or $x = \dfrac{-9 - \sqrt{57}}{2}$

    e) $x = \dfrac{8 + \sqrt{56}}{2}$ or $x = \dfrac{8 - \sqrt{56}}{2}$

    f) $x = \dfrac{-7 + \sqrt{77}}{2}$ or $x = \dfrac{-7 - \sqrt{77}}{2}$

2., 3. a) $x = \dfrac{2 + \sqrt{76}}{4}$ or $x = \dfrac{2 - \sqrt{76}}{4}$;

       $x = \dfrac{1 + \sqrt{19}}{2}$ or $x = \dfrac{1 - \sqrt{19}}{2}$;

       $x \doteq 2.68$ or $x \doteq -1.68$

    b) $x = \dfrac{7 + \sqrt{29}}{10}$ or $x = \dfrac{7 - \sqrt{29}}{10}$;

       $x \doteq 1.24$ or $x \doteq 0.16$

c) $x = -\dfrac{3 + \sqrt{137}}{8}$ or

$x = -\dfrac{3 - \sqrt{137}}{8}$

$x \doteq -1.84$ or $x \doteq 1.09$

d) $x = \dfrac{-9 + \sqrt{33}}{12}$ or

$x = \dfrac{-9 - \sqrt{33}}{12}$;

$x \doteq -0.27$ or $x \doteq -1.23$

e) $x = \dfrac{-3 + \sqrt{117}}{6}$ or

$x = \dfrac{-3 - \sqrt{117}}{6}$;

$x \doteq 1.30$ or $x \doteq -2.30$

f) $x = -\dfrac{1 + \sqrt{5}}{2}$ or $x = -\dfrac{1 - \sqrt{5}}{2}$;

$x \doteq -1.62$ or $x \doteq 0.62$

4. 77.2 m

5. a) Diagrams may vary. e..g,

Total Area = 150 m²
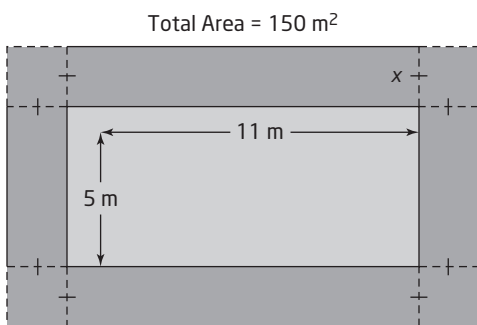
b) area $= -95 + 32x + 4x^2$

c) 2.3 m

6. a) $x \doteq 5.28$ or $x \doteq 94.72$

b) $x \doteq 2.47$ or $x \doteq -6.47$

c) $x \doteq -0.03$ or $x \doteq -0.10$

d) $x = -2$

e) $x \doteq 3.41$ or $x \doteq 0.59$

f) $x \doteq 0.54$ or $x \doteq -5.54$

7. a) $x \doteq 2.12$ or $x \doteq -0.41$

b) $x \doteq 0.72$ or $x \doteq 0.28$

c) $x \doteq 7.87$ or $x \doteq 0.13$

d) $x = -2$ or $x = -8$

e) $x \doteq 0.64$ or $x \doteq -1.24$

f) $x = \dfrac{3}{4}$ or $x = -2$

g) $x \doteq 4.64$ or $x \doteq 0.22$

h) $x \doteq 1.71$ or $x \doteq -0.88$

B

8. a) \$0 and \$5000   **b)** \$5472

c) The company must spend between
\$1500 and \$3500 on advertising.

9. 6 m/s up the river, 10 m/s down the river

C

10. $a = \dfrac{1}{2}$ and $b = 2$ or $a = \dfrac{1}{18}$ and $b = \dfrac{2}{3}$.

3.3 Real Roots of Quadratic Equations

Prerequisite Skills

1. a) $D = 0$ **b)** $D = -99$

c) $D = 81$ **d)** $D = 0$

2. a) 2 real roots; $x = -5$ or $x = 1$

b) 1 real root; $x = \dfrac{1}{2}$

c) one real root; $x = 4$

d) two real roots; $x = -1$ or $x = 9$

3. a) $x \doteq -1.68$ or $x \doteq 2.68$

b) The graph does not intersect the x-axis.

c) $x \doteq -1.52$ or $x \doteq 0.92$

d) $x = 1$

A

1. **a)** $D < 0$, so there are no real roots.

b) $D > 0$, so there are two real roots.

c) $D < 0$, so there are no real roots.

d) $D < 0$, so there are no real roots.

e) $D > 0$, so there are two real roots.

f) $D = 0$, so there is one real root.

2., 3. a) $D < 0$, so there are no real roots.

b) $D < 0$, so there are no real roots.

c) $D > 0$, so there are two real roots.

d) $D = 0$, so there is one real root.

e) $D > 0$, so there are two real roots.

f) $D > 0$, so there are two real roots.

4. **a)** $D > 0$, so there are two real roots;
$x = -1, x = 3$

b) $D > 0$, so there are two real roots;
$x = 4, x = -2$

c) $D > 0$, so there are two real roots;
$x = -1, x = \dfrac{2}{5}$

d) $D > 0$, so there are two real roots;
$x = 3, x = -3$

e) $D = 0$, so there is one real root;
$x = \dfrac{1}{2}$

f) $D > 0$, so there are two real roots;
$x = 2, x = \dfrac{1}{2}$

g) $D = 0$, so there is one real root;
$x = -1$

h) $D > 0$, so there are two real roots;
$x = 1, x = -1$

5. **a)** The x-intercepts are -1 and 2.

b) There are no x-intercepts.

c) The x-intercepts are -1 and -2.

d) The x-intercepts are 0 and -2.

6. **a)** The x-intercept is 3.

b) The x-intercepts are -1 and 8.

c) The x-intercepts are -9 and 2.

d) There are no x-intercepts.

7. **a)** The x-intercepts are 2.62 and 0.38.

b) The x-intercepts are -0.44 and -4.56.

c) The x-intercepts are -1 and $\dfrac{3}{5}$.

d) The x-intercepts are -4.61 and 0.11.

e) The x-intercepts are 3.20 and 0.11.

f) The x-intercepts are -1 and $-\dfrac{1}{7}$.

g) The x-intercepts are 7 and $-\dfrac{1}{3}$.

h) The x-intercept is -1.

B

8. **a)** $x \doteq 3.25$ or $x \doteq -0.92$

b) $x \doteq 0.23$ or $x \doteq -1.48$

c) $x \doteq 2.12$ or $x \doteq -0.79$

d) $x = 0.8$ or $x = 10$

9. 140 m by 85 m

C

10. **a)** $x^2 - 13x + 5 = 0$

b) $3x^2 - 7x - 10 = 0$

11. The discriminant cannot be zero for this equation. If $D > 0$, then $a < 0$ and if $D < 0$, $a > 0$.

3.4 Multiple Forms of Quadratic Functions

Prerequisite Skills

1. a) $f(x) = 4x^2 - 2x - 12$

    x-intercepts: 2 and $-\dfrac{3}{2}$

    $f(x)$-intercept: -12
    vertex: $(0.25, -12.25)$
    equation of axis of symmetry:
    $x = 0.25$

    $g(x) = -0.45x^2 + 0.9x + 5$
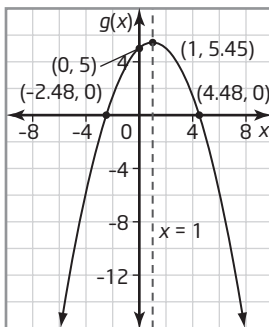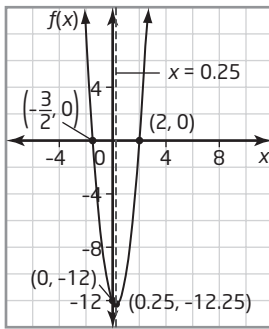   approximate x-intercepts: -2.48
    and 4.48
    $g(x)$-intercept: 5
    vertex: $(1, 5.45)$
    equation of axis of symmetry: $x = 1$

b) Sketches may vary.

2. The function is positive when $x < -5$ and $x > 1$; The function is negative when $-5 < x < 1$; The function is increasing $x > -2$; The function is decreasing when $x < -2$.

A

1., 2. a) x-intercepts: -1 and 6; y-intercept: -6; vertex: $(2.5, -12.25)$; equation of axis of symmetry: $x = 2.5$. Sketches may vary.

b) x-intercepts: -12 and -1; y-intercept: 12; vertex: $(-6.5, -30.25)$; equation of axis of symmetry: $x = -6.5$. Sketches may vary.

c) x-intercepts: -12 and 2; y-intercept: 24; vertex: $(-5, 49)$; equation of axis of symmetry: $x = -5$. Sketches may vary.

d) x-intercepts: -3 and $\dfrac{1}{3}$; y-intercept: -3;

    vertex: $\left(-\dfrac{4}{3}, -8\dfrac{1}{3}\right)$; equation of axis

    of symmetry: $x = -\dfrac{4}{3}$. Sketches may

    vary.

e) x-intercept: $-\dfrac{3}{2}$; y-intercept: 18;

    vertex: $\left(-\dfrac{3}{2}, 0\right)$; equation of axis of

    symmetry: $x = -\dfrac{3}{2}$. Sketches may

    vary.

f) x-intercept: -2; y-intercept: -18; vertex: $(-2, 0)$; equation of axis of symmetry: $x = -2$. Sketches may vary.

3., 4. a) $x \doteq 2.30$ or $x \doteq -0.80$; y-intercept: -11; vertex: approximately $(0.75, -14.38)$; equation of axis of symmetry: $x = 0.75$. Sketches may vary.

b) $x \doteq 0.33$ or $x \doteq 7.67$; y-intercept: -5; vertex: $(4, 27)$; equation of axis of symmetry: $x = 4$. Sketches may vary.

c) $x \doteq -2.38$ or $x \doteq 0.63$; y-intercept: 6; vertex: approximately $(-0.88, 9.06)$; equation of axis of symmetry: $x = -0.88$. Sketches may vary.

d) $x \doteq 5.58$ or $x \doteq -3.58$; y-intercept: 10; vertex: $(1, 10.5)$; equation of axis of symmetry: $x = 1$. Sketches may vary.

e) $x \doteq 1.94$ or $x \doteq -13.94$; y-intercept: -9; vertex: $(-6, -21)$; equation of axis of symmetry: $x = -6$. Sketches may vary.

f) $x \doteq 3.41$ or $x \doteq 0.59$; y-intercept: 1.28; vertex: approximately $(2, -1.28)$; equation of axis of symmetry: $x = 2$. Sketches may vary.

5. a) x-intercepts: 2 and $-\dfrac{5}{2}$;

vertex: $\left(-\dfrac{1}{4}, 10\dfrac{3}{8}\right)$; The function is

positive when $-\dfrac{5}{2} < x < 2$; The

function is negative when $x < -\dfrac{5}{2}$ or

$x < 2$; The function is increasing

when $x < -\dfrac{1}{4}$; The function is

decreasing when $x > -\dfrac{1}{4}$.

b) x-intercepts: -1 and 17; vertex: $(8, -81)$; The function is positive when $x < -1$ or $x > 17$; The function is negative when $-1 < x < 17$; The function is increasing when $x > 8$; The function is decreasing when $x < 8$.

c) x-intercepts: -1 and $\dfrac{7}{4}$; vertex:

$\left(\dfrac{3}{8}, 7\dfrac{9}{16}\right)$; The function is positive

when $-1 < x < \dfrac{7}{4}$; The function is

negative when $x < -1$ or $x > \dfrac{7}{4}$; The

function is increasing when $x < \dfrac{3}{8}$;

The function is decreasing when

$x > \dfrac{3}{8}$.

d) x-intercepts: -2 and 11; vertex: $(4.5, -42.25)$; The function is positive when $x < -2$ or $x > 11$; The function is negative when $-2 < x < 11$; The function is increasing when $x > 4.5$; The function is decreasing when $x < 4.5$.

e) x-intercept: 4; vertex: $(4, 0)$; The function is positive for all values of x; The function is increasing when $x > 4$; The function is decreasing when $x < 4$.

f) x-intercepts: 0 and 3; vertex: $(1.5, 20.25)$; The function is positive when $0 < x < 3$; The function is negative when $x < 0$ and $x > 3$; The function is increasing when $x < 1.5$; The function is decreasing when $x > 1.5$.

g) x-intercept: -3; vertex: $(-3, 0)$; The function is positive for all values of x; The function is increasing when $x > -3$; The function is decreasing when $x < -3$.

h) x-intercepts: -5 and 5; vertex: $(0, -125)$; The function is positive when $x < -5$ and $x > 5$; The function is negative when $-5 < x < 5$; The function is increasing $x > 0$; The function is decreasing when $x < 0$.

6. a) x-intercepts: $\dfrac{3}{2}$ and 1;

vertex: $(1.25, -0.12)$;
y-intercept: 3;
equation of axis of symmetry:
$x = 1.25$;
The function is positive
when $x < 1$ and $x > 1.5$;
The function is negative
when $1 < x < 1.5$;
The function is increasing
when $x > 1.25$;
The function is decreasing
when $x < 1.25$.

b) x-intercepts: 0.68 and 3.36;
vertex: $(2, -7)$;
y-intercept: 9;
equation of axis of symmetry: $x = 2$;
The function is positive
when $x < 0.68$ and $x > 3.36$;
The function is negative
when $0.68 < x < 3.36$;
The function is increasing when $x > 2$;
The function is decreasing
when $x < 2$.

c) x-intercepts: -6.28 and -3.72;
vertex: $(-5, 5)$;
y-intercept: -70;
equation of axis of symmetry: $x = 5$;
The function is positive
when $-6.28 < x < -3.72$;
The function is negative
when $x < -6.28$ and $x > -3.72$;
The function is increasing
when $x < -5$;
The function is decreasing
when $x > -5$.

d) x-intercepts: -1.1 and 4.6;
vertex: $(1.8, 16.1)$;
y-intercept: 10;
equation of axis of symmetry: $x = 1.8$;
The function is positive
when $-1.1 < x < 4.6$;
The function is negative
when $x < -1.1$ and $x > 4.6$;
The function is increasing
when $x < 1.8$;
The function is decreasing
when $x > 1.8$.

e) x-intercepts: -4 and 6;
vertex: $(1, 50)$;
y-intercept: 48;
equation of axis of symmetry: $x = 1$;
The function is positive
when $-4 < x < 6$;
The function is negative
when $x < -4$ and $x > 6$;
The function is increasing
when $x < 1$;
The function is decreasing
when $x > 1$.

f) x-intercepts: -1 and $\frac{1}{3}$;

vertex: $\left(-\frac{1}{3}, -\frac{4}{3}\right)$;

y-intercept: -1;

equation of axis of symmetry:
$x = -\frac{1}{3}$;
The function is positive

when $-1 < x$ and $x > \frac{1}{3}$;
The function is negative

when $-1 < x < \frac{1}{3}$;
The function is increasing

when $x > -\frac{1}{3}$;
The function is decreasing

when $x < -\frac{1}{3}$.

B
7. a) 40 cm   **b)** 190 cm   **c)** 50 cm

  d) Sketches may vary.

  e) approximately 106 cm

8. a) The h-intercepts represent when the
  whale is exiting or entering the water
  and the t-intercept represents the time
  at which the whale reaches maximum
  speed.

  b) 0.74 s

C
9. -5

10. $y = \frac{1}{6}(x - 7)^2 - 2$

3.5 Model With Quadratic Equations
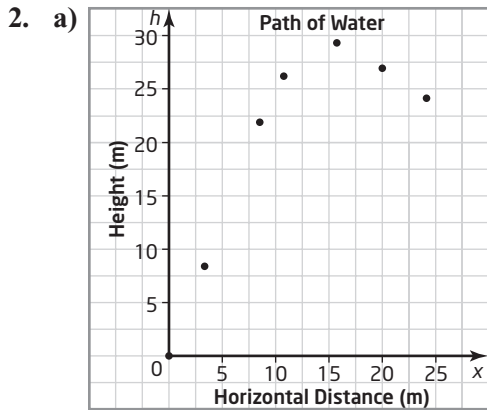
Prerequisite Skills

1. $y = \frac{3}{16}(x - 8)^2$

2. a) $h = -4.9t^2 + 49.2t$
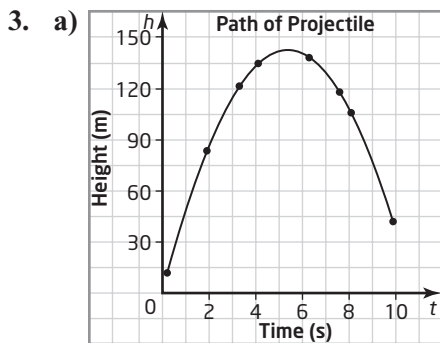
  b) Estimates may vary. e.g.,
  approximately 122 m

A

1. a) $y = -(x + 3)^2 + 5$

   b) $y = 0.25(x - 1)^2 - 4$

2. a)


Path of Water

   b) Estimates and equations may vary; vertex: (16, 29); $y = -0.113x^2 + 3.625x$

   c) $y = -0.115\,642\,59x^2 + 3.678\,995\,14x - 0.361\,067\,811\,8$

   d) The equation derived from the quadratic regression on the data is the best model. When graphed, the parabola passes through all of the points.

B

3. a)

Path of Projectile

   b) Estimates may vary; vertex: (5.5, 142)

   c) $y = -4.900\,150\,673x^2 + 52.601\,847\,94x + 1.495\,694\,353$ maximum: 142.66

   d) Answers may vary. E.g., The estimate was accurate but not as exact as the calculated maximum.

   e) 78 m

4. a)

Spread of Weeds
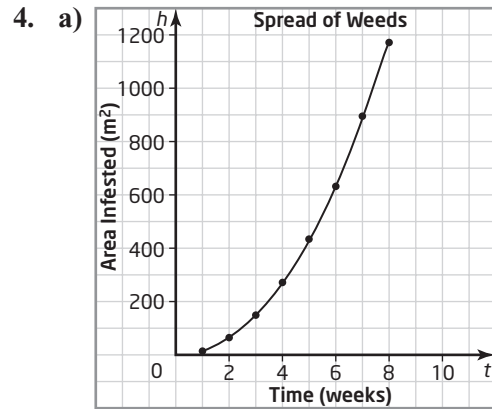
   b) Estimates and equations may vary; vertex: (0, 10); $y = 18.297x^2 + 10$

   c) $y = 20.125x^2 - 16.041\,666\,67x + 12.875$

   d) E.g., 2645 m² with equation from part b), 2718 m². The estimates are relatively close so both equations appear to model the data well.

C

5. No, the ball will pass below the crossbar.

Chapter 3 Review

1. a) $y = (x + 2)^2 - 3$

   b) $y = -3(x - 1)^2 + 6$

   c) $y = 2(x + 4)^2 - 12$

   d) $y = -\frac{1}{2}(x - 4)^2 - 1$

2. a) minimum value: 4

   b) maximum value: 9

   c) minimum value: -23

   d) maximum value: 19

3. a) Sketches may vary.

   b) approximately 12.9 m, when $t = 1.5$ s

   c) 11.7 m

4. a) $x = \dfrac{10 + \sqrt{80}}{2}$ or $x = \dfrac{10 - \sqrt{80}}{2}$;

$x = 5 + \sqrt{20}$ or $x = 5 - \sqrt{20}$;

$x \doteq 9.47$ or $x \doteq 0.53$

b) $x = \dfrac{15 + \sqrt{1185}}{24}$

or $x = \dfrac{15 - \sqrt{1185}}{24}$;

$x \doteq 2.06$ or $x \doteq -0.81$

c) $x = \dfrac{1}{2}$ or $x = -\dfrac{1}{3}$;

d) $x = \dfrac{40 + \sqrt{1420}}{90}$

or $x = \dfrac{40 - \sqrt{1420}}{90}$;

$x = \dfrac{20 + \sqrt{355}}{45}$ or $x = \dfrac{20 - \sqrt{355}}{45}$;

$x \doteq 0.86$ or $x \doteq 0.03$

5. 8.7 cm by 15 cm

6. $x = \dfrac{-5 + \sqrt{53}}{2}$

7. a) $D > 0$, so there are two real roots;

$x = \dfrac{7}{5}, x = -1$

b) $D > 0$, so there are two real roots;

$x = 0, x = -\dfrac{3}{8}$

c) $D > 0$, so there are two real roots;

$x = 3, x = -3$

d) $D = 0$, so there is one real root;

$x = -2$

8. a) The x-intercepts are -1 and $-\dfrac{2}{3}$.

b) The x-intercepts are 3.37 and -2.37.

c) The x-intercepts are $\dfrac{1}{2}$ and $\dfrac{5}{6}$.

d) The x-intercepts are 1.42 and -1.82.

9. $b > 24$

10. a) x-intercepts: -6 and 1; y-intercept: -12; vertex: $(-2.5, -24.5)$; equation of axis of symmetry: $x = -2.5$. Sketches may vary.

b) x-intercepts: $\dfrac{1}{3}$ and 5; y-intercept: -5;

vertex: $\left(\dfrac{8}{3}, \dfrac{49}{3}\right)$; equation of axis of

symmetry: $x = \dfrac{8}{3}$. Sketches may vary.

c) x-intercepts: -2 and 1; y-intercept: 4; vertex: $(-0.5, 4.5)$; equation of axis of symmetry: $x = -0.5$. Sketches may vary.

d) x-intercept: 0.5; y-intercept: 0.25; vertex: $(0.5, 0)$; equation of axis of symmetry: $x = 0.5$.

11. a) 0.04 s

b) 1.47 s

c) approximately 2.65 m, when $t \doteq 0.78$ s

d)
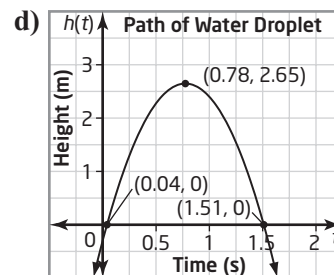
e) The function is increasing when $t < 0.78$; The function is decreasing when $t > 0.78$.

12. a) x-intercepts: -2 and 5; vertex: $(1.5, -24.5)$; The function is positive when $x < -2$ and $x > 5$; The function is negative when $-2 < x < 5$; The function is increasing when $x > 1.5$; The function is decreasing when $x < 1.5$.

b) x-intercepts: -5 and -1; vertex: $(-3, -8)$; The function is positive when $x < -5$ and $x > -1$; The function is negative when $-5 < x < -1$; The function is increasing when $x > -3$; The function is decreasing when $x < -3$.

c) x-intercepts: -1 and $\frac{11}{3}$; vertex: $\left(\frac{4}{3}, \frac{49}{3}\right)$; The function is positive when $-1 < x < \frac{11}{3}$; The function is negative when $x < -1$ and $x > \frac{11}{3}$; The function is increasing when $x < \frac{4}{3}$; The function is decreasing when $x > \frac{4}{3}$.

d) x-intercept: 1; vertex: (1, 0); The function is always negative; The function is increasing when $x < 1$; The function is decreasing when $x > 1$.

13. a) $y = -(x - 3)^2 + 2$

 b) $y = 0.5(x + 1)^2 + 1$

14. a)

Car's Distance vs. Time
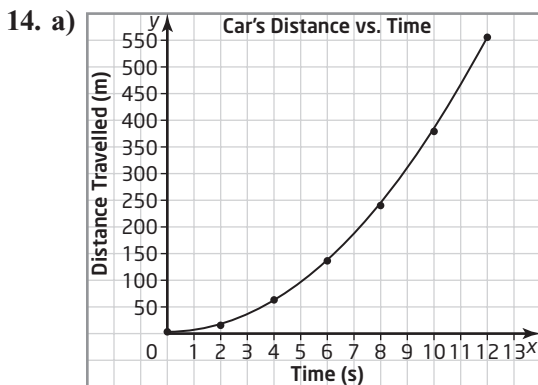
b) Equations may vary; $y = 3.848x^2$

c) e.g., 865.8 m

d) $y = 3.912\ 797\ 619x^2 - 1.130\ 357\ 143x + 1.688\ 095\ 238$

e) e.g., 865.1 m; the estimates are very close

Chapter 3 Practice Exam

1. B   **2.** D   **3.** C   **4.** B

5. a) maximum: 6   **b)** minimum: -8

 c) maximum: 18   **d)** maximum: -20

6. a) $D = 0$, so there is one real root; $x = 1$

 b) $D > 0$, so there are two real roots; $x = 1$ or $x = -2$

7. a) $x = \frac{1}{2}$ or $x = \frac{5}{2}$

 b) $x = -\frac{13}{2}$ or $x = 1$

 c) $x = -\frac{5}{3}$ or $x = 2$

 d) $x = -\frac{3}{4}$ or $x = 3$

8. a) x-intercepts: -4 and $\frac{4}{3}$; y-intercept: -16; vertex: $\left(-\frac{4}{3}, -\frac{64}{3}\right)$; equation of axis of symmetry: $x = -\frac{4}{3}$. Sketches may vary.

 b) x-intercepts: $\frac{1}{4}$ and 3; y-intercept: -3; vertex: (1.625, 7.5625); equation of axis of symmetry: $x = 1.625$. Sketches may vary.

9. a) $y = \frac{3}{2}(x + 2)^2 - 3$

 b) $y = -(x - 4)^2 + 1$

10. a) *x*-intercepts: -1 and 15; vertex: (7, -64); The function is positive when $x < -1$ and $x > 15$; The function is negative when $-1 < x < 15$; The function is increasing when $x > 7$; The function is decreasing when $x < 7$.

b) *x*-intercepts: -4 and 0; vertex: (-2, 8); The function is positive when $-4 < x < 0$; The function is negative when $x < -4$ and $x > 0$; The function is increasing when $x < -2$; The function is decreasing when $x > -2$.

11. a) 22.1 m

b) When the horizontal distance is between 2.1 and 15.9 m, the tennis ball will be at 1.5 m or more above the ground.

12. a) $(20 - 0.5x)$; $(120 + 5x)$

b) $R(x) = -2.5(x - 8)^2 + 2560$

c) \$16

d) \$2560

13. a) $4x^2 - 280x + 2784 = 0$

b) 12 mm

c) 56 mm by 36 mm by 12 mm

14. a)
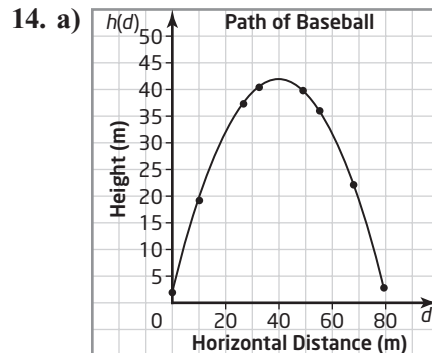
b) Estimates may vary; vertex: (40, 42), equation: $h(d) = -0.0255d^2 + 2.04d + 1.2$

c) $y = -0.252\,784\,695x^2 + 2.027\,232\,963x + 1.182\,921\,463$

d) A quadratic model fits the data well since the parabola passes through several points and the number of points above and below the parabola are equal. The parabola represents the average distances between the points.

Chapter 4 Trigonometry

Prerequisite Skills

1. $\sin A = \dfrac{8}{17}$, $\cos A = \dfrac{15}{17}$, $\tan A = \dfrac{8}{15}$

2. a) 0.7193   **b)** 0.7660
    c) 1.3763   **d)** 0.4305

3. a) 57°   **b)** 75°   **c)** 21°   **d)** 56°

4. a) 10.5 cm   **b)** 4.7 mm

5. a) 79°   **b)** 75°

6. $a = 150°$, $b = 30°$

7. a) $m = \dfrac{y - b}{x}$

    b) $b = \dfrac{a}{m}$

    c) $c = \pm\sqrt{a^2 - b^2}$

    d) $l = \dfrac{P - 2w}{2}$
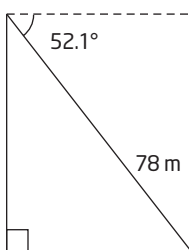
4.1 Use Trigonometry to Find Lengths

Prerequisite Skills

1. $\sin D = \dfrac{21}{29}$, $\cos D = \dfrac{20}{29}$, $\tan D = \dfrac{20}{21}$

2. a) 0.7660   **b)** 0.9563
    c) 0.4663   **d)** 0.1763

A

1. a) 6 cm   **b)** 2 m   **c)** 35 mm

2. a)   **b)** 61 m

52.1°

78 m
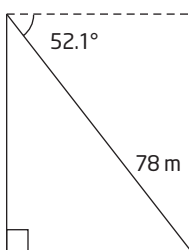
3. a) sine ratio   **b)** 97.8 m

B

4. a) 1.5 m   **b)** 2.6 m

5. a) 2224 m   **b)** 2682 m

6. a) length of DE; use the tangent ratio
       for ∠D

    b) DE = 2.87 m   **c)** 14.3 m²

C

7. 9.6 m

8. 52.9 cm

4.2 Use Trigonometry to Find Angles

Prerequisite Skills

1. a) 71°   **b)** 30°   **c)** 44°   **d)** 9°

A

1. a) 51°   **b)** 74°

2. a) ∠N = 27.1°, ∠L = 62.9°,
       $m = 83.9$ mm

    b) ∠G = 42.7°, $e = 26.2$ mm,
       $f = 35.7$ mm

3. 12.1°

B

4. 62.7°

5. 33.6°

C

6. a) 18.4°   **b)** $\dfrac{2}{5}$

7. 69°

4.3 Solve Problems Involving Two Right Triangles

Prerequisite Skills

1. 15.1 m

2.

A

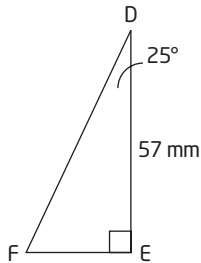**1. a)** First, use the cosine ratio to determine ∠DCE. Second, find ∠ACB by subtracting 72 and ∠DCE from 180° (supplementary angles). Finally, use the sine ratio to determine BC.

 b) 56.2 mm

2. a) First, use the tangent ratio with ∠FIG to determine IG. Second, use the cosine ratio to determine ∠GIH.

 b) 42°

B

3. 1.8 m

4. a) 5.4 cm   **b)** 90 cm

5. 18.6 km

C

6. a) 20 m, 14 m

 b) perimeter = 72 m, area = 280 m²

4.4 Investigate the Sine Law

Prerequisite Skills

1. a) $b = y - mx$

 b) $n = \dfrac{mb}{a}$

 c) $A = \sin^{-1}\left(\dfrac{a \sin B}{b}\right)$

 d) $c = \dfrac{b \sin C}{\sin B}$

2. a) ∠A = 67°   **b)** ∠F = 47°

 c) ∠K = 35°   **d)** ∠M = 47°

A

1. a) 12.5 m   **b)** 10.9 cm

2. a) 49.6°   **b)** 66.9°

3. a) ∠L ≐ 81.2°, ∠N ≐ 36.8°, n ≐ 10.2 m

 b) ∠U = 73.9°, u ≐ 69.9 mm, w ≐ 72.4 mm

 c) ∠B = 56°, a ≐ 6.7 m, c ≐ 4.3 m

4. a) Use the sine law to solve for the angle the ladder makes with the ground.

 b) 75°

5. a) Use the sine law to solve for ∠C, then subtract ∠A and ∠C from 180° to find ∠B. Use the sine law to solve for b.

 b) ∠C ≐ 32°, ∠B ≐ 78°, b ≐ 83 m

6. 4.6 m

7. Second player; e.g. use the sine law to calculate the distances between the players and the ball.

B

8. 57 cm

C

9. a) 388 m   **b)** 343 m

4.5 Investigate the Cosine Law

Prerequisite Skills

1. **a)** $a = \pm \sqrt{c^2 - b^2}$

   b) $a = \dfrac{bm}{n}$

   c) $A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right)$

   d) $a = \pm \sqrt{b^2 + c^2 - 2bc \cos A}$

2. **a)** $\angle A = 87°$ **b)** $\angle B = 110°$
   c) $\angle C = 21°$ **d)** $\angle D = 73°$

A

1. **a)** 26.6 mm   **b)** 37.9 m

2. **a)** 54.8°   **b)** 73.0°

3. **a)** RS $\doteq$ 66.4 mm, $\angle R \doteq 69.2°$,
   $\angle S \doteq 39.8°$

   b) QR $\doteq$ 4.1 cm, $\angle P \doteq 58.0°$,
   $\angle Q \doteq 81.5°$

   c) $\angle F \doteq 46.9°$, $\angle G \doteq 77.9°$,
   $\angle H \doteq 55.2°$

4. 165.1 m

B

5. Yes; e.g. the angles are 55°, 51°, and 74°.

6. **a)** First, draw and label a diagram with the given values. Second, substitute the values into the cosine law to solve for the third leg of the race. Third, use the sine law to calculate one of the other angles. Finally, subtract the two known angles from 180° to calculate the third angle.

   b) The angle at the end of the second leg is approximately 67.6°. The angle at the end of the third leg is approximately 33.4°.

c) All calculations were carried with full accuracy by using the answer key. The value for the third leg was copied with sufficient accuracy to ensure an error of less than one tenth of a degree. This value could also have been stored with full accuracy to a memory in the calculator and recalled as needed when solving for the second angle.

7. 29.2 km

C

8. 173 cm

9. 179.4 m

4.6 Make Connections With the Sine Law and the Cosine Law

Prerequisite Skills

1. **a)** $C = \sin^{-1}\left(\dfrac{c \sin B}{\sin A}\right)$

   b) $b = \dfrac{a \sin A}{a}$

   c) $B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right)$

   d) $b = \pm \sqrt{a^2 + c^2 - 2ac \cos B}$

A

1. **a)** cosine law; 27 cm
   b) sine law; 37 mm

2. **a)** sine law; 46°   **b)** cosine law; 66°

3. **a)** KM $\doteq$ 5.1 cm, $\angle K \doteq 68.3°$,
   $\angle M \doteq 62.2°$

   b) $\angle R \doteq 68.4°$, PQ $\doteq$ 19.4 mm,
   QR $\doteq$ 19.3 mm

4. **a)** cosine law; Explanations may vary.
   b) 28.7 km

B

5. a)
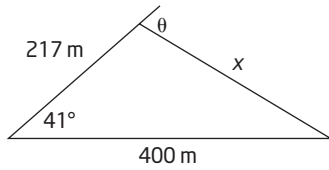
b) 275.8 m   **c)** 72.1°

6. 21 mm

7. 48.5 mm

8. 11 km

C

9. 248.8 km/h; 52.6° west of south

10. 40 cm

Chapter 4 Review

1. $e \doteq 20.2$ cm, $f \doteq 6.9$ cm

2. 8 m

3. $\angle L \doteq 52.0°$, $\angle N \doteq 38.0°$, MN $\doteq 10.2$ m

4. 20.5°

5. 29 cm

6. 62°

7. 20.5 m

8. 87.7 m

9. 82.0°

10. 32.4 mm

11. 60.3°

12. a) $\angle X \doteq 57.7°$, $\angle Y \doteq 67.6°$, $\angle Z \doteq 54.7°$

   b) $\angle F \doteq 34.0°$, $\angle E \doteq 83.0°$, $e \doteq 4.8$ cm

13. 85.5°, 41.7°, 52.8°

Chapter 4 Practice Exam

1. D   **2.** C   **3.** C   **4.** B

5. a) 90 cm   **b)** 12 m

6. a) 58°   **b)** 49°

7. 45°

8. 13 m

9. horizontal length: 11.4 m; surface length: 3.7 m

10. 38.7°

11. approximately 65.1 m

12. 196.7 m

Chapter 5 Sine Functions

Prerequisite Skills

1. **a)** 9 mm **b)** 104 cm **c)** 3 m

2. **a)** 41° **b)** 50° **c)** 55°

3. **a)** 13.5 mm **b)** 26.9 m **c)** 12.1 cm

4. **a)** 0.5000 **b)** 0.5774 **c)** 0.2588

5. **a)** −0.8660 **b)** −1.0681
   c) −5.1962 **d)** 0.4330

6. **a)**
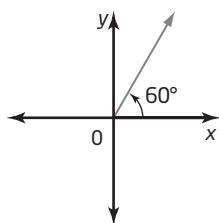
b)
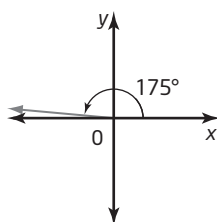
c)
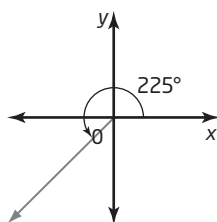
d)
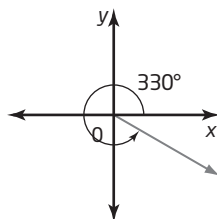
7. **a)**
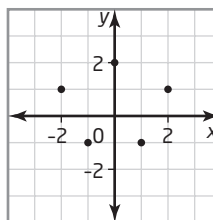
b) The points form a pentagon or a star, depending on how they are connected.

8. **a)** $y = (x + 2)^2 + 3$
   b) $y = -2x^2$
   c) $y = \frac{1}{4}(x - 4)^2 - 1$

9. Sketches may vary.
   a) Parabola with vertex $(0, 2)$, through $(2, 6)$ and $(-2, 6)$.
   b) Parabola with vertex $(-3, 1)$, through $(-2, 0)$ and $(-6, 8)$.
   c) Parabola with vertex $(2, 0)$, through $(6, -32)$ and $(-2, -32)$.

10. **a)** The domain is the set of real numbers that are greater than or equal to −5 but less than or equal to 0.
    b) The range is the set of real numbers greater than 1.

11. **a)** Parabola with vertex $(0, -3)$ through $(2, -11)$ and $(-2, -11)$. The domain is the set of real numbers. The range is the set of real numbers less than or equal to −3.
    b) Parabola with vertex $(-1, 4)$ through $(2, 5)$ and $(-2, 13)$. The domain is the set of real numbers. The range is the set of real numbers greater than or equal to 4.

5.1 Periodic Functions

Prerequisite Skills

1. a) -1.7321 **b)** -0.8660

2. a)
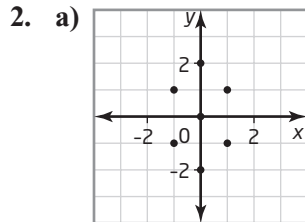
b) e.g., the points make a figure-eight pattern.

A

1. a) not periodic; function does not repeat maximum and minimum y-values.

b) periodic; a period of 1 and an amplitude of 1.5.

2. a) period = 4; amplitude = 2.5;
domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid 2 \le y \le 7\}$

b) period = 4; amplitude = 3;
domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

3. a) $f(2) = 2; f(4) = -3; f(5) = 3$

b) e.g., $f(2) = f(6) = f(10) = 2$;
$f(4) = f(0) = f(8) = -3$;
$f(5) = f(1) = f(7) = 3$

B

4. a) amplitude = 25 m, period = 30 s

b) The amplitude is the same since the size of the pool did not change. The period is less since it now takes him less time to swim a length.

5. a)
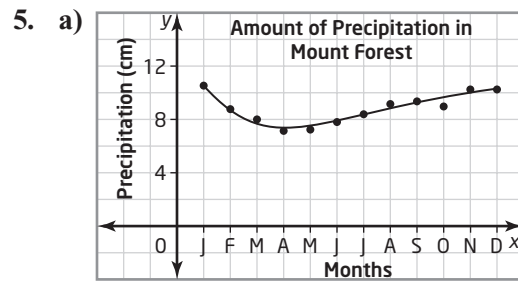
b) e.g., yes. Precipitation should be about the same every year, so the values should repeat themselves.

c) maximum: 10.5 cm, in January; minimum: 7.0 cm, in April

6. a) Predictions may vary. High temperature for August = 24°C; Low temperature for April = 4°C

b) The predictions should be fairly accurate, since the periodic trend in temperature that has been observed will continue.

7. a)
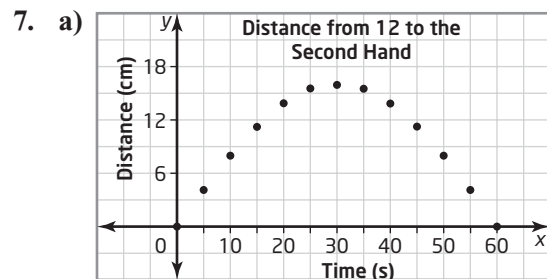
b) maximum and minimum values are evenly spaced, and the data would repeat each minute. period = 60 s, amplitude = 8.

C

8.
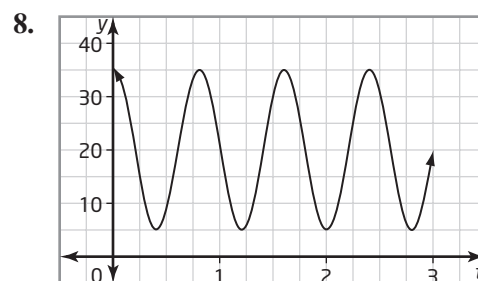
5.2 Circles and the Sine Ratio

Prerequisite Skills
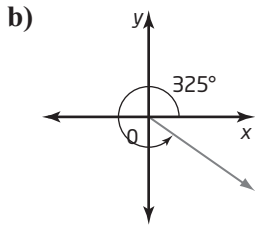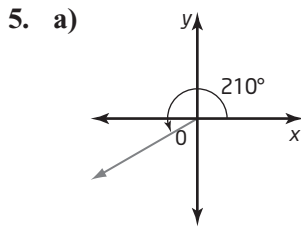
1. **a)** 5 m **b)** 7 mm

2. **a)** 55° **b)** 43°

3. **a)** 30.6 cm **b)** 7.9 m

4. **a)** 0.9659 **b)** 1.7321

5. **a)**

b)

A

1. Answers may vary.
   a) 390° and −330°
   b) 630° and −90°
   c) 330° and 690°

2. **a)** not coterminal; Their difference is not a multiple of 360°.
   b) coterminal; Their difference is 360°.
   c) coterminal; Their difference is 720°.
   d) not coterminal; Their difference is not a multiple of 360°.

3. Diagrams may vary.
   a) $r = \sqrt{45}$ or $3\sqrt{5}$; $\theta = 243.4°$ (third quadrant)
   b) $r = \sqrt{65}$; $\theta = -60.3°$ (fourth quadrant) or 299.7° (coterminal positive)

4. Diagrams may vary.
   a) $x = 0.5$, $y \doteq 0.9$
   b) $x \doteq -0.7$, $y \doteq 0.7$
   c) $x \doteq -0.6$, $y \doteq -0.8$

5. For a) and b) , $(\cos \theta)^2 + (\sin \theta)^2 = 1$.

6. **a)** 0.5774; The value is positive because the slope of the terminal arm is positive.
   b) −0.8660; The value is negative because the slope of the terminal arm is negative.
   c) −0.5000; The value is negative because the terminal point is left of the y-axis.

B

7. **a)** 54°, 414°, 126° **b)** −45°, 315°, 225°

8. **a)** 320° **b)** 255° **c)** 121°

9. **a)** 4 m **b)** 16 m **c)** 6.1 m

C

10. $\dfrac{-561}{600}$

11. 44.3 square units

5.3 Investigate the Sine Function

Prerequisite Skills

1. **a)**
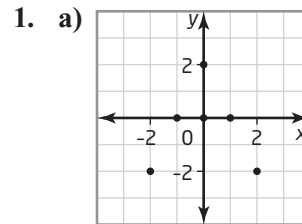
   b) The points form a capital letter A.

2. **a)** The domain is the set of real numbers that are greater than or equal to 0.
   b) The range is the set of real numbers that are greater than or equal to −2 but less than or equal to 2.

Practise

1. a), b)

x (°)	$y = \sin x$
0	0
30	0.5
60	0.8660
90	1
120	0.8660
150	0.5
180	0
210	−0.5
240	−0.8660
270	−1
300	−0.8660
330	−0.5
360	0
390	0.5
420	0.8660
450	1
480	0.8660
510	0.5
540	0
570	−0.5
600	−0.8660
630	−1
660	−0.8660
690	−0.5
720	0

e.g., the values repeat themselves after 360°, the maximum value is 1, the minimum value is −1, the values on either side of the maximum and minimum are the same.

c)
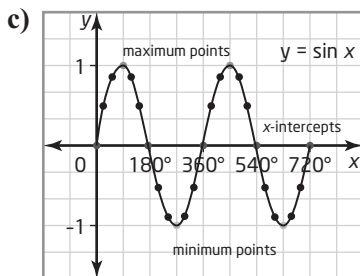
2. a)
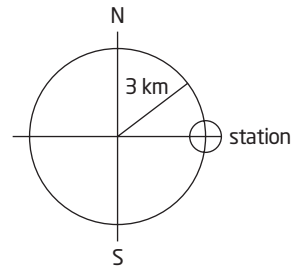
Angle (°)	Distance (km)
0	0
30	1.5
60	2.5981
90	3
120	2.5981
150	1.5
180	0
210	−1.5
240	−2.5981
270	−3
300	−2.5981
330	−1.5
360	0

b)
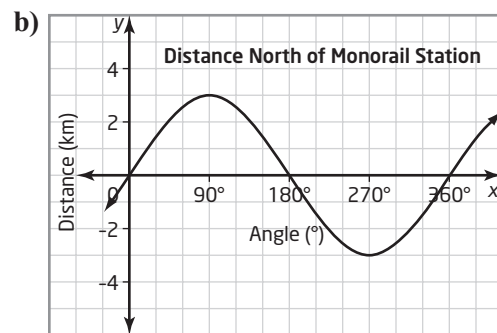
c) The period is the same, but the amplitude is 3 instead of 1.

3. a) approximately 0.77 dm

b) −0.5 dm

c) about 64° the first time, about 116° the second time.

C

4. a) $y = -1$ **b)** $y = \dfrac{1}{90}x + 2$

5. a)
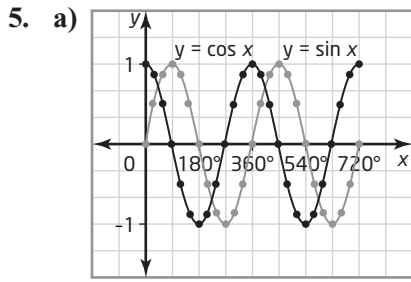
b) e.g. use the vertical line test on the graph from part a)

c) e.g., the period, amplitude, and maximum and minimum values are the same; the graph of $y = \cos x$ is the same as the graph of $y = \sin x$, just translated left 90°.

5.4 Investigate Transformations of Sine Curves

Prerequisite Skills

1. a) $y = (x - 1)^2 - 2$

   b) $y = -\dfrac{1}{3}x^2$

   c) $y = 4(x + 2)^2 + 1$

2. a) The domain is the set of real numbers less than or equal to 4.

   b) The range is the set of real numbers that are greater than −5 and less than or equal to 0.

3. Sketches may vary.

   a) domain = $\{x \in \mathbb{R}\}$;
   range = $\{y \in \mathbb{R} \mid y \geq 0\}$

   b) domain = $\{x \in \mathbb{R}\}$;
   range = $\{y \in \mathbb{R} \mid y \leq 1\}$

A

1. Sketches may vary.

   a) period = 360°; amplitude = 2;
   domain = $\{x \in \mathbb{R}\}$;
   range = $\{y \in \mathbb{R} \mid -2 \leq y \leq 2\}$

b) period = 360°; amplitude = $\dfrac{1}{4}$;

   domain = $\{x \in \mathbb{R}\}$;

   range = $\{y \in \mathbb{R} \mid -\dfrac{1}{4} \leq y \leq \dfrac{1}{4}\}$

2. a)

	$y = \sin x - 2.5$	$y = \sin x + 2.5$
Period (°)	360	360
Amplitude	1	1
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$-3.5 \leq y \leq -1.5$	$1.5 \leq y \leq 3.5$
Phase Shift	0	0
Horizontal Axis	$y = -2.5$	$y = 2.5$

b)

	$y = \sin(x + 30°)$	$y = \sin(x - 90°)$
Period (°)	360	360
Amplitude	1	1
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$-1 \leq y \leq 1$	$-1 \leq y \leq 1$
Phase Shift	30° left	90° right
Horizontal Axis	$y = 0$	$y = 0$

3. Sketches may vary.

   a) $x = 270°$ **b)** $x = 90°$

4. Sketches may vary.

   a) e.g., $x = -90°$ **b)** $x = 90°$

5. a) $y = -4 \sin x$

   b) $y = \sin x + 2$

   c) $y = \sin (x + 30°)$

6. a)

domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid -3 \leq y \leq 1\}$

b)

domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

c)
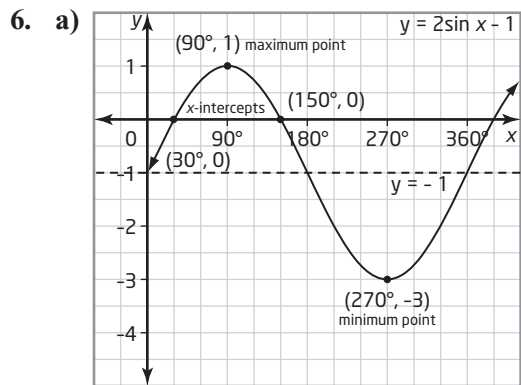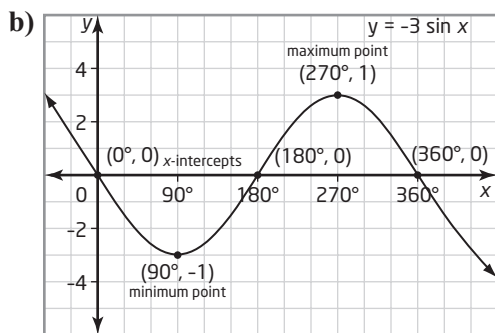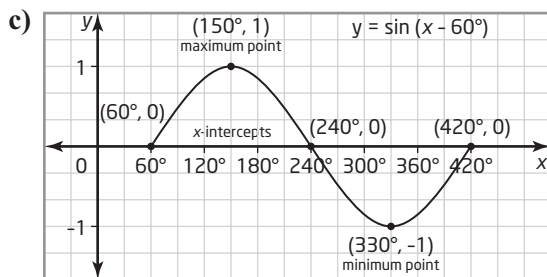
domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

B

7. period = $360°$; amplitude = 1;
phase shift = 0; domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid 54 \leq y \leq 56\}$,
equation of horizontal axis: $y = 55$

8. a) maximum = $(90°, -3)$,
minimum = $(270°, -7)$

b) maximum = $(30°, 1)$,
minimum = $(210°, -1)$

9. a) $y = 5 \sin x + 2$; The increasing
intervals are $\{0° < x < 90°\}$ and
$\{270° < x < 360°\}$, the decreasing
interval is $\{90° < x < 270°\}$.

b) $y = \sin(x - 45°)$; The increasing
intervals are $\{45° < x < 135°\}$ and
$\{315° < x < 405°\}$, the decreasing
interval is $\{135° < x < 315°\}$.

10. a)
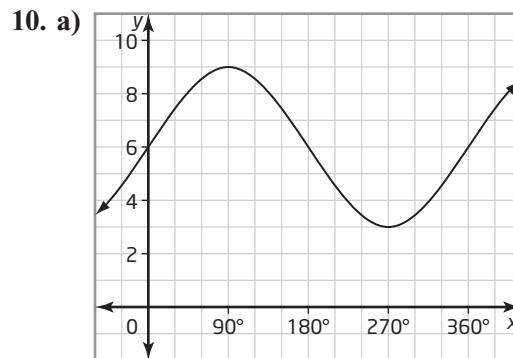
b) $y = 3 \sin x + 6$

C

11. $y = 2 \sin(x + 90°) + 10$

12. $\{..., -540°, -180°, 180°, 540°, 900°, ...\}$

5.5 Making Connections with Sine Functions

Prerequisite Skills

1. Sketches may vary.

a) domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid y \leq -5\}$

b) domain = $\{x \in \mathbb{R}\}$;
range = $\{y \in \mathbb{R} \mid y \geq 0\}$

A

1. b) 13.9 m, at about 4:30 A.M. and 4:30 P.M.

c) 10.9 m, at about 11 A.M. and 11 P.M.

d) 12.1 h

e) 7.1 h

2. a)
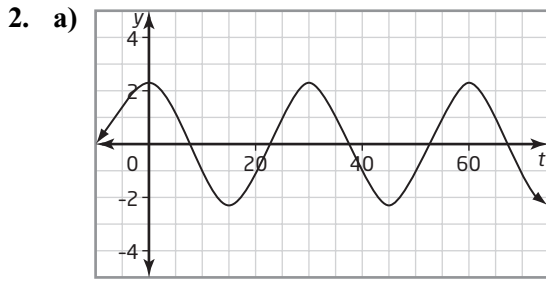
b) -2 cm, or 2 cm below the relaxed length

c) 2.4 cm

B

3. a)
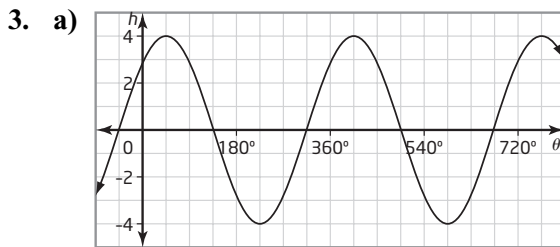
b) 2.8 cm

c) at 45° with the horizontal axis

4. a)
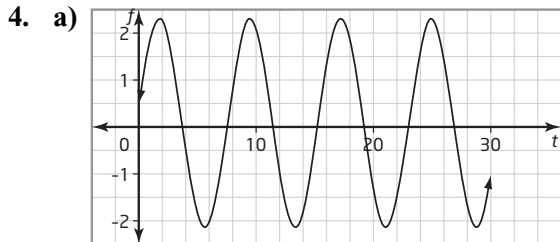
b) 2.1 cm

c) 4.2 cm

d) approximately 7.7 s

C

5. a)

b)
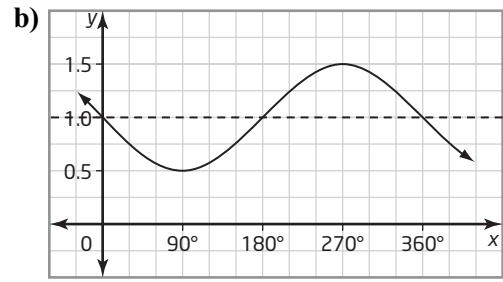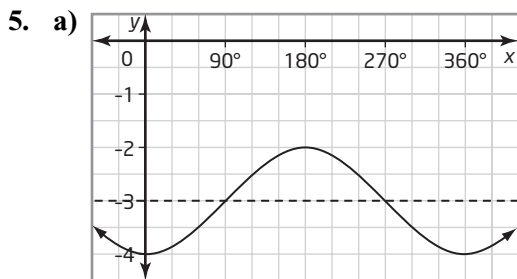
Chapter 5 Review

1. a) periodic; period $= 5$, amplitude $= 2.5$, domain $= \{x \in \mathbb{R}\}$, range $= \{y \in \mathbb{R} \mid -1 \le y \le 4\}$

b) not periodic

c) not periodic

d) period $= 2$
amplitude $= 2.5$
domain: $\{x \in \mathbb{R}\}$
range: $\{y \in \mathbb{R}\}$

2. a)
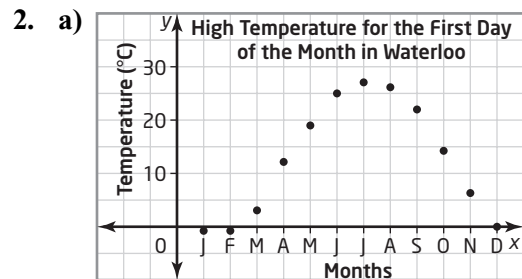
b) e.g., 16°C, 18°C

c) e.g., 13°C

3. a) $r = \sqrt{40}$ or $2\sqrt{10}$, $\theta = 71.6°$

b) $r = 5$, $\theta = 126.9°$ (second quadrant)

c) $r = \sqrt{17}$, $\theta = 256.0°$ (third quadrant)

d) $r = \sqrt{41}$, $\theta = 308.7°$ (fourth quadrant)

4. a) $x = 0.174$, $y = 0.985$

b) $x = -0.423$, $y = 0.906$

c) $x = -0.906$, $y = -0.423$

d) $x = 0.766$, $y = -0.643$

5. a) 0.5 units   **b)** 0.9848 units

   c) 0.2588 units   **d)** e.g., 210°, 330°

6. a) period = 360°, amplitude = 0.5,
phase shift = 0 , domain = $\{x \in \mathbb{R}\}$,
range = $\{y \in \mathbb{R} \mid 0.5 \le y \le 1.5\}$,
equation of horizontal axis: $y = 1$

   b) period = 360°, amplitude = 1, phase
shift = 45° right, domain = $\{x \in \mathbb{R}\}$,
range = $\{y \in \mathbb{R} \mid -3 \le y \le -1\}$,
equation of horizontal axis: $y = -2$

   c) period = 360°, amplitude = 3, phase
shift = 60° right, domain = $\{x \in \mathbb{R}\}$,
range = $\{y \in \mathbb{R} \mid -1 \le y \le 1\}$,
equation of horizontal axis: $y = 0$

7. a) $y = \sin(x + 30°) - 1$

   b) $y = 2 \sin x + 1$

   c) $y = -\sin(x + 60°)$

8. b) period = 1, amplitude = 20

   c) period would decrease since the heart
rate is faster, amplitude would not
change.

Chapter 5 Practice Exam
1. C   **2.** C   **3.** B   **4.** D

5. a) 254.8°   **b)** 315.4°   **c)** 118.4°

6. Sketches and answers may vary.

   a) −287°, 433°   **b)** −200°, 520°

   c) −100°, 620°   **d)** −20°, 700°

7. a) $x = 270°$

   b) $x = 60°, 120°$

   c) $x = 30°, 150°$

8. Sketches may vary.

9. a) $y = 2 \sin x - 0.5$; The increasing
intervals are $\{0° < x < 90°\}$ and
$\{270° < x < 360°\}$, the decreasing
interval is $\{90° < x < 270°\}$.

   b) $y = 0.5 \sin(x + 60°)$; The increasing
intervals are $\{-60° < x < 30°\}$ and
$\{210° < x < 300°\}$, the decreasing
interval is $\{30° < x < 210°\}$.

   c) $y = \sin(x - 45°) + 2$; The increasing
intervals are $\{45° < x < 135°\}$ and
$\{315° < x < 405°\}$, the decreasing
interval is $\{135° < x < 315°\}$.

10. b) maximum height = 0.54 m;
minimum height = 0.02 m

   c) 0.1064 m; 190° and 550° are
coterminal angles.

Chapter 6 Exponential Functions Answers

Prerequisite Skills

1. **a)** base 9, exponent 2; 9×9

   b) base 4, exponent 6; $4 \times 4 \times 4 \times 4 \times 4 \times 4$

   c) base 3.7, exponent 8; $3.7 \times 3.7 \times 3.7 \times 3.7 \times 3.7 \times 3.7 \times 3.7 \times 3.7$

   d) base $\left(-\dfrac{2}{7}\right)$, exponent 4; $\left(-\dfrac{2}{7}\right) \times$

   $\left(-\dfrac{2}{7}\right) \times \left(-\dfrac{2}{7}\right) \times \left(-\dfrac{2}{7}\right)$

   e) base $2a$, exponent 2; $2a \times 2a$

   f) base $\left(\dfrac{1}{5}\right)$, exponent 5; $\left(\dfrac{1}{5}\right) \times$

   $\left(\dfrac{1}{5}\right) \times \left(\dfrac{1}{5}\right) \times \left(\dfrac{1}{5}\right) \times \left(\dfrac{1}{5}\right)$

2. **a)** 6^4 **b)** $(-2)^4$ **c)** 1.8^3

3.

4. **a)** $y = -0.5x + 1$ **b)** $y = x^2 + 4x + 1$

5. **a)** $y = (x - 4)^2 + 2$

   b) $y = x^2 - 8x + 16$

6. **a)**
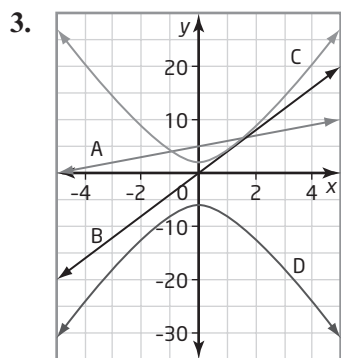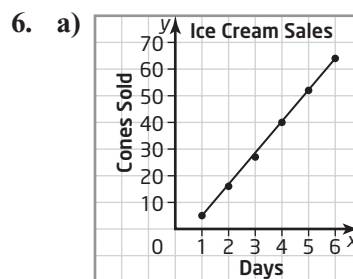
   b) linear

   c) Predictions may vary. For example: Approximately 75 cones.

7. **a)** $P = 20$ cm   **b)** $A = 8$ m

   c) $F = 14.2$ h   **d)** $V = 3054$ cm

6.1 The Exponent Rules

Prerequisite Skills

1. **a)** base 6, exponent 2

   b) base 7, exponent 5

   c) base 8.12, exponent 8

   d) base $\dfrac{1}{4}$, exponent 9

   e) base x, exponent 7

   f) base $-\dfrac{3}{8}$, exponent 4

2. **a)** 8^5 **b)** $(-5)^3$ **c)** 11^4

   d) $\left(\dfrac{2}{3}\right)^6$ **e)** 4.1^3 **f)** 6.7^{40}

3. **a)** 3×3 **b)** $(-2.5)(-2.5)(-2.5)$

   c) $\dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{3}{5}$

   d) 4.8×4.8

   e) $\left(-\dfrac{1}{6}\right)\left(-\dfrac{1}{6}\right)\left(-\dfrac{1}{6}\right)$

   f) $(-1.75)(-1.75)(-1.75)(-1.75)$

A

1. a) 21^7 **b)** 15^{11}

  c) $\left(\dfrac{3}{8}\right)^7$ **d)** $(-10)^{11}$

2. a) 17^4 **b)** 12^3 **c)** 5^{11} **d)** $(-9)^7$

3. a) 8^{15} **b)** 4^{16} **c)** 19^{54} **d)** $(-7)^{24}$

4. a) 2^{10} **b)** 6^{12} **c)** 3^6 **d)** 7^3

5. a) 9^1 **b)** 10^{11} **c)** $(-2)^{-8}$ **d)** 4^{10}

6. a) $\left(-\dfrac{2}{3}\right)^5$ **b)** $(-1.25)^2$

  c) $\left(\dfrac{1}{5}\right)^{12}$ **d)** $(-12)^6$

7. a) x^{32} **b)** $x^4 y^6$ **c)** $12x^4 y^5$
  d) $2x^5 y^4$ **e)** $5x^4 y^{20}$

8. a) $\dfrac{134\,217\,728 \times 512}{32\,768 \times 4096} = 512$

  b) $8^3 = 512$

  c) Answers may vary. For example:
   I prefer method b) because the
   calculations are simpler.

9. Answers may vary. For example:
  a) $3^{30} \times 3^6$; $3^{15} \times 3^{21}$; $3^{10} \times 3^{26}$

  b) $\dfrac{3^{38}}{3^2}$; $\dfrac{3^{40}}{3^4}$; $\dfrac{3^{50}}{3^{14}}$

  c) $(3^2)^{18}$; $(3^3)^{12}$; $(3^4)^9$
  d) $3^2 \times 3^2 \times 3^3 \times 3^3$

10. a) 81 **b)** $\dfrac{4}{49}$ **c)** $100\,000\,000$

B
11. a) Answers may vary.
   For example: $9^1 = 3^2$; $9^2 = 3^4$
  b) no; $9^3 = 3^6$

12. Answers may vary. For example:
  a) $36 = 2^2 + 2^5$
  b) $144 = 2^7 + 2^4$
  c) 788

13. a) 10^5 times more intense
  b) 10^3 times more intense

C
14. $[(2^2)^6]^3 = 2^{2\times6\times3} = 2^{36}$
  $[(2^6)^3]^2 = 2^{6\times3\times2} = 2^{36}$

15. a) $6^3 = 216$ **b)** $2^5 = 32$ **c)** $3^2 = 9$

6.2 Evaluate Powers with Integer Exponents

Prerequisite Skills

1. a) 9^8 **b)** 25^{10} **c)** 6^9

  d) $(-8)^{18}$ **e)** $\left(\dfrac{1}{4}\right)^{12}$ **f)** $\left(\dfrac{5}{3}\right)^{17}$

2. a) 11^5 **b)** 7^5 **c)** 16^6
  d) $(-3)^3$ **e)** 10^4 **f)** $(-2)^8$

3. a) 5^{18} **b)** 15^{16} **c)** 20^{45}
  d) $(-12)^{42}$ **e)** $(-4)^{40}$ **f)** $(-17)^{28}$

4. a) 14^9 **b)** 33^9 **c)** $(-5)^{-2}$
  d) $(-8)^{17}$ **e)** $(-0.75)^7$ **f)** $\left(\dfrac{2}{3}\right)^{72}$

A

1. a) $\dfrac{1}{49}$ **b)** $\dfrac{1}{16}$ **c)** $\dfrac{1}{6}$

  d) $\dfrac{1}{64}$ **e)** $\dfrac{1}{18}$ **f)** $\dfrac{1}{81}$

2. a) $\dfrac{1}{248\,832}$ **b)** $\dfrac{1}{100\,000\,000}$

  c) $\dfrac{1}{19\,683}$ **d)** $\dfrac{1}{11\,390\,625}$

3. a) 3^{-4} **b)** 2^{-6} **c)** 10^{-4} **d)** 10^{-7}

4. a) $6\frac{1}{36}$ **b)** $1\frac{1}{125}$ **c)** $\frac{15}{16}$ **d)** $\frac{8}{27}$

5. a) 8 **b)** $\frac{256}{81}$

c) 0.16 **d)** $0.000\,025$

6. a) $2^{-8};\ 4^{-4}$ **b)** $2^{-9};\ 8^{-3}$

7. a) 5^{-3} **b)** 6^{-2}
c) 3^{-8} **d)** $(-10)^{-5}$

8. a) $\left(\dfrac{2}{3}\right)^{-3}$ **b)** 0.8^2

c) $\left(\dfrac{1}{5}\right)^{8}$ **d)** $(-4)^{-6}$

9. 8×10^{-7}

10. 3^{-8}

B
11. 61.74 Hz

12. a) 6.7×10^{-3} **b)** 10^{-1}

13. a) 3.8 m/s^2 **b)** 3.7 m/s^2
c) 2.5×10^1 m/s^2

C
14. Minimum: $u = -3,\ v = 3,\ x = 2,$
  $y = -2$
  Maximum: $u = -2,\ v = 2,\ x = -3,$
  $y = 3$

15. $x = 1,\ y = 2;\ x = 2,\ y = 3;\ x = 0,\ y$ is
  any positive integer

6.3 Investigate Rational Exponents

Prerequisite Skills

1. a) $\frac{1}{27}$ **b)** $\frac{1}{64}$ **c)** $\frac{1}{256}$

d) $\frac{1}{25}$ **e)** 1 **f)** $\frac{1}{10\,000}$

2. a) $\frac{1}{15\,625}$ **b)** $\frac{1}{83\,521}$

c) $\frac{1}{7776}$ **d)** $\frac{1}{6561}$

3. a) 3^{-3} **b)** 2^{-5} **c)** 6^{-3} **d)** 5^{-2}

4. a) 2 **b)** $64\frac{1}{64}$ **c)** $\frac{35}{36}$ **d)** $\frac{3}{16}$

5. a) 25 **b)** $\frac{16}{81}$ **c)** 81 **d)** 64

6. a) $P = 36$ cm **b)** $A = 14$ m
c) $F = 18$ h **d)** $V = 754$ cm

A
1. a) 2 **b)** 6 **c)** 9
d) 5 **e)** -4 **f)** 30
g) 40 **h)** 10 **i)** -20

2. a) $\frac{1}{5}$ **b)** $\frac{9}{5}$ **c)** $\frac{4}{5}$
d) $-\frac{6}{10}$ **e)** 2 **f)** $\frac{3}{8}$

3. a) 3 **b)** 36 **c)** 4
d) 8 **e)** 32 **f)** 0.09

4. a) $\frac{27}{64}$ **b)** $\frac{8}{27}$ **c)** $\frac{4}{9}$
d) $\frac{81}{625}$ **e)** $0.000\,008$ **f)** 0.027

5. a) 5 **b)** $\frac{4}{25}$ **c)** $\frac{27}{64}$ **d)** $\frac{125}{27}$

6. a) 2.5271 **b)** 3.681 **c)** 0.427 **d)** 0.127

7. a) $14^{\frac{1}{2}}$ **b)** $14^{\frac{5}{3}}$

8. a) 11.2 **b)** 192.4

9. a) 29.47 years   **b)** 164.81 years

   c) 223 days

10. a) $\dfrac{153}{22}$ **b)** $\dfrac{12}{2}$

C

11. $x = -2, y = 3$

12. $x = 0.42$

6.4 Model Data with Exponential Functions

Prerequisite Skills

1. a)
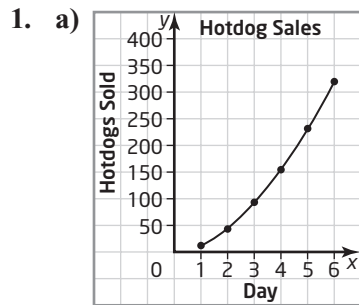
   b) quadratic

   c) Predictions may vary. For example: approximately 425 hotdogs.

2. a) first differences are not constant; second differences are 2

   b) quadratic; second differences are constant

A

1. a) exponential   **b)** linear   **c)** quadratic

2. A quadratic   **B** exponential

   C linear   **D** exponential

3. a)

Month	Uncovered Surface Area of Pond (m²)
0	2560
1	1280
2	640
3	320
4	160
5	80

b), c)
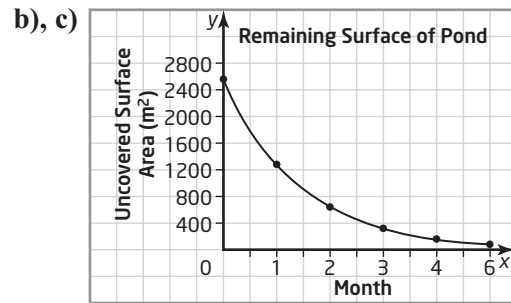
d) The graph appears to be exponential.

e) Yes. The ratios are 0.5 so this is an exponential relation.

f) Answers may vary. For example: By day 18, only 0.01 m² or 1 cm² of the pond's surface will not be covered in algae.

4. a)

Day	Distance Travelled (km)
1	150
2	75
3	37.5
4	18.75
5	9.375

b)
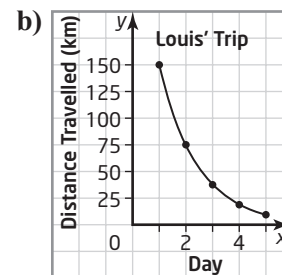
c) Answers may vary. For example: On day 9. I continued the pattern in the table and added the distance travelled.

d) Answers may vary. For example: On day 10, he has travelled 299.71 km and is 30 m away from his destination. It is unlikely he will continue travelling half the distance each day at this point.

5. a)
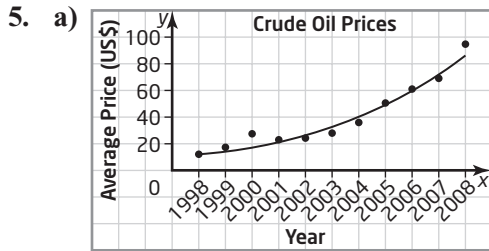
b) Graph is increasing and seems to be exponential.

6. No. Cooling is modelled by an exponential relation. The ratio between 40°C and 32°C is 0.8 so the tub would only cool to approximately 32°C × 0.8 = 25.6°C.

C

7. 4 days

6.5 Exponential Functions and Their Properties

Prerequisite Skills

1. a) quadratic **b)** linear **c)** exponential

2. a) Rani **b)** Wilson **c)** Enrico

A

1. a)
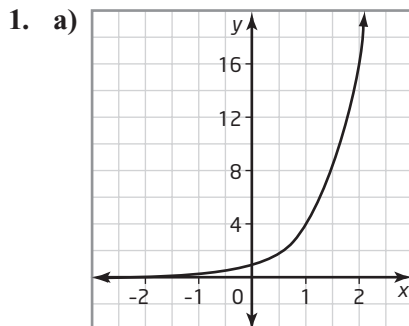
b) For every x-value, there is only one y-value.

c) $f(x) = 4^x$

d) domain:$\{x \in \mathbb{R}\}$
range: $\{y > 0, y \in \mathbb{R}\}$

e) no x-intercept; y-intercept is 1

f) As you move along the graph from left to right, the graph is always increasing.

g) The asymptote is the x-axis.

2. a) $f(x) = 9^x$ **b)** $h(x) = (0.3)^x$

c) $x = 0; y = 1$

3. A: $h(x) = (0.3)^x$; B: $f(x) = 9^x$;
C: $g(x) = 3^x$

4. $f(x) = 5.5^x$

5. $f(x) = 3.5^x$

6. a) $0 < a < 1$; the function is decreasing

b) $a > 1$; the function is increasing

c) $0 < a < 1$; the function is decreasing

B

7. a) $\dfrac{95}{100}; \dfrac{90.25}{100}$

b) $I = 0.95^d$, where I is light intensity and d is depth, in 8-m intervals.

c) 112 m

8. $I = 0.7^d$, where I is light intensity and d is depth, in 20-m intervals.

9. a) It is 1000 times as hazardous.

b) When $P = 8$, $H = 10^8 = 100\ 000\ 000$, when $P = 2$, $H = 10^2 = 100$. Therefore, an asteroid with a Palermo scale value of 8 is 1 000 000 times as hazardous as one with a scale value of 2.

C

10. 10^{14} or 100 000 000 000 000

6.6 Compare Linear, Quadratic, and Exponential Functions

Prerequisite Skills
1. **a)** linear   **b)** exponential

    c) quadratic

A
1. a), b), d) linear; c), f) quadratic;
   e) exponential

2. **a)** A: linear; B: exponential; C: quadratic

    b) D: exponential; E: quadratic; F: linear

3. **a)** quadratic   **b)** exponential

    c) exponential   **d)** linear

4. **a)** exponential, ratio is 0.6

    b) linear, first differences are 0.2

    c) none, first and second differences and ratios are not constant

    d) quadratic, second differences -2

B
5. exponential, ratio is 1.06

6. **a)** linear, first differences are -1236

    b) the regulator

7. Answers may vary. For example: Quadratic, because the formula for surface area contains values that are squared.

C
8. They are the same.

9. **a)** linear   **b)** quadratic   **c)** exponential

6.7 Exponential Growth and Decay

Prerequisite Skills
1. **a)** $f(x) = 2.25^x$ **b)** $h(x) = (0.6)^x$

2. C: $h(x) = (0.6)^x$; D: $f(x) = 2.25^x$;
   E: $g(x) = 1.5^x$

A
1. 8.9%

2. approximately 14 200 years

3. 44 days; if the bacterial culture covered 100% of the surface area after 46 days and it doubles every 2 days, it must have covered 50% of the surface area 2 days before.

4. **a)**
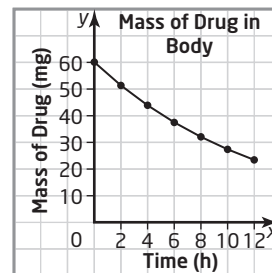
    b), c) 9 h

    d) 53 h

5. **a)**
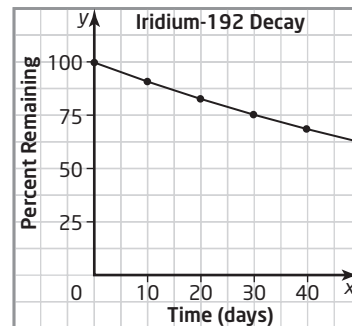
    b) approximately 30 days

    c) approximately 39%

    d) 74 days. Answers may vary. For example: It is the denominator in the exponent of the function.

6. **a)** 7000 bacteria

    b) 7507 bacteria; 8050 bacteria; 11 417 bacteria

c)
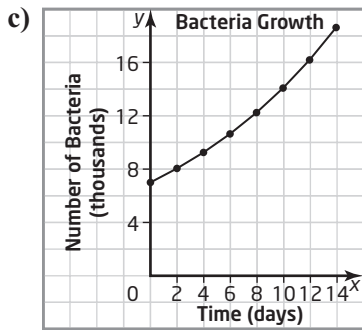
d) approximately 10 days

e) The domain is restricted to $0 \le t \le 14$. This is the time during which Portia conducted the experiment. The range is restricted to $7 \le N \le 19$. These are the numbers describing the size of the bacterial culture during the experiment.

7. a) $M(t) = 500\left(\dfrac{1}{2}\right)^{\frac{t}{30}}$, where M

represents the mass of cesium-137 remaining and t represents the time, in years.

b) approximately 420 ug

c) approximately 93 years

8. a) 19 g; 7 g; 3 g

b) $M(d) = 40\left(\dfrac{1}{2}\right)^{\frac{d}{8}}$, where M

represents the mass of iodine-131 remaining after d days.

c) 51 days

9. a) $A(n) = 750(0.688)^n$ **b)** 6 times

10. approximately 0.3%

11. a) $P = P_0(3)^{\frac{d}{8}}$ **b)** 1.116

Chapter 6 Review

1. a) 9^4 **b)** 4^8 **c)** x^5 **d)** n^6

2. a) $\dfrac{1}{36}$ **b)** 8   **c)** 126

3. a) 2.68   **b)** 0.044 72   **c)** 3.3437

4. 1.87 years

5. a) approximately 22 807 racoons

b) approximately 80 racoons

6. a)

x	y
-4	0.260308205
-3	0.364431487
-2	0.510204082
-1	0.714285714
0	1
1	1.4
2	1.96
3	2.744
4	3.8416
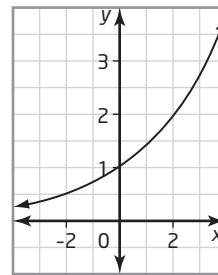
b) Domain: $\{x \in \mathbb{R}\}$; range: $\{y > 0, y \in \mathbb{R}\}$; no x-intercept, y-intercept is 1. As you move along the graph from left to right, the graph is always increasing. Asymptote at x-axis.

7. a) exponential; ratio is 6

b) quadratic; second differences are 2

c) linear; first differences are -0.33

8. a)

Mass of Drug in Body
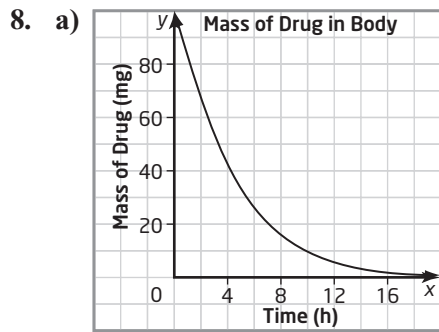
b) approximately 3 h

Chapter 6 Practice Exam

1. B **2.** C **3.** D

4. B **5.** B **6.** 6 m

7. $\dfrac{1}{11\,019\,960\,576}$

8. a) $N(g) = 6(2.5)^{g-1}$, where g is the number of generations (starting with one) and $N(g)$ is the number of fish in each generation

b) approximately 586 fish

9. $y = 8^x$

10. a) exponential, ratio is 0.93

b) Yes. The leak is best modelled by an exponential relation, therefore the sac is leaking.

11. a)

Time (min)	Number or People "Infected"
15	3
30	9
45	27
60	81
75	243

b) Sketches may vary depending on scale used.

c) As the time increases in 15-min intervals, the number of people 'infected" increases exponentially, therefore the relationship can be modelled using an exponential function.

d) $N(t) = 3t$, where t is the number of 15-min intervals and $N(t)$ is the number of people "infected".

e) $N(1) = 3$, $N(4) = 81$

f) approximately 2 h

Chapter 7 Compound Interest

Prerequisite Skills

1. a) 0.04   **b)** 0.079   **c)** −0.182

   d) −0.0225   **e)** 0.003   **f)** −0.053 75

2. Estimates may vary. For example:

   a) 90   **b)** 700   **c)** 2000

   d) 1000   **e)** 200   **f)** $150

   g) $25 000   **h)** $4000

3. a) 0.002 071   **b)** 0.001 808   **c)** 0.002 083

   d) 0.0515   **e)** 0.0012   **f)** 0.000 351

4. a) 36 months   **b)** $\frac{20}{52}$ years

   c) $\frac{82}{365}$ years   **d)** 65 weeks

   e) 2.5 months   **f)** 9 months

5. a) 104   **b)** 4   **c)** 42

   d) 4   **e)** 1.5   **f)** 0.25

6. a) 36   **b)** 0.0039   **c)** 5839.9166

   d) 21.4335   **e)** 1.0444

7. From least to greatest: $2000(0.50)^6$, $2000(0.65)^6$, $2000(0.75)^4$, $2000(1.002)^7$, $2000(1.08)^3$

8.

x	$y = 1.3^x$
0	1
1	1.3
2	1.69
3	2.197
4	2.856
5	3.713
6	4.827
7	6.275
8	8.157
9	10.604
10	13.786

Sketches may vary.

9. a) Yes; each term is approximately 2.5 times the previous.

   b) No; the common ratios vary from about 1.5 to 2.6.

7.1 Explore Simple Interest and Compound Interest

Prerequisite Skills

1. a) 0.061   **b)** 0.218   **c)** −0.059

   d) −0.0175   **e)** 0.006   **f)** −0.106 25

2. a) 0.000 081   **b)** 0.020 857   **c)** 0.002 767

   d) 0.004 25   **e)** 0.002 875   **f)** 0.007 042

3. a) 0.5 years   **b)** 48 months

   c) $\frac{300}{365}$ years   **d)** about 22 weeks

   e) 7 months   **f)** 14.5 months

4. a) 156   **b)** 10   **c)** 81

   d) 24   **e)** 2   **f)** 1.5

A

1. a) $870; $6870   **b)** $720; $3720

   c) $58; $8058   **d)** $41.44; $941.44

2. a) $49 811.53; $11 811.53

   b) $29 904.99; 5904.99

   c) $7658.61; $158.61

   d) $4415.25; $415.25

3. a) $84

   b) $3584, $3668, $3752, $3836, $3920

   c) linear

   d) Graphs may vary. $y = 84x$

   e) Graphs may vary. $y = 3500 + 84x$

4. $19.91

5. a) 6.1%   **b)** $6366

c) −5.25%; $5685

6. a) $726.11   **b)** $827.33

7. Option A: $1589; **Option B:** $1567.38;
take option A since it returns more money
in 3 years

8.

Months	Value ($)
0	1200
6	1216.80
12	1233.84
18	1255.43
24	1277.40
30	1306.14
36	1335.53

7.2 The Compound Interest Formula

Prerequisite Skills

1. a) 0.0015   **b)** 0.012 292

c) 0.048   **d)** 0.000 059

e) 0.013 286   **f)** 0.0023

2. a) 1.1729   **b)** 1.0033

c) 1.0171   **d)** 1.0396

3.

x	$y = 1.7^x$
0	1
1	1.7
2	2.89
3	4.913
4	8.352
5	14.199
6	24.138
7	41.034
8	69.758

Sketches may vary.

A

1.

	Number of Compounding Periods Per Year	Compounding Periods, n
a)	1	15
b)	2	60
c)	4	6
d)	12	78
e)	365	44

2.

	Number of Compounding Periods Per Year, n	Interest Rate Per Compounding Period, i (expressed as a decimal)
a)	1	0.09
b)	2	0.002 675
c)	4	0.004 25
d)	12	0.008 542
e)	365	0.000 616

3. a) $i = 0.06$ and $n = 5$;
$A = 2000(1 + 0.06)^5$

b) $i = 0.024$ and $n = 12$;
$A = 90\ 000(1 + 0.024)^{12}$

c) $i \doteq 0.006\ 042$ and $n = 36$;
$A = 45\ 000(1 + 0.006\ 042)^{36}$

4. a) $5737.62   **b)** $5744.41

c) $5747.87   **d)** $5750.20

B

5. a) There is a common ratio of 1.045.

b) 1.045

c) The growth factor equals $1 + i$.

6. a) $981.19 **b)** $9034.26 **c)** $22 126.33

7. a) $3671.05 **b)** $171.05 **c)** $2528.75

8. a) $0.40

   b) For example, because the interest rate is so low, you barely make any money.

9. a) $258.77 **b)** $3.96

   c) For example, the interest he is earning in the account is much less than the interest he is being charged on the credit card.

10. a) A **b)** $115.91

C
11. a) about 1.0831 **b)** $34 405.32

7.3 Present Value

Prerequisite Skills
1. a) 0.002 792 **b)** 0.334 615
   c) 0.000 034 **d)** 0.022
   e) 0.008 533 **f)** 0.008 286

2. a) 0.4823 **b)** 0.4423 **c)** 0.6274
   d) 0.006 975 **e)** 0.4989 **f)** 0.7599
   g) 0.6041 **h)** 0.5026 **i)** 0.7473
   j) 0.9200 **k)** 0.9631 **l)** 0.6768

3. From least to greatest: $5000(1.12)^{-6}$, $5000(1.06)^{-4}$, $5000(1.025)^{-3}$, $5000(1.004)^5$, $5000(1.35)^7$

A
1. a) 2007.92 **b)** 36 771.73
   c) 2154.32 **d)** 115.06

2. a) $417.50 **b)** $1250.13
   c) $29 995.78 **d)** $3625.73

3. a) $3680.51 **b)** $5.91

4. a) $293.34 **b)** $656.37
   c) $7633.34 **d)** $42 879.26

B
5. Using 182 days: $2938.56

6. $6245.55

7. $8041.32

8. a) $53 394.81 **b)** $946 605.19

9. $22 443.35

C
10. a) $7657.54

   b) Alisha will need to invest $8185.91 in the GIC for 3 years to make $10 000; if she does not invest the $5000 now, she should save about $265.49/month.

11. $9852.50

7.4 Solve Financial problems Using Technology

Prerequisite Skills
1. a) 0.013 571 **b)** 0.006 233
   c) 0.0055 **d)** 0.000 150
   e) 0.000 212 **f)** 0.004

2. a) 1.1649 **b)** 0.7058
   c) 1.3023 **d)** 0.9313
   e) 1.1003 **f)** 0.5537

3. From least to greatest: $10\ 000(0.75)^3$, $10\ 000(1.09)^2$, $10\ 000(1.065)^4$, $10\ 000(0.9)^{-5}$, $10\ 000(1.08)^{10}$

A
1. $16 554.19

2. $32 866.85

3. $4799.52

4. $16 919.74

5. a) about 5.37 years **b)** 6

6. a) about 4.78 years **b)** 58 months

7. about 1.8%

8. a) about 106%   **b)** about 0.29%

B
9. a) about 1.55 years **b)** about 0.976%

10. a) about 6.74 years **b)** about 2.048%

11. $1815.79

12. a) about 3.42%; 3.42%

   b) about 3.39%; 1.695%

   c) about 3.38%; 1.69%

   d) about 3.37%; 1.685%

13. $512.14

14. $4134.54

15. a) 4%   **b)** about 2.07%

C
16. a) Tamara: $77 009.08; Lucas: $68 537.81

   b) Tamara: $5006.06; Lucas: $4414.85

   c) $17 183.54

Chapter 7 Review
1. a)

Year	Total Interest ($)
1	18.80
2	37.60
3	56.40
4	75.20
5	94.00
6	112.80

   b) Sketches may vary.

c)

Year	Total Value ($)
1	418.80
2	437.60
3	456.40
4	475.20
5	494.00
6	512.80

   d) Sketches may vary.

2. A bank offers a 6-year term deposit that pays 4.5% annual interest, compounded annually.

   a) $520.91

   b)
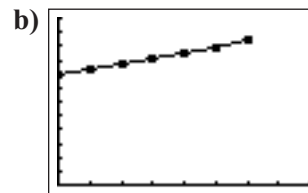
   c) $y = 400(1.045)^x$

3. compound interest; there is a common ratio of 1.075

4. Answers may vary. For example:

   a) Both are increasing.

   b) The simple interest increases at a constant rate, the compound interest increases faster as time goes on.

5. a) $i = 0.02$, $n = 10$

   b) $i = 0.014\ 25$, $n = 6$

   c) $i = 0.006$, $n = 108$

   d) $i = 0.000\ 534$, $n = 30$

6. a) $8141.34   **b)** $9797.44

   c) $47 700.39   **d)** $863.73

7. Answers may vary. For example:

   a) a $7000 investment at 6.8% interest compounding annually

   b) a $7000 investment at 13.6% annual interest compounding semi-annually

8. a) Yes, there is a common ratio of about 1.035.

b) No, it is 3.5%/month.

9. $15 500

10. No, it would have a value of $226.69; Answers may vary: Consider having most of the money now versus waiting another 2 months.

11. a) about 5.65%   **b)** about 57.76 years

c) about 5.54%   **d)** about 8.84 years

12. $15 500

Chapter 7 Practice Exam
1. C   **2.** C   **3.** B   **4.** D   **5.** B

6. a) $68.75   **b)** $568.75

7. Answers may vary. For example, a $1600 investment at 5.25% interest per compounding period.

8. a) $40 516.33

b) $1410.52

c) $524.41

9. simple interest: $3405
compound interest: $3423.50

10. $781.46

11. a) $17.23   **b)** $52.57
c) $107.91   **d)** $227.45

12. Answers may vary. For example, debts at high interest rates increase very quickly.

13. a)


```
0    3    6    9   12   15   18   21   24   27   30   33  36 months
|----+----+----+----+----+----+----+----+----+----+----+----|--->
Principal           7500(1.02125)⁴     7500(1.02125)⁸      Amount
= 7500                                                     = 7500(1.02125)¹²
```

```
0  2  4  6  8  10 12 14 16 18 20 22 24 26 28 30 32 34 36 months
|--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--|--->
Principal        7500(1.0137)⁶      7500(1.0137)¹²      Amount
= 7500                                                  = 7500(1.0137)¹⁸
```
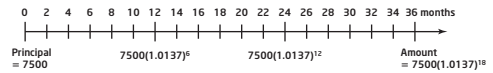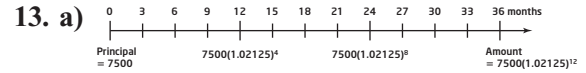
b) Loan A, it returns more money.

14. a) $5281

b) Answers may vary. Yes, because I cannot find a better deal on 3-year GICs.

15. a) about 9.9 years   **b)** about 867 years
c) about 7.4%   **d)** 8%

16. a) Pippa's: simple interest (graph is linear); Adrianne: compound interest (graph is exponential)

b) Pippa's: $5900; Adrianne: $6312

c) Pippa's: 4.5% per year simple interest Adrianne: 6% per year, compound interest

d) Adrianne's; it increases in values much faster.

Chapter 8 Annuities

Prerequisite Skills

1. **a)** $270 **b)** $1080 **c)** $5400

   d) $10 800 **e)** $12 420 **f)** $13 770

2. **a)** less; $4694.84 **b)** less; $2073.07

   c) greater; $310.25 **d)** less; $720. 64

3. **a)**
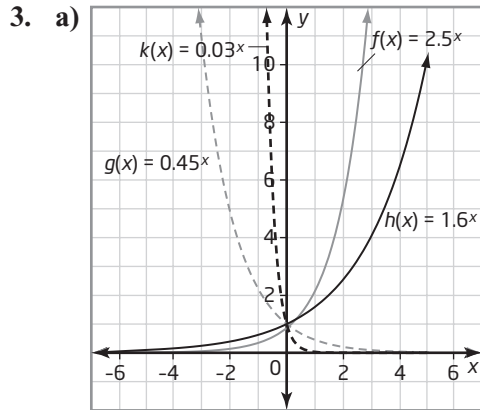
   b) All functions pass through $(0, 1)$ because $a^0 = 1$.

4.

x	**a)** $f(x) = 100 + 100(1.07)^x$	**b)** $g(x) = 100(1.07)^x$
0	200.00	100
1	207.00	107
2	214.49	114.49
3	222.50	122.50
4	231.08	131.08
5	240.26	140.26

$f(x) = 100 + 100(1.07)^x$
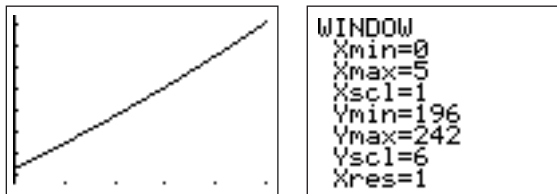
$g(x) = 100(1.07)^x$
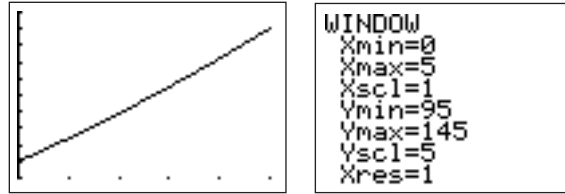
5. **a)** $58.50 **b)** $116.10

   c) $0.84 **d)** $36.40

6. **a)** $i = 0.038$; $n = 8$; $1347.66

   b) $i = 0.007\ 292$; $n = 120$; $5978.18

   c) $i = 0.000\ 021$; $n = 182.5$; $200.75

   d) $i = 0.017$; $n = 20$; $14\ 009.38

8.1 Future Value of an Ordinary Simple Annuity

Prerequisite Skills

1. **a)** $1440 **b)** $5760 **c)** $14 400

   d) $36 000 **e)** $43 200

2. **a)** $i = 0.014\ 78$, $n = 4$

   b) $i = 0.011\ 25$, $n = 72$

   c) $i = 0.000\ 033$, $n = 243.3$

   d) $i = 0.014\ 75$, $n = 28$

3. **a)** $848.35 **b)** $20 139.89

   c) $50.40 **d)** $9794.16

A

1., 2. Answers may vary slightly due to rounding.

i)
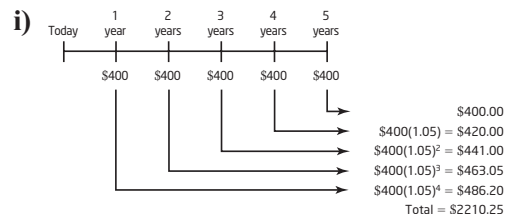
ii)

Month	Amount ($)
1	$650(1.003\ 25)^{11} = 673.62$
2	$650(1.003\ 25)^{10} = 671.44$
3	$650(1.003\ 25)^{9} = 669.26$
4	$650(1.003\ 25)^{8} = 667.09$
5	$650(1.003\ 25)^{7} = 664.93$
6	$650(1.003\ 25)^{6} = 662.78$
7	$650(1.003\ 25)^{5} = 660.63$
8	$650(1.003\ 25)^{4} = 658.49$
9	$650(1.003\ 25)^{3} = 656.36$
10	$650(1.003\ 25)^{2} = 654.23$
11	$650(1.003\ 25)^{1} = 652.11$
12	$650(1.003\ 25)^{0} = 650.00$
	Total = 7940.94

iii)

Month	Amount ($)
1	$5000(1.06)^{11} = 9\ 491.49$
2	$5000(1.06)^{10} = 8\ 954.23$
3	$5000(1.06)^{9} = 8\ 447.39$
4	$5000(1.06)^{8} = 7\ 969.24$
5	$5000(1.06)^{7} = 7\ 518.15$
6	$5000(1.06)^{6} = 7\ 092.60$
7	$5000(1.06)^{5} = 6\ 691.13$
8	$5000(1.06)^{4} = 6\ 312.38$
9	$5000(1.06)^{3} = 5\ 955.08$
10	$5000(1.06)^{2} = 5\ 618.00$
11	$5000(1.06)^{1} = 5\ 300.00$
12	$5000(1.06)^{0} = 5\ 000.00$
	Total = 84 349.69

iv)

Month	Amount ($)
1	$2000(1.005\ 625)^{3} = 2033.94$
2	$2000(1.005\ 625)^{2} = 2022.56$
3	$2000(1.005\ 625)^{1} = 2011.25$
4	$2000(1.005\ 625)^{0} = 2000.00$
	Total = 8067.75

3. i) $2210.25 **ii)** $7940.95

iii) $84 349.71 **iv)** $8067.75

4. a) PMT $= 400$, $i = 0.05$, $n = 5$,
FV$= \$2210.25$

b) PMT $= 650$, $i = 0.003\ 25$, $n = 12$,
FV$= \$7940.95$

c) PMT $= 5000$, $i = 0.06$, $n = 12$,
FV$= \$84\ 349.71$

d) PMT $= 2000$, $i = 0.005\ 625$, $n = 4$.
FV$= \$8067.75$

5. a), b) i) $3846.61

ii) $11 444.24

iii) $4565.57

B

6. a) $2787.50 **b)** $20 033.46

7. a) $2742.38

b) No, it will be slightly more than double because the investment from the first month will be accumulating interest.

c) $5490.18

8. a) $7936.30, $12 595.78, $20 386.53

b) $15 300

c) After 17 years, $12 396.17 of interest.

d) $6745.04

9. $8238.22

C

10. a) $370.32

b) approximately 70 weeks

c) $2.45 more for $9.45

8.2 Present Value of a Simple Ordinary Annuity

Prerequisite Skills

1. a) $i = 0.0475$, $n = 3$

b) $i = 0.0225$, $n = 8$

c) $i = 0.0475$, $n = 3$

d) $i = 0.0045$, $n = 7$

2. a) $6019.32 **b)** $49 280.86

c) $41 040.83 **d)** $3712.87

A

1., 2.

i)

$3000(1.075)^{-1} = \$2\,790.70$
$3000(1.075)^{-2} = \$2\,596.00$
$3000(1.075)^{-3} = \$2\,414.88$
$3000(1.075)^{-4} = \$2\,246.40$
$3000(1.075)^{-5} = \$2\,089.68$
Total $= \$12\,137.66$
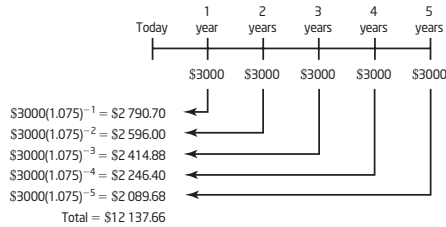
ii)

Month	Amount ($)
1	$420(1.0114\ldots)^{-1} = 415.26$
2	$420(1.0114\ldots)^{-2} = 410.57$
3	$420(1.0114\ldots)^{-3} = 405.94$
4	$420(1.0114\ldots)^{-4} = 401.35$
5	$420(1.0114\ldots)^{-5} = 396.82$
6	$420(1.0114\ldots)^{-6} = 392.35$
7	$420(1.0114\ldots)^{-7} = 387.92$
8	$420(1.0114\ldots)^{-8} = 383.54$
	Total $= 3193.75$

iii)

Month	Amount ($)
3	$600(1.016)^{-1} = 590.55$
6	$600(1.016)^{-2} = 581.25$
9	$600(1.016)^{-3} = 572.10$
12	$600(1.016)^{-4} = 563.09$
15	$600(1.016)^{-5} = 554.22$
18	$600(1.016)^{-6} = 545.49$
21	$600(1.016)^{-7} = 536.90$
24	$600(1.016)^{-8} = 528.45$
	Total $= 4472.02$

iv)

Month	Amount ($)
6	$130(1.026)-1 = 126.71$
12	$130(1.026)-2 = 123.49$
18	$130(1.026)-3 = 120.37$
24	$130(1.026)-4 = 117.32$
30	$130(1.026)-5 = 114.34$
	Total $= 602.23$

3. Answers may vary slightly due to rounding.

   i) $12 137.65   **ii)** $3193.75

   iii) $4472.05   **iv)** $602.22

4. a) i) PMT $= 3000$, $i = 0.075$, $n = 5$

     ii) PMT $= 420$, $i = 0.011\,417$, $n = 8$

     iii) PMT $= 600$, $i = 0.016$, $n = 8$

     iv) PMT $= 130$, $i = 0.026$, $n = 5$

  b) i) $12 137.65   **ii)** $3193.75

    iii) $4472.05   **iv)** $602.22

5. a) $17 716.67   **b)** $13 912.42

  c) $53 632.10

B

6. a) $9898.50   **b)** $8731.57

  c) $1166.93

7. $520 317.46

8. a) $9842.19   **b)** $1416.81

9. $16 624.91

10. a) $263 031.91   **b)** $480 000

  c) 216 968.09

C

11. a) PMT $= \mathrm{PV}\left[\dfrac{i}{1 - (1 + i)^{-n}}\right]$

  b) $533.09   **c)** $6397.08

12. $1134.03

8.3 Payments and Total Interest

Prerequisite Skills

1. a) greater; 303.62

  b) greater; 109 160.54

  c) greater; 17 472.4

  d) greater; 233 631.81

  e) greater; 148 148 148.15

  f) less; 10 432.45

2. a) present value   **b)** future value

  c) future value   **d)** present value

3. Answers may vary slightly due to rounding.

a) $871.67   b) $23 407.78

c) $1605.01   d) $3425.68

A

1. a) $PV = 16\ 000$, $i = 0.022\ 5$, $n = 16$

b) $PV = 21\ 000$, $i = 0.005\ 208$, $n = 60$

c) $PV = 35\ 000$, $i = 0.089\ 5$, $n = 7$

d) $PV = 1900$, $i = 0.006\ 083$, $n = 18$

2. a) $1201.87   **b)** $408.43

c) $6930.85   **d)** $111.76

3. a) $1201.87   **b)** $408.43

c) $6930.85   **d)** $111.76

4. a) $PMT = FV\left[\dfrac{i}{(1 + i)^{-n} - 1}\right]$

b) $4092.44

5. a) $7520   **b)** $6600

c) $40 976   **d)** $6336

6. a) $300 007.20   **b)** $523

c) $778.20   **d)** $1872

7. a) $842 038.11   **b)** $379.27   **c)** $259.20

B

8. a) $263.82   **b)** $9497.52   **c)** $997.52

9. a) $180.91   **b)** $341.84

10. a) $481.62; $419.93

b) $17 338.32; $15 117.48; the second sedan ends up costing less

c) $238.32; $1617.48

d) Answers may vary; Rebecca should consider safety ratings, fuel consumption, predicted reliability, depreciation rates, performance, and comparative features.

11. a) $180.77   **b)** $2373.46

12. a) $579.23

b) Answers may vary; Assume that Trevor's predictions with respect to vehicle prices and interest rates are valid.

13. $187.45

14. a) $1282.34   **b)** $384 702   **c)** $204 802

d) $s = 179\ 900(1.12)^n$, where s represents the future selling price, in dollars, and n represents the number of years after the purchase is made.

e) $252 746.55   **f)** $72 846.55

C

15. Answers may vary.

8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity

Prerequisite Skills

1. a) present value   **b)** future value

c) future value   **d)** present value

e) present value   **f)** future value

g) future value   **h)** present value

2. a) $681.39   **b)** $102 375.25

c) $3102.97   **d)** $5568.41

e) $16 935.77   **f)** $10 647.69

g) $97 709.78   **h)** $1270.53

A

1. a) $876.84   **b)** $459.15   **c)** $320.33

d) $251.23   **e)** $210.02   **f)** $149.11

2. a) $522.08   **b)** $1019.60   **c)** $1531.88

d) $2059.04   **e)** $2601.20   **f)** $4314.56

3. a) 166   **b)** 77   **c)** 47   **d)** 38

4. a) $367.79   **b)** $370.53

    c) $377.42   **d)** $381.60

5. a) $1240.44   **b)** $1339.08

    c) $1587.12   **d)** $1737.60

B

6. a) option 1   **b)** $5.59

7. Answers may vary due to rounding.

    a) 21 months   **b)** $176.22

    c) $4229.28   **d)** $32.16

8. a) i) $376 583.46   **ii)** $246 691.74

    iii) $137 285.89

    b) i) $41 400   **ii)** $62 000

    iii) $60 000

    c) i) $335 183.46   **ii)** $184 691.74

    iii) $77 285.89

9. Answers may vary due to rounding.

    a) $184 264.20   **b)** $161 102.40

    c) $160 896.00   **d)** $147 787.64

    e) $36 476.56

10. a) $193.99   **b)** $1311.52

    c) $44.69   **d)** $1295.52

    e) When payments are made more frequently, the remaining principal is reduced more frequently, and therefore the interest is calculated on a smaller amount.
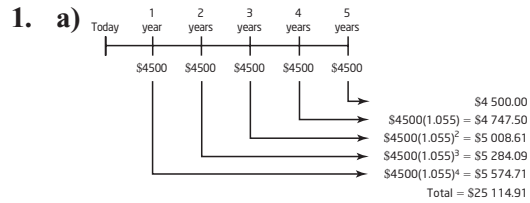
11. a) $301.40   **b)** $21 700.80

    c) $4700.80   **d)** $823.20

    e) $243.79

C

12. a) $611.62   **b)** $3669.72

    c) $16 924.88   **d)** $1423.72

13. a) $14 950.33   **b)** $12 228.03

    c) $11 528.02

Chapter 8 Review

1. a)

b)

Year	Amount ($)
1	$4500(1.0055)^4 = 5\ 574.71$
2	$4500(1.0055)^3 = 5\ 284.09$
3	$4500(1.0055)^2 = 5\ 008.61$
4	$4500(1.0055)^1 = 4\ 747.50$
5	$4500(1.003\ 25)^0 = 4\ 500.00$
	Total $= 25\ 114.91$

    c) $25 114.91   **d)** $25 114.91

2. a) $13 235.12   **b)** $12 725.64

3. a)
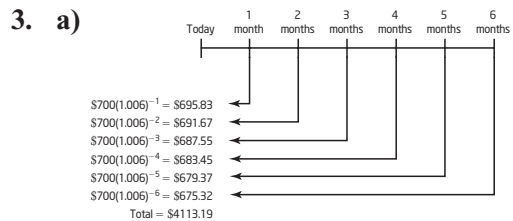
b)

Month	Amount ($)
1	$700(1.006)^{-1} = 695.83$
2	$700(1.006)^{-2} = 691.67$
3	$700(1.006)^{-3} = 687.55$
4	$700(1.006)^{-4} = 683.45$
5	$700(1.006)^{-5} = 679.37$
6	$700(1.006)^{-6} = 675.32$
	Total $= 4113.19$

    c) $4113.19   **d)** $4113.19

4. a) $6585.52   **b)** $89 186.28

5. a) $i = 0.006$; $n = 60$

    b) $i = 0.0325$; $n = 30$

6. **a)** PMT = $457.60

    b) PMT = $3160.90

7. **a)** $4456.00   **b)** $34 827.00

8. **a)** $219.97

    b) i) $5279.28   **ii)** $220.72

9. **a)** $1 299 338; $1 298 436
    b) semi-annual option

10. **a) i)** $543 031.74   **ii)** $275 999.79

    b) i) $70 500   **ii)** $87 000

    c) i) $472 531.74   **ii)** $188 999.79

    d) Because the growth of every individual
    deposit is an exponential function, the
    longer a deposit is allowed to earn
    interest, the faster it grows in value.

Chapter 8 Practice Exam

1. D   2. B   3. B   4. C

5. $148.78

6. **a)** $520 000   **b)** $373 791.51

7. yes

8. **a)** monthly option   **b)** $83 464.61

9. **a)** $1577.29   **b)** $94 637.40
    c) $1640.31   **d)** $488 311.80
    e) $v = 250\ 000(1.09)^n$, where v represents
    the value of the condominium in
    dollars after n years.
    f) $910 620; $2 155 770

10. **a)** $8535.15   **b)** $585.27
    c) $8779.05   **d)** $10 260
    e) $1480.95