Chapter 3 Represent Quadratic Functions

Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 122–123 of the *Functions and Applications 11* textbook.

Lowest Common Multiple and Rational Numbers

1. Find the lowest common multiple of each set of numbers.

a) 3, 4 **b)** 3, 7 **c)** 8, 6 **d)** 2, 9 **e)** 4, 5, 6 **f)** 3, 4, 9

2. Evaluate without using a calculator.

a) $\frac{2}{5} + \frac{1}{2}$ **b)** $\left(-\frac{5}{8}\right) \times \frac{4}{5}$ **c)** $\frac{5}{6} - \frac{3}{4}$

d) $\frac{7}{4} \div 4$ **e)** $\frac{1}{7} + \frac{4}{5}$ **f)** $-\frac{3}{10} \times 10$

Evaluate Expressions and Radicals

3. Evaluate each expression for $a = 3$, $b = -2$, and $c = -1$.

a) $2a + 3b^2c$ **b)** $\frac{4ac - b^2}{4a}$ **c)** $(a + 2b)^2$

d) $a^2 - 2a + 4bc$ **e)** $3a^2 + 2b + c$ **f)** $(-2a + b - c)(c - a)$

4. Evaluate. Round your answers to two decimal places, if necessary.

a) $\sqrt{169}$ **b)** $\sqrt{21}$ **c)** $-\sqrt{125}$

d) $\sqrt{6^2 + 64}$ **e)** $\sqrt{7^2 - 4(-1)(8)}$ **f)** $-\sqrt{11^2 - 4(13)(-2)}$

Greatest Common Factor and Factor Quadratic Expressions

5. Determine the greatest common factor of each set.

a) $12x, 4x^2$ **b)** $9ab, 15bc$ **c)** $4y^2, 8y^2$

d) $10yz^2, 20xy^2, 28xyz$ **e)** $30d^2e^2, 36d^2e, 12e$ **f)** $35g^2h^3, 14g^2h, 28g^3h^2$

6. Factor.

a) $12x^2 + 9x$ **b)** $8x^2 - 4x$ **c)** $-5x^2 - 20x$

d) $8x^2 + 6x$ **e)** $4x^2 - 28x$ **f)** $15xy + 9y$

7. Factor.

Use algebra tiles or the decomposition factoring procedure.

a) $x^2 + 11x + 28$ **b)** $x^2 - 15x + 36$ **c)** $x^2 + 7x - 18$

d) $x^2 + x - 2$ **e)** $x^2 - 16$ **f)** $x^2 - 2x + 1$

8. Factor.

a) $7x^2 + 4x - 3$ **b)** $3x^2 - x + 10$ **c)** $8x^2 - 9x + 1$

d) $5x^2 + 10x + 5$ **e)** $8x^2 - 2x - 3$ **f)** $3x^2 - 75$

3.1 Complete the Square

Textbook pp. 124–134

Prerequisite Skills

1. Rewrite each quadratic function in vertex form by completing the square. Then, use a graphing calculator to verify that the two forms are equivalent.

a) $y = x^2 - 8x + 9$ **b)** $y = 2x^2 + 12x + 20$

2. Find the maximum or minimum value of each quadratic function by rewriting it in the vertex form. Then, use technology to verify your answers.

a) $y = -2x^2 - 3x - 1$ **b)** $y = \frac{1}{4}x^2 - 3x + 1$

3. The path of a discus thrown from a height of 1.5 m above the ground can be approximated by the quadratic function $h(x) = -0.0625x^2 + x + 1.5$, where x is the horizontal distance travelled, in metres, and $h(x)$ is the height, in metres.

a) Sketch the graph.

b) Find the maximum height of the discus by completing the square.

c) At what horizontal distance does the discus reach its maximum height?

4. Zandra sells smoothies at a market for \$6 each. At this price, she usually sells 120 smoothies a day. She plans to decrease the price to generate more sales. A survey indicated that for every \$0.15 decrease in price, she can expect to sell 5 more smoothies. What price should Zandra charge to maximize her revenue? What is the maximum revenue?

A

1. Use algebra tiles to rewrite each quadratic function in the vertex form, $y = a(x - h)^2 + k$, by completing the square.

a) $y = x^2 + 6x + 5$

b) $y = x^2 + 2x + 5$

c) $y = x^2 + 8x - 2$

d) $y = x^2 + 4x - 4$

e) $y = x^2 + 10x + 15$

f) $y = x^2 + 6x - 5$

2. Find the value of k that makes each expression a perfect square trinomial.

> You can work backwards from $(x + c)^2 = x^2 + 2cx + c^2$.

a) $x^2 - 4x + k$ **b)** $x^2 + 10x + k$

c) $x^2 - 2x + k$ **d)** $x^2 + 11x + k$

e) $x^2 - 22x + k$ **f)** $x^2 - 15x + k$

3. Rewrite each quadratic function in the vertex form, $y = a(x - h)^2 + k$, by completing the square.

a) $y = x^2 + 12x + 18$

b) $y = x^2 + 14x + 7$

c) $y = x^2 - 2x - 6$

d) $y = 2x^2 + 16x + 16$

e) $y = 4x^2 - 8x$

f) $y = -3x^2 - 6x - 9$

4. Use a graphing calculator to verify that the two forms of each function in question 3 are equivalent.

5. Find the maximum or minimum value of each quadratic function by completing the square.

a) $y = \frac{2}{5}x^2 - 4x + 10$

b) $y = 3x^2 + 2x - 2$

c) $y = -x^2 + 4x - 5$

d) $y = 5x^2 - 13x + 6$

e) $y = -\frac{1}{3}x^2 - 6x + 19$

f) $y = -4x^2 - 4x$

6. Use technology to verify your answers to question 5.

7. The path of a football can be modelled by the function $h(d) = -0.024d^2 + 1.2d + 1.8$, where $h(d)$ is the height, in metres, and d is the horizontal distance, in metres, that the football travelled from the quarterback.

a) What is the maximum height of the football?

b) What is the horizontal distance from the quarterback at the maximum height?

c) If the football is intercepted by a player 49.5 m down the field, how high will it be at the moment it is intercepted?

d) How high was the football when it was first thrown?

8. A toy rocket powered by air compression is launched vertically upward off of a 3-m tall platform. The height of the rocket, $h(t)$, in metres, can be approximated by the function $h(t) = -5t^2 + 20t + 3$, where t is the time, in seconds, after the rocket is launched.

a) Sketch the graph.

b) How high is the rocket after 3 s?

c) Find the maximum height of the rocket by completing the square.

d) How long does it take to reach its maximum height?

B

9. Tim's Kayaks offers kayak rentals for $45 per hour. The business' weekly revenue is $1350. An informal survey shows that for every $2.50 decrease in price per hour they can expect an additional 3-hour rental.

a) Write expressions for the price of a kayak rental per hour and the number of rental hours per week.

b) Write an equation for the weekly revenue using your expressions from part a).

c) What kayak rental price per hour would maximize weekly revenue?

d) What is the maximum weekly revenue that Tim's Kayaks could receive?

C

10. A parabola in the form $y = ax^2 + 2x + c$ has a y-intercept of -12 and passes through the point (6, 18). Find the coordinates of the vertex.

The y-intercept tells you the y-value when $x = 0$.

11. Consider the quadratic function $y = 2x^2 + 6bx + 10$.

a) Write the function in vertex form.

b) Write an expression for the coordinates of the vertex.

3.2 The Quadratic Formula

Textbook pp. 135–144

Prerequisite Skills

1. **a)** Find the roots of $3x^2 + 5x - 2 = 0$.

b) Use a graphing calculator to verify that the roots are the x-intercepts of the related quadratic function.

2. Solve $\frac{3}{4}x^2 - \frac{1}{3}x - 4 = 0$ using the quadratic formula. Round your answers to the nearest hundredth.

3. A geyser erupts, shooting water upward at a velocity of 31 m/s from an initial height of 0.8 m. The equation that models the height of a drop of water shot from the geyser is $h(t) = -4.9t^2 + 31t + 0.8$, where t is the time, in seconds, and $h(t)$ is the height, in metres.

a) After how many seconds, to the nearest hundredth, does the drop of water hit the ground?

When the drop of water hits the ground, $h(t) = 0$.

b) For how long, to the nearest hundredth of a second, is the drop of water more than 30 m high?

c) How high is the drop of water after 3 s, to the nearest hundredth of a metre?

4. A rectangular park has an area of 575 m^2. The park is to be surrounded by a walkway that is the same width on all sides of the park. The park and walkway together are to measure 30 m by 24 m.

a) Sketch and label a diagram to represent this situation.

b) Write an equation for the width of the walkway.

c) Determine the width of the walkway, to the nearest tenth of a metre.

A

1. Use the quadratic formula to find the roots of each equation. Express your answers as exact roots.

a) $x^2 + 4x - 8 = 0$

b) $x^2 - 6x - 5 = 0$

c) $x^2 - 3x + 1 = 0$

d) $x^2 + 9x + 6 = 0$

e) $x^2 - 8x + 2 = 0$

f) $x^2 + 7x - 7 = 0$

2. Use the quadratic formula to find the roots of each equation. Express answers as exact roots and as approximate roots to the nearest hundredth, where appropriate.

a) $2x^2 - 2x - 9 = 0$

b) $-5x^2 + 7x - 1 = 0$

c) $-4x^2 - 3x + 8 = 0$

d) $6x^2 + 9x + 2 = 0$

e) $3x^2 + 3x - 9 = 0$

f) $-x^2 - x + 1 = 0$

3. Use a graphing calculator to verify that the roots of each equation in question 2 are the x-intercepts of the related function.

4. When a ski-jumper launches off of a take-off ramp, her path can be modelled by $h(d) = -0.005d^2 + 0.32d + 5.1$, where d is the horizontal distance from the ramp, in metres, and $h(d)$ is his height, in metres. How far away from the end of the ramp did he land, to the nearest tenth of a metre?

5. An above-ground pool measures 5 m by 11 m and is to be surrounded by a wooden deck. The width of the deck is to be equal on all four sides of the pool. The total area of the pool plus the deck is to be 150 m^2.
   a) Sketch and label a diagram to represent this situation.
   b) Write an equation for the total area.
   c) Find the width of the deck, to the nearest tenth of a metre.

6. Solve each equation. Round answers to the nearest hundredth, if necessary.
   a) $-0.05x^2 + 5x - 25 = 0$
   b) $0.04x^2 + 0.16x - 0.64 = 0$
   c) $-7x^2 - 0.875x - 0.0175 = 0$
   d) $\frac{1}{8}x^2 + \frac{1}{2}x + \frac{1}{2} = 0$
   e) $\frac{1}{4}x^2 - x + \frac{1}{2} = 0$
   f) $\frac{1}{5}x^2 + x - \frac{3}{5} = 0$

7. Solve. Round answers to the nearest hundredth, if necessary.
   a) $7x^2 - 6 = 12x$
   b) $5x^2 - 5x = -1$
   c) $-x^2 = -8x + 1$
   d) $(2x + 4)(x - 7) = (3x - 6)(x + 2)$
   e) $\frac{x^2}{3} + \frac{x}{5} = \frac{4}{15}$
   f) $-3(x - 1)(x + 4) = x^2 - 4x + 6$
   g) $(4x - 4)^2 = (3x + 3)^2$
   h) $\frac{x + 3}{2} - \frac{3 - x}{3} = x^2 - 1$

B

8. The manager of a graphic design company wishes to spend money on advertising to increase the company's profit. The company's increase in profit is modelled by the function $P(x) = -0.0012x^2 + 6x$, where $P(x)$ is the increase in profit, in dollars, and x is the amount spent on advertising, in dollars.
   a) Which amounts of advertisement spending result in no profit increase?
   b) What is the increase in profit if the company spends \$1200 on advertising?
   c) If the manager wishes to increase the company's profit by at least \$6300, what range must their advertisement spending be in?

9. A boat travelled 1800 m up a river at a constant speed. On the trip back down the river, the current increased the boat's speed by 4 m/s. If the entire trip up the river and back took 8 min, how fast was the boat travelling each way?

C

10. If one of the roots of $ax^2 + bx - 6 = 0$ is -6 and the y-coordinate of the vertex is -8, find the possible values of a and b.

First, determine an expression for the y-coordinate of the vertex in terms of a and b by completing the square. Then, solve a system of equations involving your equation and the given equation.

3.3 Real Roots of Quadratic Equations

Textbook pp. 145–152

Prerequisite Skills

1. Calculate the value of the discriminant for each equation.

a) $x^2 + 8x + 16 = 0$ **b)** $x^2 - x + 25 = 0$

c) $2x^2 + x - 10 = 0$ **d)** $18x^2 - 12x + 2$

2. Determine how many real roots each equation has, then solve.

a) $x^2 + 4x - 5 = 0$ **b)** $8x^2 - 8x = -2$

c) $2x^2 = 16x - 32$ **d)** $x^2 - 8x - 9 = 0$

3. Find the x-intercepts of each quadratic function. Round your answers to the nearest hundredth, when necessary.

> You need to solve for x when $y = 0$ to find the x-intercepts.

a) $y = 2x^2 - 2x - 9$ **b)** $y = 3x^2 + 8x + 6$

c) $y = 5x^2 + 3x - 7$ **d)** $y = 10x^2 - 20x + 10$

A

1. Identify the number of roots of the quadratic equation given the following values for the discriminant, D.

a) -1

b) 10

c) -0.001

d) $-\frac{1}{10}$

e) 1.11

f) 0

2. Determine how many real roots each equation has.

> When determining the number of real roots, find the discriminant, $D = b^2 - 4ac$.

a) $x^2 - 3x + 3 = 0$

b) $6x^2 - 10x + 5 = 0$

c) $-2x^2 + x + 12 = 0$

d) $-x^2 + 2x - 1 = 0$

e) $4x^2 + 8x + 3 = 0$

f) $-x^2 - x + 1 = 0$

3. Verify your answers to question 2 graphically, with or without technology.

4. Determine the number of real roots each equation has. Then, find the roots of each equation by factoring.

a) $2x^2 - 4x = 6$

b) $x^2 = 2x + 8$

c) $5x^2 + 3x - 2 = 0$

d) $-6x^2 + 54 = 0$

e) $-12x^2 = -12x + 3$

f) $-2x^2 + 5x - 2 = 0$

g) $7x^2 + 14x = -7$

h) $11x^2 - 11 = 0$

5. Find the x-intercepts of each quadratic function by graphing, with or without technology.

a) $y = x^2 - x - 2$

b) $y = -4x^2 + 4x - 2$

c) $y = 2x^2 + 6x + 4$

d) $y = -3x^2 - 6x$

6. Find the x-intercepts of each quadratic function by graphing, with or without technology.

a) $y = 2x^2 - 12x + 18$

b) $y = -x^2 + 7x + 8$

c) $y = x^2 + 7x - 18$

d) $y = x^2 - x + 1$

7. Find the x-intercepts of each quadratic function. Round your answers to the nearest hundredth, when necessary.

a) $y = -x^2 + 3x - 1$

b) $y = x^2 + 5x + 2$

c) $y = 10x^2 + 4x - 6$

d) $y = -2x^2 - 9x + 1$

e) $y = 4x^2 - 15x + 7$

f) $y = 7x^2 + 8x + 1$

g) $y = -3x^2 + 20x + 7$

h) $y = -3x^2 - 6x - 3$

B

8. Solve. Round your answers to the nearest hundredth, when necessary.

a) $x^2 - 3 = \frac{7}{3}x$

b) $x^2 + \frac{5}{4}x - \frac{1}{3} = 0$

c) $\frac{x^2}{2} - \frac{2x}{3} - \frac{5}{6} = 0$

d) $\frac{x^2}{4} - 2 = \frac{23}{10}x$

9. A field has a perimeter of 450 m and an area of 11 900 m^2. Find the dimensions of the field, to the nearest metre.

C

10. Find a quadratic equation with each pair of roots.

The values of a and b can be quickly found by comparing each pair of roots to the quadratic formula.

a) $x = \frac{13 \pm \sqrt{149}}{2}$

b) $x = \frac{7 \pm 13}{6}$

11. If $y = a(x - h)^2 + k$ has vertex $(h, 5)$, explain under what circumstances $D = 0$, $D > 0$, or $D < 0$.

3.4 Multiple Forms of Quadratic Functions

Textbook pp. 153–163

Prerequisite Skills

1. Consider the functions $f(x) = 4x^2 - 2x - 12$ and $g(x) = -0.45x^2 + 0.9x + 5$.

> You can simplify the functions by removing a common factor first.

a) Find these features of each graph:
- x-intercepts
- $f(x)$-intercept or $g(x)$-intercept
- coordinates of the vertex
- equation of the axis of symmetry.

b) Sketch each graph. Label the features.

2. Determine the intervals for which the function $y = 2x^2 + 8x - 10$ is positive or negative and the intervals for which the function is increasing or decreasing.

A

1. These quadratic functions can be factored. Find the x-intercepts, the y-intercept, the coordinates of the vertex, and the equation of the axis of symmetry. Then, sketch the graph and label the features.

a) $y = x^2 - 5x - 6$

b) $y = x^2 + 13x + 12$

c) $y = -x^2 - 10x + 24$

d) $y = 3x^2 + 8x - 3$

e) $y = 8x^2 + 24x + 18$

f) $y = -4.5x^2 - 18x - 18$

2. Verify your answers to question 1 using graphing technology.

> The discriminant can be used to verify the number of x-intercepts.

3. These quadratic functions cannot be factored. Find the x-intercepts, the y-intercept, the coordinates of the vertex, and the equation of the axis of symmetry of each function. Then, sketch the graph and label the features.

a) $y = 6x^2 - 9x - 11$

b) $y = -2x^2 + 16x - 5$

c) $y = -4x^2 - 7x + 6$

d) $y = -\frac{1}{2}x^2 + x + 10$

e) $y = \frac{1}{3}x^2 + 4x - 9$

f) $y = 0.64x^2 - 2.56x + 1.28$

4. Verify your answers to question 3 using graphing technology.

5. For each quadratic function, find the x-intercepts and the coordinates of the vertex. Then, sketch the graph and use it to identify the intervals for which the function is positive or negative and the intervals for which it is increasing or decreasing.

a) $y = -2x^2 - x + 10$

b) $y = x^2 - 16x - 17$

c) $y = -4x^2 + 3x + 7$

d) $y = x^2 - 9x - 22$

e) $y = 6x^2 + 48x + 96$

f) $y = -3x^2 + 9x$

g) $y = (x + 3)^2$

h) $y = 5x^2 - 125$

6. For each quadratic function, find the x-intercepts and the coordinates of the vertex. Then, sketch the graph and use it to identify the intervals for which the function is positive, the intervals for which the function is negative, the y-intercept, and the equation of the axis of symmetry.

a) $y = (2x - 3)(x - 1)$

b) $y = 4(x - 2)^2 - 7$

c) $y = -3(x + 5)^2 + 5$

d) $y = -2x^2 + 7x + 10$

e) $y = -(x - 6)(2x + 8)$

f) $y = 3x^2 + 2x - 1$

B

7. A science student presented a project in which she launched a ball into the air using a lever and it landed in the centre of a basket, as predicted. The predicted model of the ball's path was $h(x) = -0.06x^2 + 6x + 40$, where $h(x)$ is the height of the ball above the floor, in centimetres, and x is the horizontal distance from the lever, in centimetres.

a) From what height did the ball start?

b) What was the maximum height of the ball?

c) How far away from the lever did the ball begin to fall?

d) Sketch a graph of the path of the ball.

e) Determine the horizontal distance from the lever to the centre of the basket.

8. Whales are known to "breach," or leap out of the water. The breach of a Humpback whale can be approximated by the equation $h = -4.9t^2 + 5.9t - 1.1$, where h represents the height from the surface of the water, in metres, and t is the time since the whale reaches maximum speed, in seconds.

a) What do the h- and t-intercepts represent?

b) How long is the whale above the surface of the water, to the nearest hundredth of a second?

First, determine the two times $h(t) = 0$.

C

9. Find the y-intercept of the parabola with vertex (6, 4) and that passes through the point (10, 0).

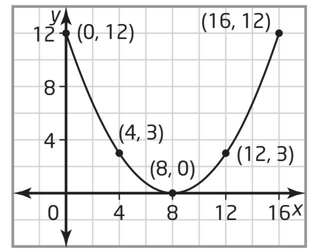
10. Find the equation of the parabola with vertex (7, −2) and that passes through the point (1, 4).

3.5 Model With Quadratic Equations

Textbook pp. 164–173

Prerequisite Skills

1. The graph models the parabolic cross section of a parabolic reflector in a car headlight. In the graph, x represents the horizontal distance from one side of the reflector, in centimetres, and y represents the height of the headlight measured from the vertex, in centimetres. Determine an equation that models the data.



2. A fireworks company is collecting data on a new firework rocket. The table shows data collected on the path of the rocket, where h is the rocket's height, in metres, and t is the time, in seconds.

a) Determine an equation for the rocket's flight path with and without technology. Compare the models.

b) The company is considering designing this rocket to explode after 5.5 s. Use both models you found in part a) to estimate the height at which the rocket explodes.

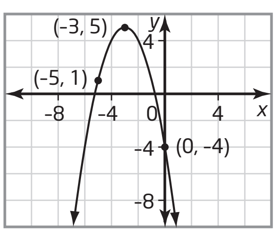
Time, t (s)	Height, h (m)
0	0
1	44
2	79
4	118
7	104
8	80
10	0

A

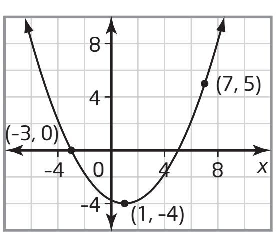
1. Write an equation for each parabola shown.

Check your answers by substituting a point on the parabola for x and y in your equations.

a)



b)



2. The table shows the path of a jet of water that is shot into the air from a fountain nozzle. x is the horizontal distance from the nozzle and h is the height of the jet above the fountain, both in metres.

x (m)	h (m)
0	0
3	9
8	22
11	26
16	29
20	27
23	23

a) Make a scatter plot of the data.

b) Estimate the coordinates of the vertex and find an equation that models the data.

c) Use graphing technology to create a scatter plot and find a quadratic equation that models the data.

d) Compare the two equations. Which equation best models the data? Explain.

B

3. The table shows the height of a projectile shot into the air over time.

Time (s)	Height (m)
0.2	11.82
1.9	83.75
3.3	121.72
4.1	134.79
6.3	138.40
7.6	118.24
8.1	106.07
9.9	41.99

a) Make a scatter plot of the data and draw a curve of best fit.

b) Estimate the coordinates of the vertex.

c) Use graphing technology to find a quadratic equation that models the data. Then, find the maximum value of the function.

d) Compare the calculated maximum value from part c) to the estimated value in part b).

e) Use the equation to predict the height of the projectile after 9 s.

4. The table shows data collected on the spread of weeds in a field.

Time (weeks)	Area Infested (m^2)
1	14
2	64
3	149
4	271
5	435
6	632
7	895
8	1171

a) Make a scatter plot of the data and draw a curve of best fit.

b) Estimate the coordinates of the vertex and find an equation that models the data.

c) Use graphing technology to find a quadratic equation that models the data.

d) Use both equations to estimate the area of the field infected after 12 weeks. How close are the two estimates?

Estimate the coordinates of the vertex and find an equation that models the data.

C

5. Manuel kicks a football at the goalposts. The table shows the horizontal distance, in metres, and the height above ground, in metres, of the path of the football.

Horizontal Distance (m)	Height (m)
0.0	0.1
2.0	4.2
3.9	7.6
5.5	10.1
7.3	12.7
9.0	14.7
10.9	16.6
12.8	18.1
14.4	19.0
16.2	19.7

The goalposts are 36 m away and the crossbar is 3.05 m high. Will the ball clear the crossbar?

Chapter 3 Review

3.1 Complete the Square

Textbook pp. 124–134

1. Rewrite each quadratic function in the vertex form, $y = a(x - h)^2 + k$, by completing the square.

a) $y = x^2 + 4x + 1$

b) $y = -3x^2 + 6x + 3$

c) $y = 2x^2 + 16x + 20$

d) $y = -\frac{1}{2}x^2 + 4x - 9$

2. Find the maximum or minimum value of each quadratic function.

a) $y = 3x^2 - 6x + 7$

b) $y = -4x^2 - 4x + 8$

c) $y = 2x^2 + 12x - 5$

d) $y = -x^2 + 8x + 3$

3. A basketball is thrown up into the air. Its height, $h(t)$, in metres, after t seconds, is given by $h(t) = -4.9t^2 + 14.7t + 1.9$.

a) Sketch the graph.

b) Determine the maximum height of the basketball and the time it takes to reach it.

c) How high is the basketball after 2 s?

3.2 The Quadratic Formula

Textbook pp. 135–144

4. Solve the following equations using the quadratic formula. Express answers as exact roots and as approximate roots, rounded to the nearest hundredth.

a) $x^2 - 10x + 5 = 0$

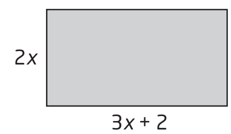
b) $\frac{3}{5}x^2 - \frac{3}{4}x - 1 = 0$

c) $-6x^2 + x + 1 = 0$

d) $-0.9x^2 + 0.8x - 0.02 = 0$

5. Find the dimensions of the rectangle if its area is 130 cm^2. Round your answers to the nearest tenth of a centimetre.

> Write an equation using the given information.



6. Find the positive number that is 5 less than 7 times its reciprocal.

3.3 Real Roots of Quadratic Equations

Textbook pp. 145–152

7. Determine how many real roots each equation has. Then, find the roots of each equation by factoring.

a) $5x^2 - 2x - 7 = 0$

b) $8x^2 + 3x = 0$

c) $7x^2 - 63 = 0$

d) $x^2 + 4x + 4 = 0$

8. Find the x-intercepts of each quadratic function. Round answers to the nearest hundredth, when necessary.

a) $y = 3x^2 + 5x + 2$

b) $y = x^2 - x - 8$

c) $y = 12x^2 - 16x + 5$

d) $y = 5x^2 + 2x - 13$

9. If $y = 2bx^2 + bx + 3$ has two x-intercepts, determine all possible values of b.

> The discriminant, $D = b^2 - 4ac$, can be used to determine the restrictions on the value of b.

3.4 Multiple Forms of Quadratic Functions
Textbook pp. 153–163

10. Determine the key features of each graph. Find the x-intercepts, the y-intercept, the coordinates of the vertex, and the equation of the axis of symmetry. Then, sketch the graph and label the features.

a) $y = 2x^2 + 10x - 12$

b) $y = -3x^2 + 16x - 5$

c) $y = -2x^2 - 2x + 4$

d) $y = x^2 - x + \frac{1}{4}$

11. Water is shot from a nozzle that is buried 30 cm in the ground. The height of a water droplet shot from the nozzle can be approximated by the equation $h(t) = -4.9t^2 + 7.6t - 0.3$, where $h(t)$ is the height of the drop of water, in metres, and t is the time, in seconds.

a) How long will it take for the drop of the water to reach ground level, to the nearest hundredth of a second?

b) For how long is the drop of water in the air, to the nearest hundredth of a second?

c) Find the maximum height of the drop of water and the time it takes to reach this height.

d) Sketch a graph of the path of the drop of water. Label key features.

e) Determine the time intervals for which the drop of water is increasing and decreasing in height.

12. For each quadratic function, find the x-intercepts and the coordinates of the vertex. Sketch the graph and identify the intervals for which the function is positive or negative and increasing or decreasing.

a) $y = (x - 5)(2x + 4)$

b) $y = 2(x + 3)^2 - 8$

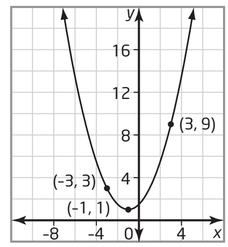
c) $y = -3x^2 + 8x + 11$

d) $y = -(x - 1)^2$

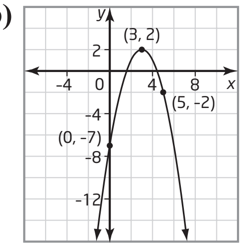
3.5 Model With Quadratic Equations
Textbook pp. 164–173

13. Write an equation for each parabola shown.

a)



b)



14. The table shows data collected on a car's distance over time.

Time (s)	Distance Travelled (m)
0	0.0
2	15.7
4	63.1
6	136.4
8	240.3
10	379.0
12	554.1

a) Make a scatter plot of the data and draw a curve of best fit.

b) Use the coordinates of the vertex to find an equation that models the data.

c) Use the algebraic equation to estimate the distance the car travelled after 15 s.

d) Use graphing technology to find a quadratic equation that models the data.

e) Use the quadratic regression equation to estimate the distance the car travelled after 15 s. Compare the result with your answer for part c).

Chapter 3 Practice Exam

For questions 1 to 4, choose the best answer.

1. What are the roots of the equation $-3x^2 + 16x + 12 = 0$?

A -6 and $\frac{2}{3}$ **B** 6 and $-\frac{2}{3}$

C 6 and $\frac{2}{3}$ **D** -6 and $-\frac{2}{3}$

2. What are the intervals for which the function $y = -4x^2 + 12x - 9$ is increasing or decreasing?

A always increasing

B always decreasing

C increasing when $x > \frac{3}{2}$

decreasing when $x < \frac{3}{2}$

D increasing when $x < \frac{3}{2}$

decreasing when $x > \frac{3}{2}$

3. What is the discriminant, D, of the quadratic equation $y = 8x^2 - 5x - 1$?

A $\frac{5}{16}$ **B** -57

C 57 **D** -7

4. What is the vertex of the quadratic function $y = -x^2 + 8x + 1$?

A $(-4, -17)$ **B** $(4, -17)$

C $(0, 1)$ **D** $(4, 17)$

5. Find the maximum or minimum value of each quadratic function.

a) $y = 2x^2 + 4x + 8$

b) $y = \frac{3}{5}x^2 + 6x + 7$

c) $y = -x^2 - 8x + 2$

d) $y = -5x^2 + 20x$

6. Determine the number of real roots each equation has. Then, find the roots of each equation by factoring.

> The discriminant, $D = b^2 - 4ac$, can tell you the number of real roots.

a) $y = 3x^2 - 6x + 3$

b) $y = 3x^2 + 3x - 6$

7. Solve each equation using the most appropriate method.

a) $4x^2 - 12x + 5 = 0$

b) $2x^2 + 11x - 13 = 0$

c) $3x^2 - x - 10 = 0$

d) $5(x^2 - 3x) = (x - 3)^2$

8. Determine the key features of each graph. Find the x-intercepts, the y-intercept, the coordinates of the vertex, and the equation of the axis of symmetry. Then, sketch the graph, labelling the key features.

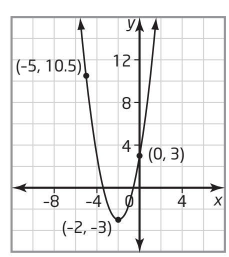
a) $y = 3x^2 + 8x - 16$

b) $y = -4x^2 + 13x - 3$

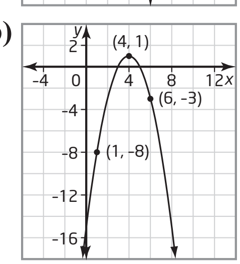
9. Write an equation for each parabola given its graph.

> The y-intercept is the constant value in $y = ax^2 + bx + c$.

a)



b)



10. For each quadratic function, find the x-intercepts and the coordinates of the vertex. Then, sketch the graph and use it to identify the intervals for which the function is positive or negative and the intervals for which it is increasing or decreasing.

a) $y = x^2 - 14x - 15$

b) $y = -2x^2 - 8x$

11. A tennis ball is hit from a height of 1.1 m above the ground. The function $h(d) = -0.012d^2 + 0.216d + 1.1$ models the path of the tennis ball, where d is the horizontal distance, in metres, and $h(d)$ is the height, in metres.

a) How far has the tennis ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?

b) For what horizontal distance, to the nearest tenth of a metre, is the tennis ball higher than 1.5 m?

12. Aaron sells scrapbooking materials. He charges \$20 per album and sells 120 albums a month at this price. He plans to decrease the price to generate more sales. A survey indicated that for every \$0.50 decrease in price, he can expect to sell 5 more albums.

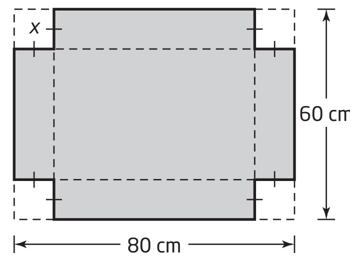
a) Write expressions for the price of an album and the number of albums sold in one month.

b) Write an equation for the revenue using your expressions from part a).

c) What price will generate the maximum revenue?

d) What is the maximum revenue he can generate in one month?

13. An open-topped box is to be made from a piece of aluminum measuring 80 mm by 60 mm. The sides of the box are formed when four congruent square corner pieces are cut out. The base area of the box is to be 2016 mm^2.



a) Determine an equation for the base area of the box.

b) Find the side length, x, of the square cut from each corner.

c) Find the dimensions of the box.

14. A baseball is hit from an initial height of 1.2 m. The table shows the horizontal distance, in metres, and the height above the ground, in metres, of the path of the baseball.

Horizontal Distance (m)	Height (m)
0.0	1.2
10.2	19.2
26.7	37.3
32.6	40.4
49.1	39.8
55.3	36.0
68.0	22.1
79.4	2.8

a) Make a scatter plot and draw a curve of best fit.

b) Estimate the coordinates of the vertex and find an equation that models the data.

c) Use graphing technology to find a quadratic equation that models the data.

d) Compare the two equations. Which equation best models the data? Explain.