Chapter 2 Factor Quadratic Expressions

Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 62–63 of the *Functions and Applications 11* textbook.

Key Features of Quadratic Functions

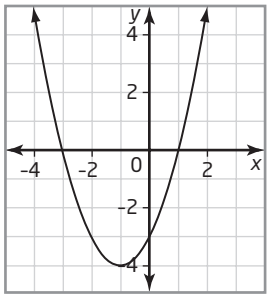
1. For each function

- create a table of values
- find the first and second differences
- decide whether the function is linear, quadratic, or neither
- graph the function using your table of values

a) $f(x) = 3x + 9$ **b)** $g(x) = x^3 + 2$ **c)** $h(x) = x^2 - x - 4$

2. For the given parabola, identify

- the direction of opening (upward or downward)
- the coordinates of the vertex
- the equation of the axis of symmetry
- the x-intercepts
- the y-intercept



3. Use the information to sketch a graph of each parabola.

a) vertex $(-1, -8)$, x-intercepts at -4 and 2 **b)** vertex $(-3, -4)$, y-intercept = 1

Expand and Simplify Algebraic Expressions

4. a) Select an x-tile and two unit tiles to model the expression $x - 3$.

b) Create two additional groups of $x - 3$ using algebra tiles.

c) Assemble the groups to form a rectangle with width 3 and length $x - 3$.

d) Use this model to explain why $3(x - 3) = 3x - 9$.

5. Expand. Simplify if necessary.

a) $2(x + 5)$ **b)** $-4(a + 1)$ **c)** $(y + 6)\left(\frac{1}{2}y\right)$ **d)** $-3(2k^2 + 4k - 4)$

e) $-\frac{3}{4}(5b^2 - 12b + 6)$ **f)** $(h - 3)(h - 2)$ **g)** $(n + 5)(n - 3)$ **h)** $6s(s - 1) - 2s$

Factors of Polynomials

6. Factor. Use algebra tiles to find the factors if necessary.

a) $12y + 3$ **b)** $6x^2 - 2x$ **c)** $5n^2 + 15n + 20$ **d)** $3(m-6) - 7(m-6)$

e) $q^2 - 5q - 6$ **f)** $a^2 + 4a - 21$ **g)** $b^2 - 7b + 10$ **h)** $g^2 - g - 42$

2.1 Quadratic Functions: Exploring Forms

Textbook pp. 64–75

Prerequisite Skills

1. Graph each quadratic function and determine:

- the direction of opening (upward or downward)
- the coordinates of the vertex
- the equation of the axis of symmetry
- the x-intercepts
- the y-intercept

a) $f(x) = x^2 + 2x - 6$ **b)** $g(x) = -x^2 - 6x + 9$ **c)** $h(x) = x^2 - x - 6$

2. Use the given information to sketch a graph of each parabola.

a) vertex (0, 12), x-intercepts at -4 and 4

b) vertex (6, 5), y-intercept $= -2$

A

1. For each quadratic function, identify

- the values of a, b, and c
- the direction of opening
- whether the vertex is a maximum or a minimum
- the y-intercept

a) $f(x) = -x^2 + 9x + 20$

b) $y = 3x^2 - 9x + 6$

2. For each quadratic function, identify

- the direction of opening
- the x-intercepts
- the coordinates of the vertex, and whether it is a maximum or a minimum value
- the axis of symmetry
- the y-intercept

a) $f(x) = (x - 2)(x - 8)$

b) $y = -5(x - 1)(x - 3)$

c) $f(x) = (x - 2)^2 + 3$

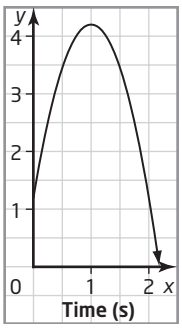
d) $y = -2(x - 4)^2 - 6$

3. Graph each function in questions 1 and 2. Identify the intervals for which the function is

- positive or negative
- increasing or decreasing

B

4. This graph shows the height-time relationship of a bottle rocket.



a) From what height above ground was this bottle rocket launched?

b) What is the approximate maximum height reached?

c) Estimate the hang time of the rocket.

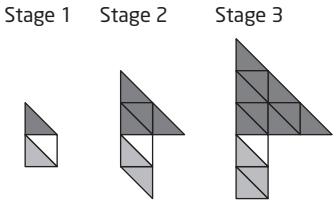
5. a) Extend the graph in question 4 to the left of the y-axis to locate a second x-intercept. Does this point have any meaning in the given situation? Explain.

b) Find an approximate value for the vertex. Then determine the vertex form of a quadratic function that models this graph.

c) Suppose that the launching platform were raised. Describe how this would affect

- the y-intercept of the corresponding graph
- the location of the vertex and the maximum height achieved
- the x-intercept of the corresponding graph and the hang time of the rocket

6. Look at the following growing pattern.



a) Draw the next two stages.

b) Describe how the number of white triangles changes from one stage to the next.

c) Repeat part b) for the number of light grey and the number of dark grey triangles.

d) Copy and complete the table.

Stage	1	2	3	4	5
White Triangles					
Light Grey Triangles					
Dark Grey Triangles					
Total Triangles					

e) Which of these relations is

- constant? • linear? • quadratic?

Justify your answers.

Use a table of first and second differences. Then verify your answers by using a graphing calculator.

7. a) Build or draw a growing pattern of squares that represents the quadratic relationship $T(n) = n^2 + 5n - 6$. Use different colours to illustrate how different parts of the pattern are growing.

b) Explain how the coefficients a, b, and c are related to your model.

C

8. a) Create a quadratic function that

- is decreasing for $x < -3$ and increasing for $x > -3$
- has a minimum y-value of -2
- has a y-intercept of 16

b) Is your answer to part a) the only correct answer? If no, find another function that satisfies these conditions, if yes, explain.

9. a) What can you tell about a function in the form $f(x) = a(x - r)^2(x - s)$?

b) Use a graphing calculator to check your prediction in part a) using the function $f(x) = 0.5(x + 2)^2(x - 6)$. Was your prediction correct? Identify the intervals for which this function is increasing or decreasing.

2.2 Quadratic Functions: Comparing Forms

Textbook pp. 76–85

Prerequisite Skills

1. Use algebra tiles to expand $3(5x - 2)$.

2. **a)** Use algebra tiles to model each of the following expressions:

i) $x + 5$ **ii)** $x + 3$

b) Using algebra tiles, build a rectangle with length $x + 5$ and width $x + 3$.

3. Expand and simplify.

a) $(x + 5)(x + 4)$ **b)** $(y - 1)(y - 7)$

c) $4h(h + 3) - 6$ **d)** $2n - (n + 3)(n - 2)$

A

1. Expand and simplify. Use algebra tiles or diagrams to help you.

a) $3(x + 5)$ **b)** $(y + 4)6$

c) $2a(a - 1)$ **d)** $(b + 7)(b + 4)$

e) $(n + 2)(3n + 1)$ **f)** $(q + 4)^2$

2. Expand and simplify by applying the distributive property.

a) $2(x - 10)$ **b)** $-3(b + 7)$

c) $4w(8 - w)$ **d)** $(y - 9)(y - 5)$

e) $(3n - 3)(n + 2)$ **f)** $(a - 4)^2$

g) $(3x + 8)^2$ **h)** $(2q - 3)(2q + 3)$

3. Write each quadratic function in standard form and identify the y-intercept. Then, sketch a graph of the function. Check your graphs using a graphing calculator.

a) $f(x) = (x - 1)(x + 6)$

b) $y = (x - 3)(x - 7)$

c) $y = 5(x + 1)(x - 5)$

d) $f(x) = (x + 3)^2 - 1$

e) $f(x) = (x - 4)^2 - 6$

f) $y = -3(x - 2)^2 + 5$

4. Expand and simplify the expression $3x(x + 2) + (x + 4)(x + 5)$ using algebra tiles or diagrams.

a) Illustrate the first product, $3x(x + 2)$. What is the expanded form of this expression?

b) Illustrate the second product, $(x + 4)(x + 5)$. What is the expanded form of this expression?

c) Add your answers from parts a) and b) to find the simplified form of $3x(x + 2) + (x + 4)(x + 5)$.

5. Use the method from question 4 to expand and simplify each expression.

a) $5(x + 3) + 2(x + 4)$

b) $7y(y + 1) + 3y(y + 3)$

c) $(4k + 5)(k + 6) + (k + 3)(2k + 2)$

d) $9(t + 2) - 4(t + 1)$

6. Expand and simplify.

a) $5(x - 3) + 3(x - 9)$

b) $6(2 - a) - 4(a + 1)$

c) $(y + 4)(y + 5) - (y - 2)(y - 2)$

d) $(n + 7)(n - 3) - (n - 1)(n - 2)$

e) $(b + 9)^2 - (b - 3)(b - 5)$

f) $2(s - 3)^2 - 4(s + 2)(s - 5)$

B

7. A firework rocket is set off over a lake. The equation relating height and time for the rocket is $h = -14.7(t - 0)(t - 6)$, where h is the height, in metres, and t is the time, in seconds.

a) In which form is this quadratic function expressed? Justify your answer.

b) Find the zeros, or t-intercepts, of this function. Explain their significance.

c) Write the function in standard form.

> Standard form is $y = ax^2 + bx + c$. Use the distributive property.

d) Find the coordinates of the vertex. Is this vertex a maximum or minimum? Explain the significance of this point.

> Half of the distance between the x-intercepts represents the x-coordinate of the vertex. Substitute this x-value into either of the equations to find y.

e) Graph the function.

8. A company wants to sell sweatshirts. Research suggests that if each sweatshirt costs \$40, the company will sell 150 sweatshirts. For every dollar increase in price, 2 fewer sweatshirts will be sold.

a) Explain why revenue is given by $R(x) = (40 + x)(150 - 2x)$, where $R(x)$ is the revenue and x is the increase in price, both in dollars.

b) Use technology to graph the function and find the y-intercept. What is the significance of the y-intercept?

c) Write the function in standard form. Explain how the standard form can be used to verify your answer to part a).

d) Find the selling price of a sweatshirt that will maximize revenue. Find the total expected revenue if sweatshirts are sold at that price.

9. Refer to question 8. Research suggests that for every dollar decrease in price, the company will sell 5 more sweatshirts.

a) Write the equation that expresses revenue as a function of x, where x is the price reduction in dollars.

b) If the company decreases the price, what is the price of a sweatshirt that will maximize revenue? What is the maximum revenue at that reduced selling price?

c) Compare the results from part b) to your results from question 9, part d). Should the company increase or decrease the price? Explain why.

C

10. The sum of the base and height of a triangle is 30 cm.

a) What is the maximum possible area of this triangle?

b) What are the lengths of the base and height for the triangle?

2.3 Factor Quadratic Expressions of the Form $ax^2 + bx + c$

Textbook pp. 88–97

Prerequisite Skills

1. **a)** Select algebra tiles to represent the polynomial $3x + 6$.

b) Arrange these tiles to form a rectangle.

c) What are the expressions for the length and width of the rectangle?

d) Explain how this model illustrates how to factor $3x + 6$.

2. Use algebra tiles to find the factors of each polynomial.

a) $2x + 8$ **b)** $4y + 16$

3. Factor out the greatest common factor.

a) $3r + 12$ **b)** $5x^2 + 2x$ **c)** $7q^2 - 14q$

d) $2n^2 + 6n - 32$ **e)** $8(x - 3) + 5(x - 3)$ **f)** $m(m - 4) - 3(m - 4)$

4. Factor each trinomial.

a) $y^2 - 11y + 18 = (y - \square)(y - \square)$

b is negative and c is positive, so the factors of 18 are both negative.

b) $b^2 + 3b - 70 = (b + \square)(b - \square)$

b is positive and c negative, so the greater factor of 72 is positive, and the other is negative.

c) $x^2 - 10x + 24 = (x - \square)(x - \square)$

b is negative and c positive, so both factors of 24 are negative.

d) $q^2 - 7q - 18 = (q - \square)(q + \square)$

Both b and c are negative, so the greater factor of 18 is negative and the other is positive.

A

1. Factor. Use algebra tiles or diagrams if necessary.

a) $x^2 + 10x + 16$ **b)** $y^2 + 10y + 9$

c) $k^2 + 8k + 15$ **d)** $b^2 - 3b - 54$

e) $n^2 + 5n - 24$ **f)** $a^2 - 9a - 36$

g) $t^2 + t - 56$ **h)** $m^2 - 13m + 42$

2. Pick one expression from question 1 and check your result by expanding.

Apply the distributive property.

3. Factor. Use algebra tiles if necessary.

a) $2x^2 + 9x + 9$ **b)** $4k^2 + 16k + 7$

c) $6y^2 + 7y + 2$ **d)** $3b^2 - 2b - 16$

e) $2a^2 - 15a + 25$ **f)** $3t^2 + t - 14$

g) $5m^2 - 6m + 8$ **h)** $8n^2 + 2n - 3$

i) $7p^2 + 19p + 10$ **j)** $9q^2 - 15q - 6$

4. Pick two expressions from question 3 and check your results by expanding.

5. Write each quadratic function in factored form. Then, identify the x-intercepts.

a) $t(x) = x^2 + 16x - 17$

b) $y = x^2 + 18x + 45$

$= (x + \square)(x + \square)$

Both b and c are positive, so the factors of 45 are positive.

c) $y = x^2 - x - 72$

$= (x - \square)(x + \square)$

b is negative and c negative, so the greater factor of 72 is negative, and the other is positive.

d) $y = 2x^2 + 9x + 10$

$= 2x^2 + \square + \square + 10$

$= (\square) + (\square)$

$= \square(\square) +$

$\square(\square)$

$= (\square)(\square)$

$9x$ can be expressed as $4x + 5x$.

e) $y = 3x^2 + 4x - 15$

f) $f(x) = 5x^2 - 13x + 6$

g) $g(x) = 7x^2 - 10x - 8$

h) $h(x) = 4x^2 + 14x + 12$

B

6. Refer to question 5. For each function,

a) identify the y-intercept

b) use the x-intercepts, the y-intercept, and the axis of symmetry to sketch the graph of the quadratic function.

7. Factor each polynomial. Look for common factors first.

a) $3x^2 + 15x + 18$ **b)** $5m^2 + 20m + 15$

c) $2q^2 - 2q - 12$ **d)** $8a^2 - 28a + 20$

8. Pick one polynomial from question 7 and check your result by expanding.

9. Create your own factoring question.

a) First, build or draw the rectangle that corresponds to your polynomial. Then, find the factors.

b) Trade questions with a classmate. Solve each other's factoring question and check your answers.

10. Explain why the following polynomials are not factorable, using algebra tiles, diagrams, or algebraic reasoning.

a) $x^2 + 7x + 9$ **b)** $3x^2 + 8x + 2$

11. a) Create two quadratic expressions that are factorable. Factor the expressions.

b) Create two quadratic expressions that are not factorable. Explain why they are not factorable.

c) Reorder the expressions you created and trade them with a classmate. Factor the expressions that are factorable and identify the expressions that are not factorable.

C

12. Fully factor each expression.

a) $(x - 2)x^2 - (x - 2)x - (x - 2)30$

b) $(x + 1)x^2 - (x + 1)4$

c) $(x^2 + x - 6) - (2x - 4) + (x^2 - 2x)$

d) $(2x^2 - 32) - (x^2 + x - 20)$

13. Factor, assuming that the coefficients can have fractional values.

a) $x^2 + \frac{5}{6}x + \frac{1}{6}$ **b)** $x^2 + \frac{5}{2}x + 1$

c) $x^2 - \frac{1}{3}x - \frac{2}{9}$ **d)** $x^2 - \frac{9}{25}$

2.4 Select and Apply Factoring Strategies

Textbook pp. 98–107

Prerequisite Skills

1. Calculate.

a) 5^2 **b)** 9^2 **c)** 6^2 **d)** 12^2

e) 13^2 **f)** 7^2 **g)** 11^2 **h)** 8^2

2. Factor out the greatest common factor.

a) $4x^2 + 7x$ **b)** $9r + 27$ **c)** $2q^2 - 18q$

d) $3n^2 + 12n - 6$ **e)** $m(m - 5) - 2(m - 5)$ **f)** $2(x - 1) + 4x(x - 1)$

3. Factor each trinomial.

a) $x^2 - 6x + 7$ **b)** $y^2 - 2y + 15$ **c)** $b^2 + 6b - 40$ **d)** $q^2 - 4q - 21$

e) $n^2 + 4n - 32$ **f)** $p^2 - 15p + 26$ **g)** $a^2 + 3a - 54$ **h)** $m^2 - 15m + 44$

A

1. Factor each perfect square trinomial. Use algebra tiles or diagrams, if necessary.

a) $y^2 + 10y + 25$

b) $x^2 + 8x + 16$

c) $9q^2 + 48q + 64$

> Take the square root of a and the square root of c. Since b is positive, the operation between the values will addition.

d) $p^2 - 2p + 1$

> Take the square root of a and the square root of c. Since b is negative, the operation between the values will subtraction.

e) $16z^2 + 24z + 9$

f) $4a^2 + 36a + 81$

g) $25m^2 - 70m + 49$

h) $4k^2 + 40k + 100$

i) $9b^2 + 30b + 25$

j) $36n^2 + 24n + 4$

2. Factor each difference of squares.

a) $x^2 - 9$ **b)** $9k^2 - 100$

c) $z^2 - 225$ **d)** $4a^2 - 169$

e) $36m^2 - 64$ **f)** $81b^2 - 144$

g) $49q^2 - 25$ **h)** $121p^2 - 1$

3. Factor each polynomial by first selecting a suitable strategy. If it cannot be factored, write "not factorable."

a) $b^2 + 9b + 14$ **b)** $2a^2 - 25$

c) $16x^2 + 32x + 16$ **d)** $9m^2 + 1$

e) $4k^2 - 169$ **f)** $4n^2 + 35n + 49$

g) $25p^2 - 30p - 7$ **h)** $r^2 + 16r + 64$

4. Consider the polynomial $14x^2 - 7x - 42$.

a) Factor out the greatest common factor.

b) Inspect the remaining trinomial factor. Can it be factored further? If so, factor it using an appropriate strategy.

c) Use the method of decomposition to factor this polynomial, without first factoring.

d) Inspect the binomial factors. Can either be factored further? Use an appropriate strategy to factor the binomial factors, where possible.

e) Compare your answers to parts b) and d). Explain what you notice.

f) Which of these strategies did you find easier to apply? Explain your answer.

5. Consider the polynomial $81y^2 - 180y - 100$.

a) Factor the polynomial by applying the pattern for a perfect square trinomial.

b) Use the method of decomposition, or another method of your choice, to factor this polynomial.

c) Compare these results. Explain what you notice.

d) Which strategy did you find easier to apply? Explain your answer.

6. Consider the quadratic function $y = x^2 - 25$.

a) Use algebraic reasoning to show that

i) the x-intercepts are opposite values

ii) the axis of symmetry is the y-axis

b) Sketch a graph of this function.

7. Consider the quadratic function $y = x^2 - 3x$.

a) Use algebraic reasoning to show that

i) the x-intercepts are 0 and 3

ii) the y-intercept is equal to one of the x-intercepts.

b) Sketch a graph of this function.

8. Identify a factoring strategy that you could use to factor each polynomial. Identify polynomials for which more than one strategy is possible.

a) $6x^2 - 96$

b) $9q^2 + 30q + 63$

c) $2p^2 - 8p + 8$

d) $160z^2 + 560z + 490$

e) $14b^2 + 42b + 28$

f) $a^2 - 289$

g) $4m^2 - 36m - 19$

h) $3k^2 + 6k - 72$

i) $7z^2 - 700$

j) $18n^2 + 18n + 4$

k) $32k^2 - 50$

l) $3a^2 + 18a + 15$

B

9. Fully factor each polynomial in question 8 by applying your chosen strategies.

10. Is $x^2 + x + 1$ factorable? Use algebra tiles, diagrams, graphical or algebraic reasoning to justify your answer.

C

11. For what integer values of a will a quadratic function of the form $y = (x + a)^2 + a$ have

a) two x-intercepts?

b) one x-intercept?

c) no x-intercepts?

Use algebraic and graphical reasoning to support your answers.

2.5 Solve Quadratic Equations by Factoring

Textbook pp. 108–113

Prerequisite Skills

1. Factor.

a) $y = x^2 - 81$

b) $y = x^2 - 2x + 15$

c) $y = 6x^2 + 13x + 7$

d) $y = 4x^2 - 16x + 16$

e) $y = 3x^2 + 15x + 18$

f) $y = 25 - 4x^2$

g) $y = 7x^2 + 15x + 2$

h) $y = 16x^2 - 49$

2. **a)** Sketch a graph of each quadratic equation in question 1.

b) Which form of the quadratic function was easiest to use when sketching the graph: the simplified form or the factored form?

A

1. Find the roots of each equation. Verify your answers.

a) $x(x + 2) = 0$

b) $(y - 1)(y + 4) = 0$

c) $(a - 7)(3a + 2) = 0$

d) $(2c + 3)(4c - 1) = 0$

e) $(5k + 10)(6k + 7) = 0$

f) $(2n - 6)(n + 9) = 0$

2. Find the roots of each equation. Verify your answers.

a) $y^2 + 20y + 19 = 0$

b) $k^2 + 8k - 33 = 0$

c) $p^2 - 5p + 36 = 0$

d) $2a^2 + 6a - 20 = 0$

e) $4n^2 - 121 = 0$

f) $5b^2 - 13b + 6 = 0$

g) $8h^2 - 10h - 25 = 0$

h) $3m^2 - 27 = 0$

3. A rock is thrown from the top of a cliff on an uncharted planet. The height of the rock above the planet's surface as a quadratic function is given by $h(t) = -3t^2 + 9t + 30$, where t is the time, in seconds, and $h(t)$ is the height of the rock, in metres.

a) Find the zeros of the function and explain their significance. Reject any inadmissible solutions.

b) Graph the function. Which parts of the graph have no meaning in this situation?

B

4. **a)** Create a quadratic function in standard form that has zeros at -3 and 11.

b) Explain how you produced this function.

c) Is your function the only possible correct answer to part a)? Explain.

5. Consider $x^2 - 36 = 0$.

a) Solve this equation by factoring.

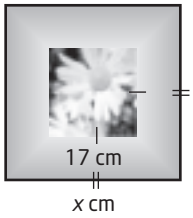
b) Solve this equation by graphing the corresponding quadratic function.

c) Solve this equation by rearranging and taking square roots.

d) Do these methods all produce the same solutions? Explain.

e) Which method or methods do you prefer? Explain why.

6. A square picture with a side length of 17 cm is surrounded by a square mat. The mat has a side length of x cm.



a) Write an algebraic expression, in standard form, for the area of the mat.

b) Use the difference of squares pattern to find the value of x for which the area of the mat is

i) 195 cm^2 **ii)** 495 cm^2

7. Refer to question 3.

a) Predict what would happen to the zeros of this function if the rock were thrown:

- with a lesser initial velocity
- from a lower altitude
- from the surface of a planet having a weaker force of gravity

Justify your reasoning in each case.

b) In which of these scenarios would you expect one of the zeros to be inadmissible? Explain your conclusions.

8. The curved mirror of a headlight is modelled by $y = \frac{1}{8}x^2 - 3x$, where x is the distance from the left side of the headlamp, in centimetres, and y is opposite of the distance from the lens, in centimetres.

a) Find the zeros of this function and explain their significance.

b) What is the width of the headlamp? Explain how you know.

c) What is the depth of the headlamp? Explain how you know.

d) Are any negative values for y admissible in this situation? Explain your reasoning.

C

9. If a quadratic equation is in the form $ax^2 + bx + \frac{b^2}{4a} = 0$, where a, b, and c are integers, how many roots will it have? Use a CAS to explore several cases. Write a brief report of your findings.

Chapter 2 Review

Work with a classmate to verify your answers. Use technology where appropriate.

2.1 Quadratic Functions: Exploring Forms

Textbook pp. 64–75

1. Consider the quadratic function $f(x) = x^2 + 3x - 10$.

a) In which form is this function?

b) Identify the values of a, b, and c. Does this parabola open upward or downward?

c) What is the y-intercept?

d) Create a table of values and graph the function. Use enough values to get a good idea of the shape of the graph.

e) Estimate the values of the x-intercepts and the vertex.

f) Is the vertex a maximum or a minimum? Explain.

2. Consider this quadratic function, $f(x) = -(x - 1)(x + 9)$.

a) In which form is this function? Does this parabola open upward or downward?

b) What are the x-intercepts?

c) Use the x-intercepts to find the coordinates of the vertex. Is the vertex a maximum or a minimum? Explain.

d) What is the axis of symmetry?

e) What is the y-intercept?

f) Graph the function.

g) Identify the intervals for which the function is

i) positive or negative

ii) increasing or decreasing

3. Consider this quadratic function. $y = 3(x - 2)^2 - 7$.

a) In which form is this function? Does this parabola open upward or downward?

b) What are the coordinates of the vertex? Is the vertex a maximum or a minimum? Explain.

c) What is the axis of symmetry?

d) What is the y-intercept?

e) Graph the function.

f) Identify the intervals for which the function is increasing or decreasing.

4. a) Build or draw a growing pattern of squares that represents the quadratic relationship $T(n) = n^2 + 3n + 2$. Use different colours to illustrate how different parts of the pattern are growing.

b) Explain how the coefficients a, b, and c are related to the model you designed.

2.2 Quadratic Functions: Comparing Forms

Textbook pp. 66–85

5. Consider each quadratic function.

i) $y = (x + 3)(x + 7)$

ii) $g(x) = (x - 4)^2 + 5$

a) Build or draw an algebra tile model to represent the quadratic function.

b) Use the model to express this function in standard form.

c) Identify the coefficients a, b, and c.

d) Identify the y-intercept.

6. Expand and simplify each expression.

a) $5(x + 2) + 3(x - 7)$

b) $-2(m + 1) + (3m - 4)$

c) $(6y - 4)(y + 2) - (y + 8)(2y - 3)$

d) $(2w + 4)^2 - (5w - 5)^2$

7. **a)** Create a quadratic function expressed in

i) factored form

ii) vertex form

b) Write each quadratic function in standard form.

2.3 Factor Quadratic Expressions of the Form $ax^2 + bx + c$

Textbook pp. 88–97

8. Factor each expression. Use algebra tiles or diagrams to illustrate, if necessary.

a) $x^2 + 5x + 4$

b) $3x^2 + 7x + 2$

9. Factor, if possible. If not possible, write "not factorable" and explain why.

a) $y^2 + 12y + 27$

b) $k^2 + 10k + 16$

c) $r^2 + 2r - 10$

d) $5n^2 + 9n + 2$

e) $2v^2 - 5v - 25$

f) $9x^2 + 6x + 1$

10. Write each quadratic equation in factored form. Look for common factors first. Then, identify the x-intercepts.

a) $y = 2x^2 + 14x + 12$

b) $y = 9x^2 + 24x + 15$

c) $y = 4x^2 - 24x + 36$

d) $y = 21x^2 - 15x - 6$

e) $y = 34x^2 - 32x + 2$

f) $y = 8x^2 - 64x - 56$

11. **a)** Create your own factoring question using algebra tiles.

b) Build the rectangle that models your polynomial. Then, find the factors.

2.4 Select and Apply Factoring Strategies

Textbook pp. 98–107

12. **a)** Identify the factors of $x^2 + 10x + 25$.

b) Explain why this polynomial is called a perfect square trinomial, using words, diagrams, and/or algebraic reasoning.

13. **a)** Create a perfect square trinomial.

b) Write the expression as an expanded polynomial and as a product of factors.

14. Fully factor each expression. Look for common factors first.

a) $2x^2 + 12x + 18$

b) $5n^2 - 20$

c) $7g^2 - 70g + 175$

d) $4m^2 - 64m + 256$

e) $27w^2 - 147$

f) $242v^2 - 50$

2.5 Solve Quadratic Equations by Factoring

Textbook pp. 108–113

15. Find the roots of each equation.

a) $p^2 + 17p + 52 = 0$

b) $4x^2 + 13x - 17 = 0$

c) $3r^2 - 13r + 14 = 0$

d) $2q^2 - 21q + 19 = 0$

e) $5n^2 + 6n - 8 = 0$

f) $3m^2 - m - 14 = 0$

16. An arrow is shot from a raised platform. Its height as a function of time is given by the equation $h(t) = -4t^2 + 32t + 28$, where $h(t)$ is the height of the arrow, in metres, and t is the time, in seconds.

a) Write this equation in factored form.

b) What is the hang time of the arrow? Explain how you found your answer.

Chapter 2 Practice Exam

For questions 1 to 5, choose the best answer.

1. Which statement is true for $f(x) = (x - 1)2 + 2$?

A The vertex is (1, 2).

B The vertex is (−1, 2).

C The x-intercepts are 1 and 2.

D The x-intercepts are −1 and 2.

2. Which statement is true for $y = 3(x - 4)(x + 6)$?

A The vertex is (−4, 6).

B The vertex is (4, −6).

C The x-intercepts are −4 and 6.

D The x-intercepts are 4 and −6.

3. Which statement is true for $m(x) = 2x^2 - 7x + 5$?

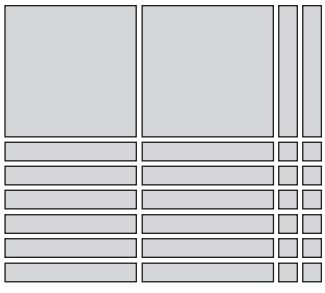
A The y-intercept is 2 and the parabola opens upward.

B The y-intercept is 2 and the parabola opens downward.

C The y-intercept is 5 and the parabola opens upward.

D The y-intercept is 5 and the parabola opens downward.

4. Which polynomial and factors do these tiles represent?



A $2x^2 + 14x + 12 = (x + 2)(2x + 6)$

B $2x^2 + 12x + 14 = (x + 2)(2x + 6)$

C $2x^2 + 14x + 12 = (2x + 2)(x + 6)$

D $2x^2 + 12x + 14 = (2x + 2)(x + 6)$

5. Which is the factored form of $x^2 - 4$?

A $(x - 2)(x + 2)$

B $(x - 2)^2$

C $x(x + 2)$

D This expression is not factorable.

6. **a)** Build or draw an algebra tile area model for the expression $3x^2 + 11x + 6$.

b) Use the model to identify the factors of the polynomial.

7. Write each quadratic function in standard form. Then, identify the y-intercept.

a) $f(x) = 4(x + 1)(x - 5)$

b) $g(x) = -2(x + 7)^2 - 12$

8. Factor each expression, if possible. If it is not possible, write "not factorable" and explain why.

a) $2a^2 + 9a - 11$ **b)** $p^2 + 9p + 18$

c) $y^2 - 24$ **d)** $k^2 + 2k - 32$

e) $3w^2 - 10w - 7$ **f)** $h^2 - 8h + 15$

9. Fully factor each expression. Look for common factors first.

a) $8f^2 - 24f + 14$

b) $3q^2 - 75$

10. Write each quadratic function in factored form. Then, identify the x-intercepts.

a) $h(x) = x^2 - 12x + 27$

b) $f(x) = 2x^2 - x - 15$

11. A disc is thrown from a raised platform. Its height as a function of time is given by $h(t) = -2t^2 + 20t + 9$, where t is the time, in seconds, and $h(t)$ is the height, in metres. What is the maximum height that the disc reaches, to the nearest metre?

Use technology.

12. Consider the quadratic function $w(x) = -2(x - 6)(x + 2)$.

a) In which form is this equation? Does this parabola open upward or downward?

b) What are the x-intercepts?

c) Find the coordinates of the vertex. Is the vertex a maximum or a minimum? Explain.

d) What is the axis of symmetry?

e) What is the y-intercept?

f) Graph the function.

g) Identify the intervals for which the function is

i) positive or negative

ii) increasing or decreasing

13. Consider the quadratic function $y = (x - 3)^2 + 1$.

a) In which form is this function? Does this parabola open upward or downward?

b) What are the coordinates of the vertex? Is the vertex a maximum or a minimum? Explain.

c) What is the axis of symmetry?

d) What is the y-intercept?

e) Graph the function.

f) Identify the intervals for which the function is increasing or decreasing.

14. The shape of a halogen lamp is modelled by $y = 2(x - 3)^2 + 9$, where x is the distance, in centimetres, from the left side of the lamp, and y is the opposite of the depth of the lamp, both in centimetres.

a) What are the coordinates of the vertex?

b) Express this function in standard form.

c) Use the standard form of this function to express the function in factored form.

d) What are the coordinates of the x-intercepts?

e) Graph the function.

f) If a halogen bulb measures 8 cm in height, will it fit into the lamp? What is the maximum height of a bulb that will fit the lamp?

g) Which parts of the domain and range of this function provide no information about the lamp?

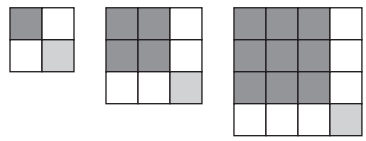
15. A day pass at an amusement park sells for \$60. Research has shown that for each \$10 increase in price, 250 fewer day passes will be sold. The revenue generated from sales can be represented by the function $R(x) = (60 + 10x)(2000 - 250x)$, where $R(x)$ is the revenue, in dollars, and x is the number of \$10 price increases.

a) What are the x-intercepts of this function?

b) Find the equation of the axis of symmetry and the coordinates of the vertex. Is the vertex a maximum or a minimum?

c) How many \$10 price increases should there be in order to maximize revenue? What is the selling price that gives the maximum revenue?

16. Consider this growing pattern of squares:



a) Describe how each coloured section of squares is growing from one stage to the next.

b) Which of these patterns (colours) is growing as a

- quadratic function?
- linear function?
- constant function?

Explain your thinking.

c) Write a function in standard form that gives the total number of coloured squares, T, as a function of the stage, n.

d) Explain how each coefficient, a, b, and c, relates to the tiles in the pattern.

e) Write this function in factored form. Explain how the factors in the expression relate to the physical model.