Chapter 4 Trigonometry

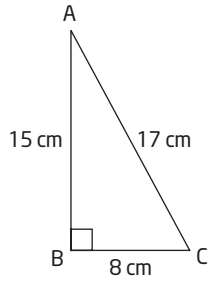
Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 184–185 of the *Functions and Applications 11* textbook.

Trigonometric Ratios

In a right triangle, the sine, cosine and tangent ratios of $\angle\theta$ are

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



1. For $\triangle ABC$, label the hypotenuse and the opposite and the adjacent sides relative to $\angle A$. Then, find the sine, cosine, and tangent ratios for $\angle A$. Express the ratios as fractions in lowest terms

2. Use your calculator to evaluate each trigonometric ratio. Round to four decimal places.

a) sin 46° **b)** cos 40° **c)** tan 54° **d)** sin 25.5°

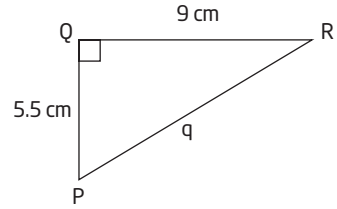
3. Determine the measure of each acute angle, to the nearest degree.

a) sin X = 0.8415 **b)** cos A = 0.2596 **c)** tan B = 0.3874 **d)** cos C = 0.5623

Pythagorean Theorem

4. Determine the measure of the unknown side in each triangle. Round answers to the nearest tenth of a unit.

a)



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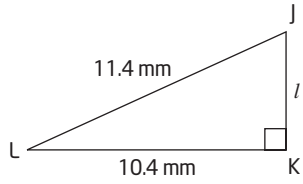
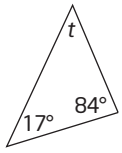


Geometry

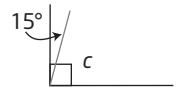
Complementary angles add to 90°. Supplementary angles add to 180°. The three angles in a triangle add to 180°.

5. Find the measure of each unknown angle.

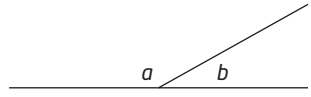
a)



b)



6. Two angles are supplementary. If one angle is five times the measure of the other, find the measure of the two angles.



Rearrange Formulas

7. Rearrange each formula to solve for the indicated variable.

a) $y = mx + b$ for m **b)** $m = \frac{a}{b}$ for b **c)** $a^2 = b^2 + c^2$ for c **d)** $P = 2l + 2w$ for l

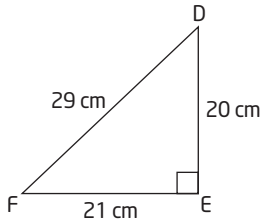
4.1 Use Trigonometry to Find Lengths

Textbook pp. 186–191

Prerequisite Skills

The hypotenuse of a right triangle is the longest side, and is always opposite the right angle.

1. For $\triangle DEF$, label the hypotenuse and the opposite and the adjacent sides relative to $\angle D$. Then, find the sine, cosine, and tangent ratios for $\angle D$. Copy and complete the statements. Express the ratios as fractions in lowest terms.



$\sin D = \frac{\square}{DF}$ $\cos D = \frac{\square}{DF}$ $\tan D = \frac{EF}{\square}$

$= \square$ $= \square$ $= \square$

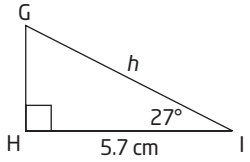
2. Use your calculator to evaluate each trigonometric ratio. Round to four decimal places.

a) sin 50° **b)** cos 17° **c)** tan 25° **d)** tan10°

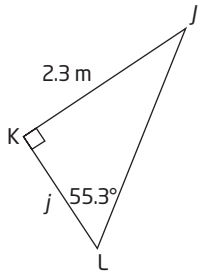
A

1. Find the length of the indicated side. Round to the nearest unit.

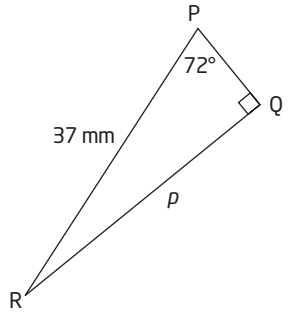
a)



b)



c)



2. The angle of depression from the top of a toboggan hill to the bottom is 52.1°. The length of the hill was measured at 78 m.

a) Draw a diagram to represent this situation. Label it fully.

b) What is the drop in altitude of the hill, to the nearest metre?

3. The angle of elevation from Rachel to her kite is 78°. She has let out 100 m of string.

a) Which trigonometric ratio can you use to calculate how high her kite is flying? Explain.

b) Ignoring Rachel's height, how high is her kite flying, to the nearest tenth of a metre?

B

4. A 3-m slide is being installed on a platform in a new playground. For safety reasons, the slide's angle of inclination must be no more than 60°.

a) What is the minimum distance from the platform that the base of the slide can be placed, to the nearest tenth of a metre?

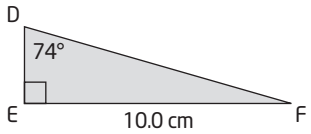
b) Using the answer from part a), how tall must the platform be for the slide to reach from the platform to the ground?

5. A hot air balloon is flying at an altitude of 1500 m. The angle of depression from the balloon to a landmark on the ground is 34°.

a) What is the balloon's horizontal distance to the landmark, to the nearest metre?

b) What is the balloon's direct distance to the landmark, to the nearest metre?

6. **a)** What information is needed to find the area of △DEF shown? What steps do you need to use to get that information?



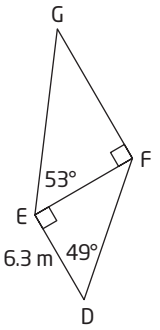
b) Determine the required information.

c) Calculate the area of the triangle.

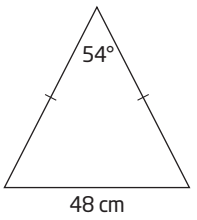
C

7. The diagram shows the readings that were taken in order to determine height of a tree on the other side of a river. What is the height of the tree (FG), to the nearest tenth of a metre?

Find FE first.



8. A piece of stone is to be cut into an isosceles triangle shape. The unequal side is to be 48 cm in length and is to be opposite a 54° angle. What are the lengths of the other two sides?



4.2 Use Trigonometry to Find Angles

Textbook pp. 192–196

Prerequisite Skills

1. Determine the measure of each angle, to the nearest degree.

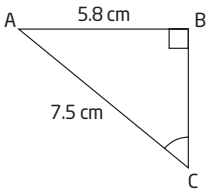
Use the $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ functions on your calculator.

a) $\tan A = 2.8941$ **b)** $\sin B = 0.4982$ **c)** $\cos C = 0.7221$ **d)** $\sin D = 0.1558$

A

1. Find the measure of the marked angle, to the nearest degree. Copy and complete the solutions.

a)

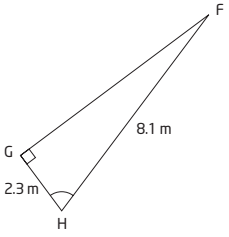


$\square\ C = \frac{5.8}{7.5}$

$\angle C = \square^{-1}\left(\frac{5.8}{7.5}\right)$

$\angle C = \square$

b)



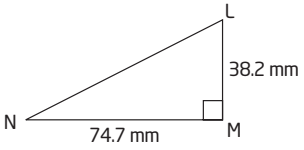
$\square\ H = \frac{2.3}{8.1}$

$\angle H = \square^{-1}\left(\frac{2.3}{8.1}\right)$

$\angle H = \square$

2. Solve each triangle. Round your answers to the nearest tenth of a unit.

a)



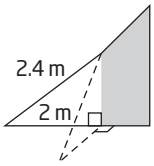
b) In $\triangle EFG$, $\angle F = 90°$, $g = 24.2$ mm, and $\angle E = 47.3°$.

3. Determine the angle of inclination of a wooden ramp that has a run of 14 m and a rise of 3 m.

B

4. A 6-m ladder rests against a wall. What angle does the ladder make with the ground, if the bottom of the ladder is 2.75 m from the wall?

5. A tent is supported by 2.4 m cables that are staked in the ground 2 m from the corner of the tent. What is the angle of inclination of the cables?



C

6. Roofs are classified by their pitch, or slope.

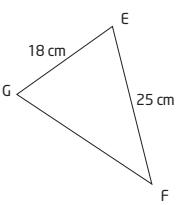
Like the slope of a line, the pitch of a roof equals $\frac{\text{rise}}{\text{run}}$.

a) Determine the angle of inclination of a roof with pitch $= \frac{1}{3}$.

b) What is the pitch of a roof with an angle of inclination of 21.8°? Write your answer as a fraction.

7. The area of $\triangle EFG$ is 210 cm^2. Side EF has a length of 25 cm and side EG has a length of 18 cm. Find $\angle FEG$ to the nearest degree.

Think: If EF or EG is the base of $\triangle EFG$, what is the height?



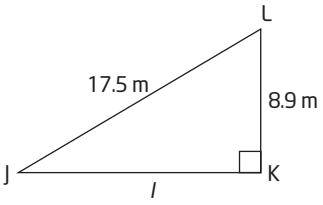
4.3 Solve Problems Involving Two Right Triangles

Textbook pp. 197–201

Prerequisite Skills

1. Determine the measure of the unknown side to the nearest tenth of a metre.

You can use trigonometry or the Pythagorean theorem to find *l*.



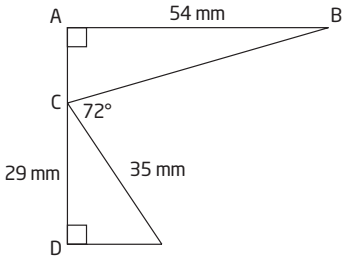
2. Draw and label $\triangle DEF$ with $\angle D = 25°$, $\angle E = 90°$, and DE = 57 mm.

A

1. **a)** Write a plan to determine the length of BC.

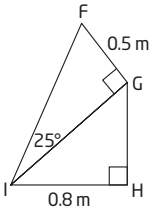
b) Carry out your plan. Round to the nearest millimetre.

The three angles around C have a sum of 180°.



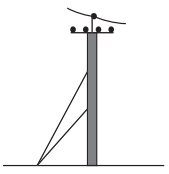
2. **a)** Write a plan to determine the measure of $\angle GIH$.

b) Carry out your plan. Round to the nearest degree.



B

3. An electricity pylon is supported by two cables, as shown. The cables are anchored on the ground on one side, 3 m from the base of the pylon. The angle of inclination of the shorter cable is 40°. The angle of inclination of the longer cable is 55°. The installers must determine the distance between the points where the cables are attached to the pylon. Round your answer to the nearest tenth of a metre.



4. A high jumper's take off point is 150 cm from the high jump bar. When she leaps her take-off angle is 43.1° and her landing angle is 57.2°.

Assume the jumper moves in a straight line going up and down. Assume she reaches her highest point above the bar.

a) The high jump bar is currently at 135 cm. By what distance will the high jumper clear the bar, to the nearest tenth of a centimetre?

b) How far past the bar does the hurdler land, to the nearest centimetre?

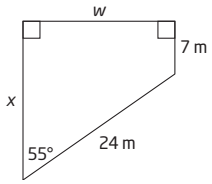
5. An airplane is flying between two towns at an altitude of 8 km. One town is to the west of the plane, and the other is to the east. The angle of depression to one town is 34.1° and to the other town is 49.7°. Determine the distance between to the two towns, to the nearest tenth of a kilometre.

C

6. A surveyor has mapped out a property, as shown.

a) How long is side *w*? side *x* ?

b) What are the perimeter and area of the property?



You can divide the figure into shapes that are easier to work with.

4.4 Investigate the Sine Law

Textbook pp. 202–209

Prerequisite Skills

1. Rearrange each formula to solve for the indicated variable.

a) $y = mx + b$ for b

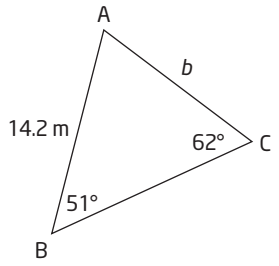
b) $\frac{m}{n} = \frac{a}{b}$ for n

c) $\frac{\sin A}{a} = \frac{\sin B}{b}$ for A

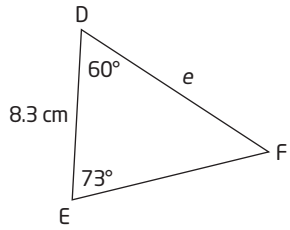
d) $\frac{b}{\sin B} = \frac{c}{\sin C}$ for c

2. Determine the measure of the missing angle.

a)



b)



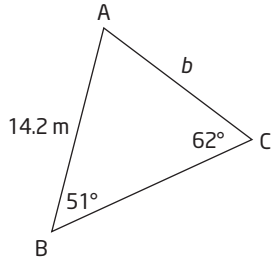
c) In $\triangle JKL$, $\angle J = 10°$ and $\angle L = 135°$.

d) In $\triangle MNO$, $\angle N = 74°$ and $\angle O = 59°$.

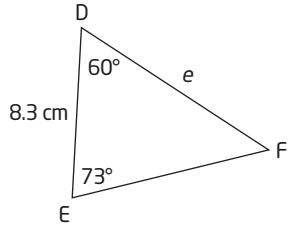
A

1. Find the length of the indicated side in each triangle, to the nearest tenth of a unit.

a)

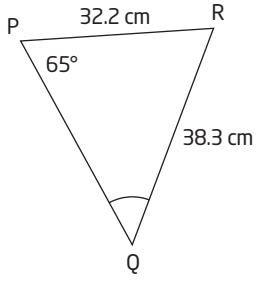


b)



2. Find the measure of the marked angle, to the nearest tenth of a degree.

a)



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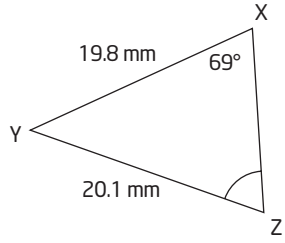


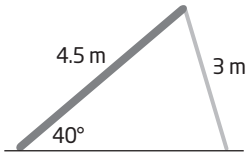
3. Sketch then solve each triangle. Round your answers to the nearest tenth of a unit, if necessary.

a) In $\triangle LMN$, $\angle M = 62°$, $l = 16.9$ cm, $m = 15.1$ cm.

b) In $\triangle UVW$, $\angle W = 83.9°$, $\angle V = 22.2°$, $v = 27.5$ mm.

c) In $\triangle ABC$, $\angle A = 84°$, $\angle C = 40°$, $b = 5.6$ m.

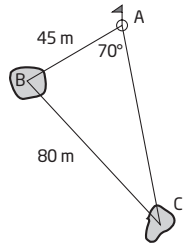
4. A 4.5-m long slide is attached to a 3-m ladder. The slide makes an angle of 40° with the ground.



a) Outline a method to calculate the angle the ladder makes with the ground.

b) Use your method to calculate the angle the ladder makes with the ground, to the nearest degree.

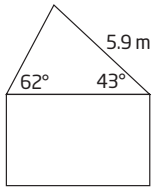
5. When planning a local golf course, the designers wanted to include two water hazards near one of the holes. The initial planned measurements are shown.



a) Describe a method to determine the remaining lengths and angle measurements.

b) Use your method to determine the remaining lengths and angle measures. Round to the nearest unit.

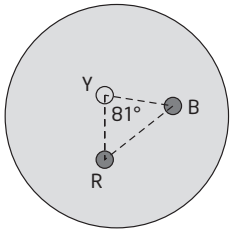
6. The roof of a shed makes angles of 62° and 43° with the horizontal. The longer roof rafter is 5.9 m long. Determine the length of the shorter roof rafter, rounded to the nearest tenth of a metre.



7. Two soccer players are 30 m apart and are both running towards the ball. The ball is located between the two players at an angle of 37° from the first player and 54° from the second player. Which player is closer to the ball? Justify your answer.

B

8. Three marbles are lying inside the playing circle, as shown.The red marble (R) is 71 cm from the blue marble (B) and 52 cm from the yellow marble (Y). The sight line from the yellow marble to the red marble and the sight line from the yellow marble to the blue marble form an angle of 81°. How far apart are the yellow and blue marbles, to the nearest centimetre?



C

9. As Ahmed is biking along the lakefront, he notices a boat anchored in the water 43° right of his travel direction. After biking 550 m straight ahead, he spots the boat 118° to the right of his travel direction.

Draw a diagram. He is closest to the boat when it is directly to his right.

a) How far from the boat is Ahmed now?

b) How close to the boat did he get in his travels?

4.5 Investigate the Cosine Law

Textbook pp. 210–215

Prerequisite Skills

1. Rearrange each formula to solve for the indicated variable.

a) $c^2 = a^2 + b^2$ for a

b) $\frac{m}{n} = \frac{a}{b}$ for a

c) $a^2 = b^2 + c^2 - 2bc \cos A$ for A

d) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ for a

2. Determine the measure of each angle, to the nearest degree.

a) $\cos A = 0.048$

b) $\cos B = -0.3385$

c) $\cos C = 0.48^2 + 0.84^2$

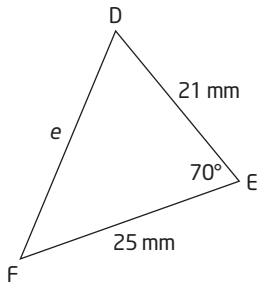
d) $\cos D = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8}$

A

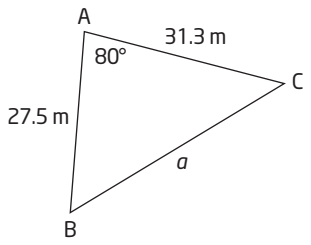
1. Find the length of the unknown side, to the nearest tenth of a unit.

Notice that sine law will not solve these problems.

a)

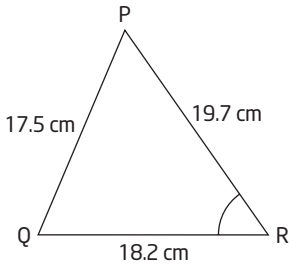


b)

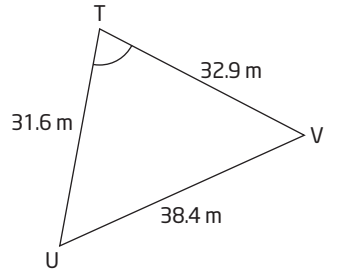


2. Find the measure of each marked angle, to the nearest tenth of a degree.

a)



b)



3. Sketch then solve each triangle using the cosine law. Round your answers to the nearest tenth of a unit.

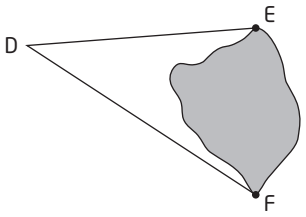
> You can draw diagrams to organize how to solve each problem.

a) In $\triangle RST$, RT = 58 mm, ST = 71 mm, and $\angle T = 61°$.

b) In $\triangle PQR$, PR = 6.3 cm, QR = 5.4 cm, and $\angle R = 40.5°$.

c) In $\triangle FGH$, GH = 5.6 m, FH = 7.5 m, and FG = 6.3 m.

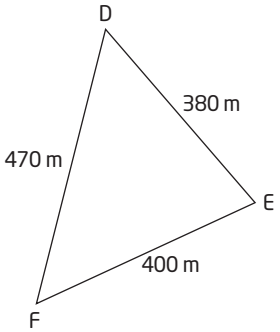
4. To measure the length of a muddy area, stakes are placed at points D, E, and F and measures DE to be 157 m, DF to be 183 m, and $\angle D$ to be 57.5°.



What is the length of the muddy area, to the nearest tenth of a metre?

B

5. The three markers in a triangular sailing course are shown in the diagram. The instructors would like each of the angles to be less than 75°. Will this design be appropriate? Justify your answer.



6. The course at a cross-country track meet is to be triangular. The first two legs are to be 75 m and 126 m long, joining at an angle of 79°.

a) Describe a plan to determine the angle measurements the runners need to turn through at the other two corners.

b) Use your plan to calculate these angles.

c) How will you ensure that your answers are precise?

7. Two helicopters are approaching the same landing area. The first helicopter is due west, 10.8 km from the landing area, at an angle of elevation of 38°. The second helicopter is due east, 14.7 km from the landing area, at an angle of 57°. How far apart are the helicopters, to the nearest tenth of a kilometre?

C

8. A table in the shape of a regular hexagon has sides 1 m in length. What is the distance between opposite sides? Round your answer to the nearest centimetre.

9. From the top of a 40-m building, the angle of depression to a restaurant, at 5° west of north, is 13°. The angle of depression to a second restaurant, due west of the tower, is 32°. How far apart are the two restaurants?

4.6 Make Connections With the Sine Law and the Cosine Law

Textbook pp. 216–221

Prerequisite Skills

1. Rearrange each formula to solve for the indicated variable.

a) $\frac{\sin A}{a} = \frac{\sin C}{c}$ for C

b) $\frac{b}{\sin B} = \frac{a}{\sin A}$ for b

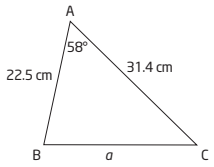
c) $b^2 = a^2 + c^2 - 2ac \cos B$ for B

d) $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ for b

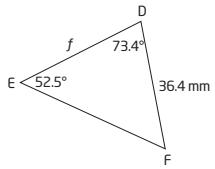
A

1. Determine whether the sine law or the cosine law is appropriate to find the length of the indicated side. Then, find the length of the side, to the nearest unit.

a)

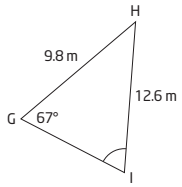


b)

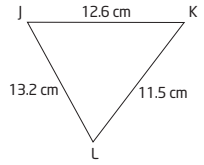


2. Determine whether the sine law or cosine law is appropriate to find the measure of the marked angle. Then, determine the measure of the angle, to the nearest degree.

a)



b)



3. Draw then solve each triangle. Round your answers to the nearest tenth of a unit.

a) In $\triangle$KLM, KL = 5.9 cm, LM = 6.2 cm, and $\angle$L = 49.5°.

b) In $\triangle$PQR, $\angle$P = 68.2°, $\angle$Q = 43.4°, PQ = 14.3 mm.

4. Two boats depart from the same dock. A pedal boat travels at 5 km/h in a direction 15° west of north. At the same time, a houseboat travels at 18 km/h in a direction of 22° east of north.

a) Which trigonometric law should you use to determine how far apart the two boats are after 2 h? Explain.

b) How far apart are the boats?

B

5. A cross-country skier's rest stop is 400 m away. To avoid a wooded area, the skier skis at an angle of 41° away from the rest stop, for a distance of 217 m. She then turns toward the rest stop.

a) Draw a diagram to model this problem.

b) How far does the skier need to travel to get to the rest stop?

c) Through what angle does she need to turn?

6. In $\triangle$LMN, $\angle$M = 65°, $\angle$N = 87°, and n = 13 mm. Determine the perimeter of the triangle to the nearest millimetre.

7. A regular decagon has a perimeter of 150 mm. What is the length of each diagonal?

> A regular decagon has ten equal sides and five equal diagonals sides.

8. An airplane is flying west of two towns. The angle of depression to the closer town is 49.1° and to the farther town is 37.5°. If the distance between the two towns is 5 km, determine how high the plane is flying, to the nearest kilometre.

C

9. An airplane's course is set at 60° west of south, at a speed of 285 km/h. A 50-km/h wind is blowing at 10° south of east. What are the resulting speed and direction of the airplane's path?

10. A regular octagon has a diagonal of 13 cm. Calculate its perimeter.

Chapter 4 Review

Work with a classmate to verify your answers. Use technology where appropriate.

4.1 Use Trigonometry to Find Lengths

Textbook pp. 186–191

1. In $\triangle DEF$, $\angle E = 90°$, $\angle F = 20°$, and $d = 19$ cm.
Draw the triangle. Then, find the length of each unknown side, to the nearest tenth of a centimetre.

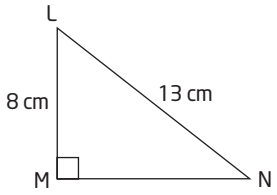
2. The angle of elevation from John to the top of his house is 57°. He is standing 5 m from the base of his house. Ignoring John's height, how tall is his house, to the nearest metre?

Use the correct trigonometric ratio for the given information.

4.2 Use Trigonometry to Find Angles,

Textbook pp. 192–196

3. Solve the following triangle. Round answers to the nearest tenth of a unit.



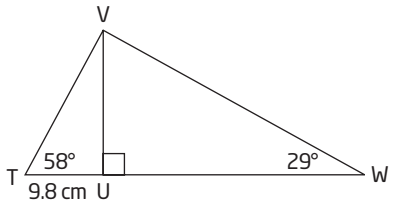
4. A building has a height of 40 m, and casts a shadow of 107 m. What is the angle of elevation to the sun?

4.3 Solve Problems Involving Two Right Triangles,

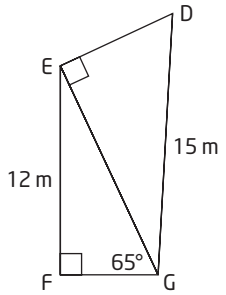
Textbook pp. 197–201

Make a plan before beginning any calculations.

5. Determine the length of side UW, to the nearest centimetre.



6. Determine the measure of $\angle EDG$, to the nearest degree.



7. A mast is fixed on the deck of a boat. From a point 20 m out on the water, the angle of elevation to the deck of the boat is 22°, and to the top of the mast is 55°. What is the height of the mast, to the nearest tenth of a metre?

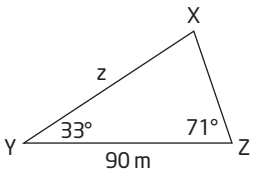
Draw and label a diagram using the given in formation.

4.4 Investigate the Sine Law

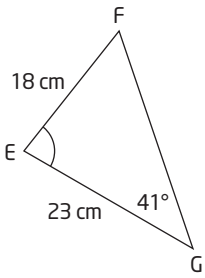
Textbook pp. 202–209

Write out the formula using variables, then substitute in the given information.

8. Find the length of the indicated side, to the nearest tenth of a metre.



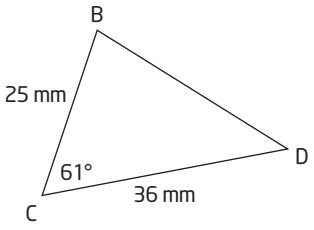
9. Find the measure of the marked angle, to the nearest tenth of a degree.



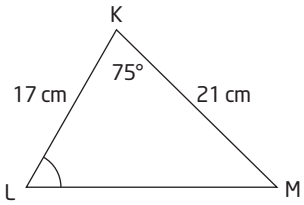
4.5 Investigate the Cosine Law

Textbook pp. 210–215

10. Find the length of the unknown side, to the nearest tenth of a millimetre.



11. Find the measure of the marked angle, to the nearest tenth of a degree.



4.6 Make Connections With the Sine Law and Cosine Law

Textbook pp. 216–221

Determine whether to use the sine law or cosine law.

12. Draw then solve each triangle. Round your answers to the nearest tenth of a unit.

a) In $\triangle XYZ$, $x = 18.1$ m, $y = 19.8$ m, $z = 17.5$ m.

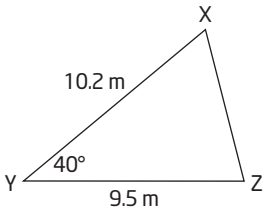
b) In $\triangle DEF$, $\angle D = 63°$, $d = 4.3$ cm, $f = 2.7$ cm.

13. A triangular tile needs to be cut with side lengths of 10 cm, 12 cm, and 15 cm. What angle measures are needed for the tile to have these side lengths? Round answers to the nearest tenth of a degree.

Chapter 4 Practice Exam

For questions 1 to 4, choose the best answer.

1. Which trigonometric ratio tool can be used to find the length of side XZ?



A sine ratio

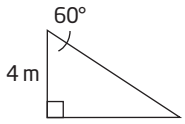
B sine law

C cosine ratio

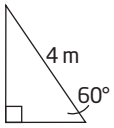
D cosine law

2. A ladder is leaning 4-m up a wall so that the ladder's angle of inclination is 60°. Which diagram models this situation?

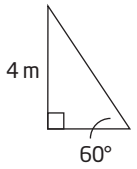
A



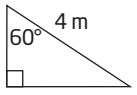
B



C



D



3. The result when evaluating

$$\cos^{-1}\left[\frac{6.2^2 + 5.7^2 - 4.9^2}{2(6.2)(5.7)}\right] \text{ is}$$

A 131.6°

B 47.1°

C 48.4°

D error

4. Which statement is true?

> You can start by eliminating the responses you know are wrong.

A The angles of elevation and depression are supplementary angles.

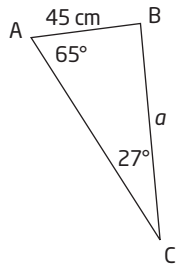
B The angle of elevation from point X to point Y equals the angle of depression from point Y to point X.

C The sum of the angles of elevation and depression is always less than 90°.

D The angles of elevation and depression are complementary angles.

5. Find the length of the indicated side, to the nearest unit.

a)



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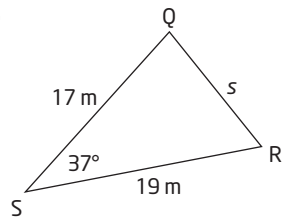
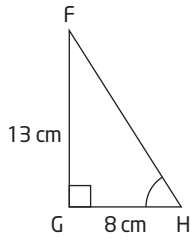
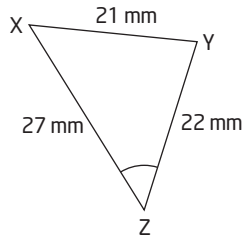


6. Find the measure of the marked angle, to the nearest degree.

a)



b)



7. In $\triangle DEF$, DE = 11 m, DF = 8 m, and $\angle F = 76°$. Sketch the triangle and find the measure of $\angle E$, to the nearest degree.

8. A squirrel is running toward a 10-m tall tree. The angle of elevation from the squirrel to the top of the tree is 37°. How far is the squirrel from the tree, to the nearest metre?

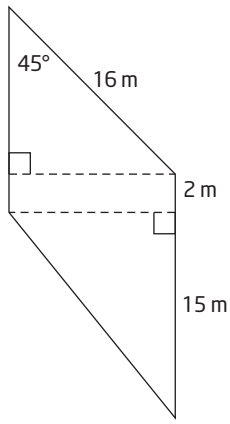
9. A loading ramp is to have an angle of inclination of 17°. The ramp needs to rise 3.5 m. Determine the horizontal length and the surface length of the ramp, to the nearest tenth of a metre.

10. A parasailer is attached to a boat using an 80-m long rope. When the parasailer is at an altitude of 50 m, what angle does the rope make with the water?

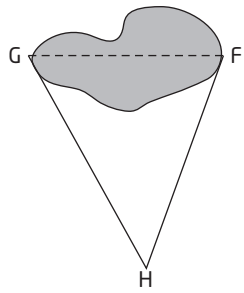
> Assume that the rope attaches to the boat at the water level. Assume the rope forms a straight line.

11. The diagram shows the measurements of a garden. How much edging is needed to surround the garden?

> Start by find the length of the top left side of the garden using trigonometric ratios.



12. A straight footbridge is going to be built across a pond. The following measurements are taken from the endpoints, F and G, of the bridge to a third point, H.



GH = 251 m

FH = 232 m

$\angle FGH = 61°$

How long will the bridge be, to the nearest tenth of a metre?