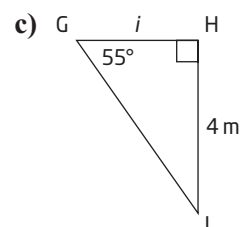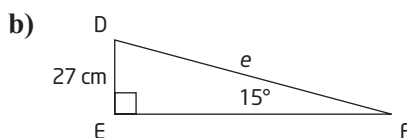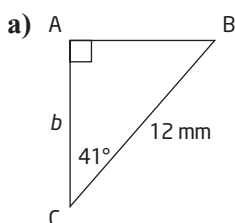# Chapter 5 Sine Functions

Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 230–231 of the *Functions and Applications 11* textbook.
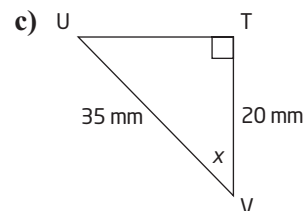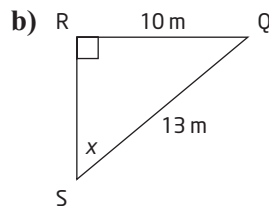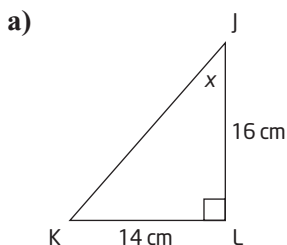
Trigonometry

In a right triangle, the sine, cosine, and tangent ratios of $\angle \theta$ are

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

1. Use the sine, cosine, or tangent ratio to determine the length of the side indicated, to the nearest unit.

a)

b)

c)

2. Determine the measure of angle x, to the nearest degree.

a)

b)

c)

3. Use the Pythagorean theorem to determine the length of each unknown side, to the nearest tenth of a unit.

a)

b)

c)
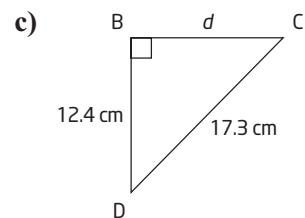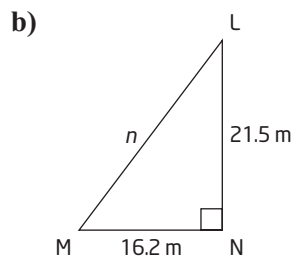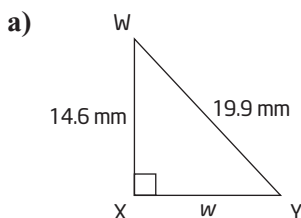
Evaluate Trigonometric Expressions

4. Evaluate with a calculator, to four decimal places.

a) $\cos 60°$

b) $\tan 30°$

c) $\sin 15°$

5. Evaluate each expression for the value indicated. Round to four decimal places.

a) $-\cos\theta$, for $\theta = 30°$

b) $2\sin\theta - 3$, for $\theta = 75°$

c) $-3\tan(\theta + 15°)$, for $\theta = 45°$

d) $-\dfrac{1}{2}\cos(2\theta - 30°)$, for $\theta = 90°$

Draw Angles

6. Use a protractor to draw each angle.

a) $60°$

b) $175°$

c) $225°$

d) $330°$

Plot Ordered Pairs

7. **a)** Plot the ordered pairs $(0, 2)$, $(1, -1)$, $(2, 1)$, $(-1, -1)$, and $(-2, 1)$.

b) Describe the pattern made by the points.

Transformations of Quadratics

8. Write an equation for the quadratic function that results from each transformation.

Hint: upward, right → positive; downward, left → negative

a) The graph of $y = x^2$ is translated 3 units upward and 2 units to the left.

b) The graph of $y = x^2$ is reflected in the x-axis and then stretched vertically by a factor of 2.

c) The graph of $y = x^2$ is compressed vertically by a factor of $\dfrac{1}{4}$ and then translated 4 units to the right and 1 unit down.

9. Use transformations to sketch a graph of each parabola. Label the vertex of each parabola.

Determine the coordinates of the vertex using the vertical translation and/or phase shift, then plot the vertex and sketch the rest of the parabola.

a) $y = x^2 + 2$ **b)** $y = (x + 3)^2 - 1$ **c)** $y = -2(x - 2)^2$

Domain and Range

10. Write the meaning of each statement.

a) domain $= \{x \in \mathbb{R} \mid -5 \le x \le 0\}$

b) range $= \{y \in \mathbb{R} \mid y > 1\}$

11. Sketch a graph of each function. Then, write the domain and range for each function.

a) $y = -2x^2 - 3$

b) $y = (x + 1)^2 + 4$

5.1 Periodic Functions

Textbook pp. 232–238

Prerequisite Skills

1. Evaluate each expression for the value indicated. Round to four decimal places.
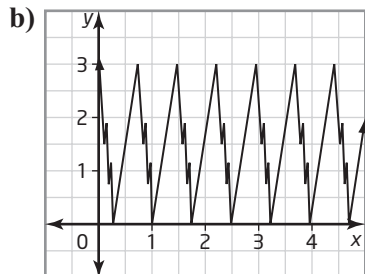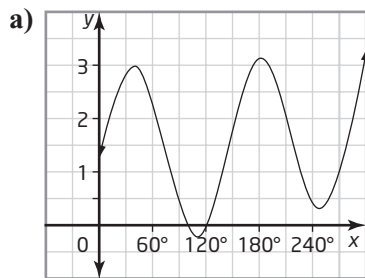
   a) $-2 \sin \theta$, for $\theta = 60°$

   b) $\frac{1}{2} \tan(2\theta + 30°)$, for $\theta = 45°$

2. **a)** Plot the ordered pairs $(0, 2)$, $(-1, -1)$, $(-1, 1)$, $(0, -2)$, $(1, -1)$, and $(1, 1)$.

   b) Describe the pattern made by the points.

A

1. Determine whether or not each graph is periodic. Justify your decision.

   a)

   b)

2. Determine the period, amplitude, domain, and range for each periodic function.

   a)

   b)
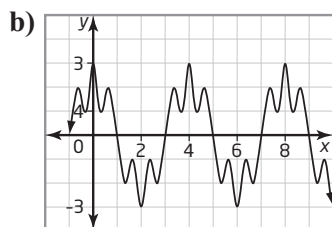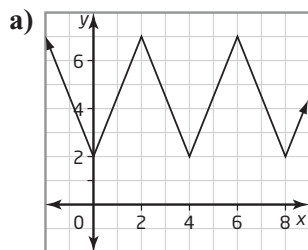
3. $y = f(x)$ is a periodic function.
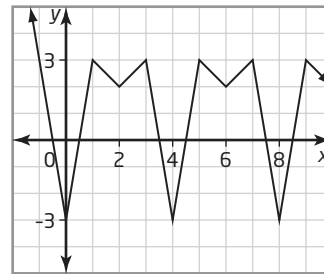
   a) Find the value of $f(2)$, $f(4)$, and $f(5)$.

   b) Find two other values of x that give you the same values of f as you found in part a).

B

4. Daniel is training by swimming lengths of a 50-m pool at a constant speed. The graph shows the distance from his starting point, in metres, over six cycles, in seconds.
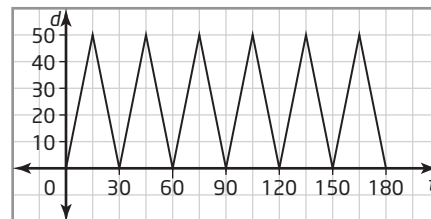
   a) Determine the amplitude and period of this function.

   b) After a month of training, Daniel increases his speed by 15%. If he swims six lengths of the pool, how do the period and amplitude compare with the values from part a)? Explain.

5. The table shows the mean monthly amounts of precipitation for Mount Forest, ON, using recorded data from over 100 years.

Month	Precipitation (cm)
January	10.5
February	8.7
March	7.9
April	7.0
May	7.1
June	7.7
July	8.3
August	9.1
September	9.3
October	8.9
November	10.2
December	10.2

a) Make a scatter plot of the data and draw a curve of best fit.

b) Does this data represent a periodic function? Explain your reasoning.

c) What are the maximum and minimum amounts of precipitation for Mount Forest? When do they occur?

6. The graph shows the mean monthly high and low temperatures, in degrees Celsius, for Windsor, ON, using recorded data from 50 years.

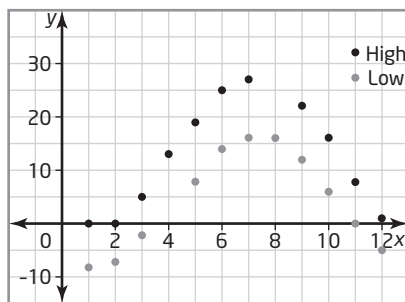**a)** Predict the average high temperature for August, and the average low temperature for April.

b) How accurate would your prediction be? Explain your reasoning.

7. The table shows the distance, in centimetres, from the number 12 on an analog clock to the tip of the second hand.

Time (s)	Distance (cm)
0	0
5	4.1
10	8.0
15	11.3
20	13.9
25	15.6
30	16.0

a) Assume that the data are periodic. Make a scatter plot of the data and extend it to 60 s.

b) How does the graph show that the data are periodic? Determine the period and amplitude of the function.

C

8. Carlos is cycling at a constant speed. The bicycle pedals have a maximum height of 35 cm and a minimum height of 5 cm from the ground. Carlos makes one revolutions in 0.8 s.

Sketch a graph representing a pedal's height above the ground, relative to time, for a 3-s period starting with the pedal at the minimum height.

5.2 Circles and the Sine Ratio
Textbook pp. 239–247

Prerequisite Skills

1. Use the sine, cosine, or tangent ratio to determine the length of the side indicated, to the nearest unit.

a)

b)

(Triangle with vertices E, F, G. Right angle at F. Angle at E is 39°. Side EF = 9.1 mm. Side FG labeled e.)

2. Determine the measure of angle x, to the nearest degree.

a)

(Triangle with vertices I, H, J. Right angle at H. Side IH = 8 mm. Angle at I is x. Side IJ = 14 mm.)

b)

(Triangle with vertices L, M, N. Right angle at N. Side ML = 25.1 cm. Side LN = 17.2 cm. Angle at M is x.)

3. Use the Pythagorean theorem to determine the length of each unknown side, to the nearest tenth of a unit.

a)

(Triangle with vertices P, Q, R. Right angle at Q. Side PQ = 24 cm. Side QR = 19 cm.)

b)

(Triangle with vertices W, X, Y. Right angle at Y. Side XW = 11.3 m. Side WY = 8.1 m.)
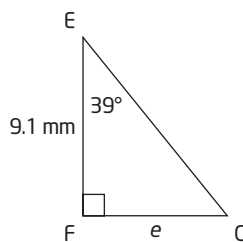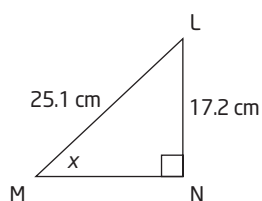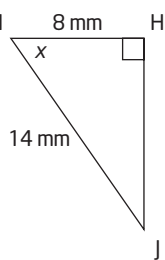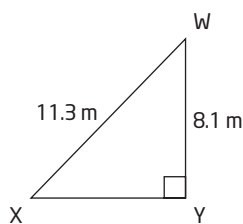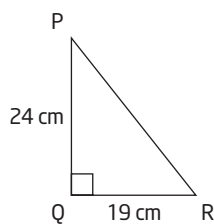
4. Evaluate with a calculator, to four decimal places.

a) $\cos 15°$

b) $\tan 60°$

5. Use a protractor to draw each angle.

a) $210°$

b) $325°$

A

1. Find two coterminal angles for each given angle. Draw each set of coterminal angles on the same sex of axes in standard position.

   a) 30°   **b)** 270°   **c)** −60°

2. Which pairs of angles are coterminal? Justify your answer.

   a) 45° and 450°   **b)** −60° and 300°

   c) 180° and 900°   **d)** 271° and −239°

3. An angle is in standard position and has terminal point P. For each set of coordinates for P, find the radius of the circle in exact form and the measure of the angle to the nearest tenth of a degree. Include a diagram illustrating each angle.

   > Plot the point, draw a right triangle using the terminal arm with point P as the hypotenuse, and the *x*-axis as the base of the triangle. Complete the triangle. Apply the Pythagorean theorem.

   a) P(−3, −6)   **b)** P(4, −7)

4. Find the coordinates of a terminal point, P(*x*, *y*), for each angle in standard position on the unit circle. Round to one decimal place. Draw a diagram to illustrate each angle.

   > Determine the sine and cosine of the given angle for a unit circle. Determine the values of *x* and *y* using the Pythagorean theorem.

   a) 60°   **b)** 135°   **c)** 310°

5. Verify that $(\cos\theta)^2 + (\sin\theta)^2 = 1$ for each of the following angles.

   a) $\theta = 21°$ **b)** $\theta = 315°$

6. Evaluate. Round to three decimal places. Explain why the sign of the result is either positive or negative.

   a) tan 210°   **b)** sin 300°   **c)** cos 120°

B

7. Find the measure of angle θ and find one coterminal angle with the same value of sin θ. Then find one non-coterminal angle that gives the same value of sin θ. Round your answers to the nearest degree.

   a) $\sin\theta = 0.8090$ **b)** $\sin\theta = -0.7071$

8. Determine the measure of angle θ in standard position, to the nearest degree.

   a) $\sin\theta = -0.6427$ if θ is in the fourth quadrant

   b) $\cos\theta = -0.2588$ if θ is in the third quadrant

   c) $\tan\theta = -1.6643$ if θ is in the second quadrant

9. The radius of a carousel is 8 m. Starting at the entrance, a rider rotates around the circumference of the carousel. Find the distance from the rider to the entrance of the carousel at each of the following angles.

   > Draw a diagram.

   a) $\theta = 30°$ **b)** $\theta = 190°$ **c)** $\theta = 310°$

C

10. Given angle θ in standard position with terminal arm in the third quadrant and $\sin\theta = -\dfrac{24}{25}$, determine the value of $\cos\theta + \sin\theta + \tan\theta$.

11. A circle is drawn on graph paper with the origin at its centre. What is the area of the smaller sector bound by the radii reaching to (6, 2) and (−2, 6), to the nearest tenth?

5.3 Investigate the Sine Function

Textbook pp. 248–253

Prerequisite Skills

1. **a)** Plot the ordered pairs $(0, 2)$, $(0, 0)$, $(-1, 0)$, $(2, -2)$, $(1, 0)$, and $(-2, -2)$.

   b) Describe the pattern made by the points.

2. Write the meaning of each statement.

   a) domain $= \{x \in \mathbb{R} \mid x > 0\}$

   b) range $= \{y \in \mathbb{R} \mid -2 \le y \le 2\}$

A

1. **a)** Make a table of values for $y = \sin x$, using $x = 0°, 30°, 60°, \dots, 360°$. Round the y-values to four decimal places.

   b) Extend the table to $720°$. What do you notice?

   c) Use your table to draw a sketch of $y = \sin x$. Label all intercepts and all maximum and minimum points.

2. A monorail operates at an amusement park to transport guests from one side of the park to the other. The monorail travels on a circular track around the perimeter of the park. The park has a radius of 3 km. One station is located at the easternmost point of the park.

   a) Draw the train track and station and collect data representing the distance of the monorail train north or south of the station, relative to the angle of rotation.

   b) Use your data to sketch a graph of the angle versus distance north or south of the station for one revolution around the track.

   c) How does the graph compare to that of $y = \sin x$?

B

3. A jack-in-the-box has a crank handle with a length of 1 dm (10 cm), and is rotated counterclockwise from a horizontal position.

Draw a diagram.

   a) What is the height of the handle, in decimetres, relative to its centre of rotation after it rotates $50°$?

   b) What is the height of the handle, in decimetres, relative to its centre of rotation after it rotates $210°$?

   c) How many degrees has the handle rotated through when its height is 0.9 dm relative to its centre of rotation?

C

4. **a)** Find an equation of the line that passes through all the minimum points of $y = \sin x$.

   b) Find an equation of the line that passes through the points on $y = \sin x$ where $x = 90°$ and $x = 180°$.

5. **a)** Sketch the graph of $y = \cos x$ on the same axes as $y = \sin x$.

   b) How can you verify that $y = \cos x$ is a function?

   c) Compare the graphs of $y = \sin x$ and $y = \cos x$. How are they the same? How are they different?

# 5.4 Investigate Transformations of Sine Curves

Textbook pp. 254–263

Prerequisite Skills

1. Write an equation for the quadratic function that results from each transformation.

   a) The graph of $y = x^2$ is translated 2 units downward and 1 unit to the right.

   b) The graph of $y = x^2$ is reflected in the x-axis and then compressed vertically by a factor of $\frac{1}{3}$.

   c) The graph of $y = x^2$ is stretched vertically by a factor of 4 and then translated 2 units to left and 1 unit up.

2. Write the meaning of each statement.

   a) domain $= \{x \in \mathbb{R} \mid x \le 4\}$

   b) range $= \{y \in \mathbb{R} \mid -5 < y \le 0\}$

3. Sketch a graph of each function. Then, write the domain and range for each function.

   a) $y = 3(x - 1)^2$

   b) $y = -2x^2 + 3$

A

1. Sketch a graph of each function for $0° \le x \le 360°$. Determine the period, amplitude, domain, and range.

   a) $y = 2 \sin x$ **b)** $y = -\frac{1}{4} \sin x$

2. Compare the graphs of each pair of functions for $0° \le x \le 360°$. Determine the period, amplitude, domain, range, phase shift, and equation of the horizontal axis.

   a) $y = \sin x - 2.5$ and $y = \sin x + 2.5$

   b) $y = \sin(x + 30°)$ and $y = \sin(x - 90°)$

3. Draw a sketch of $y = -5 \sin x$ for one period.

   a) Locate all the points where $y = 5$ and give the values of x.

   b) Locate all the points where $y = -5$ and give the values of x.

4. Draw a sketch of $y = \sin(x + 180°)$ for one period.

   a) Locate all the points where $y = 1$ and give the values of x.

   b) Locate all the points where $y = -1$ and give the values of x.

5. Write an equation for each sine function

   a)

   b)

   c)
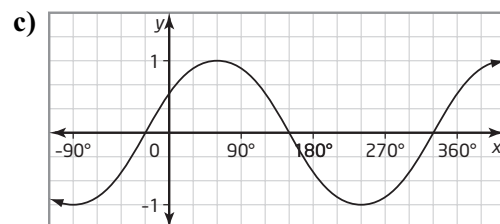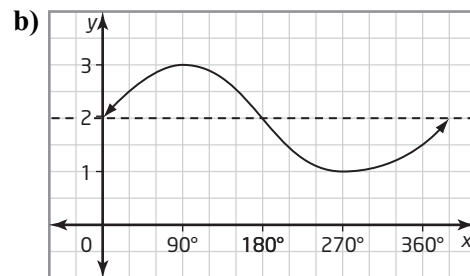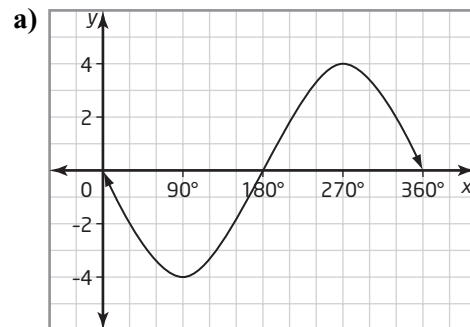
6. Graph one cycle of each function. Label the x-intercepts, the maximum points, the minimum points, and the equation of the horizontal axis. Write the domain and range of the cycle.

   a) $f(x) = 2 \sin x - 1$

   b) $f(x) = -3 \sin x$

   c) $f(x) = \sin(x - 60°)$

B

7. Without graphing, consider the function $y = \sin x + 55$. Identify the period, amplitude, phase shift, domain, range, and the equation of the horizontal axis.

8. For each of the functions, find the coordinates of the maximum and minimum points.

   a) $y = 2 \sin x - 5$

   b) $y = -\sin(x + 60°)$

9. Write an equation for each sine function. Indicate the intervals in which the function is increasing and decreasing over one period.

   a) amplitude $= 5$, horizontal axis along $y = 2$

   b) amplitude $= 1$, horizontal axis along the x-axis, phase shift of $45°$ to the right

10. A windmill with 3-m long blades has a centre 6 m above the ground. One of the blades starts out parallel to the ground.

   a) Sketch a graph that represents the height of the tip of the blade relative to the angle it forms with the horizontal as it rotates once fully. The blades rotate counterclockwise.

   b) Determine an equation that represents the height of the tip of the blade with respect to the ground.

C

11. The graph of a sine curve passes through the points $(0°, 12)$, $(90°, 10)$, $(180°, 8)$, and $(270°, 10)$. Determine an equation that represents this function.

12. A sine function of the form $y = \sin(x - d)$ has the same graph as $y = -\sin x$. Find all values of d.

5.5 Make Connections With Sine Functions
Textbook pp. 264–267

Prerequisite Skills
1. Sketch a graph of each function. Then, write the domain and range for each function.

    a) $y = -x^2 - 5$ **b)** $y = 2(x + 1)^2$

A

1. The depth of water, $d(t)$ metres, in a seaport can be approximated by the equation $d(t) = 1.5 \sin(29.52t - 44.28) + 12.4$, where t is the time in hours ($t = 0$ at 12:00 A.M.).

    a) Use a graphing calculator to graph the function for 24 h.

    b) Determine high tide and when it occurs.

    c) Determine low tide and when it occurs.

    d) Determine the period of the function.

    e) A cruise ship needs at least 12 m of water to dock safely. For how many hours in each period can the ship dock safely? Round your answer to the nearest tenth of an hour.

2. An object attached to the end of a spring is oscillating up and down. The displacement of the object is given by $y = 2.4 \sin(12t + 90.6)$, where t is the time, in seconds, and y is the distance, in centimetres. When the displacement is 0, the spring is at its relaxed length.

    a) Sketch a graph of the function for 60 s.

    b) What is the displacement from the spring's relaxed length after 12 s?

    c) What is the amplitude of the function? In what units is the amplitude measured?

B

3. A hand crank for a window rotates such that the height of the handle, h, in centimetres is given by $h(\theta) = 4 \sin(\theta + 45°)$, where θ is the rotational angle relative to the horizontal.

    a) Sketch a graph of this function for two rotations.

    b) At what height was the handle when the rotation began?

    c) What was the position of the handle relative the horizontal, when the rotation began?

4. A bridge will sway slightly if a large number of automobiles drive over it at once. During a period of heavy traffic, the displacement of the bridge's centre, in centimetres, relative to the normal position, is modelled by $f(t) = 2.1 \sin(47t)$, where t is the time, in seconds.

    a) Sketch a graph that shows the swaying of the bridge over 30 s.

    b) What is the displacement after 2 s?

    c) How wide is the sway?

    d) What is the period of the sway?

C

5. Without the use of technology, sketch a graph of each function for one period.

    a) $f(x) = \sin(x - 90°) - 3$
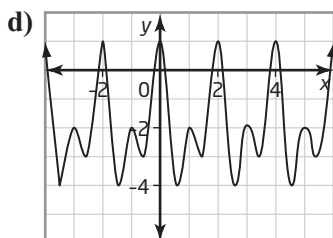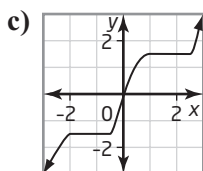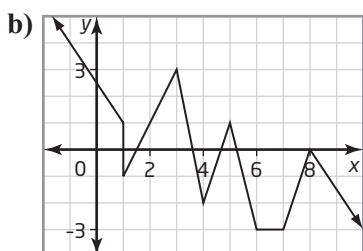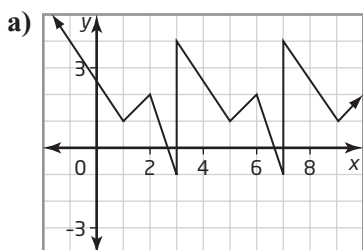
    b) $f(x) = -\dfrac{1}{4} \sin x + 1$

Chapter 5 Review

5.1 Periodic Functions

Textbook pp. 232–238

1. Determine whether or not each graph is periodic. If it is, determine the period, amplitude, domain, and range.

   a)

   b)

   c)

   d)

2. The table shows the high temperature in Waterloo, Ontario, on the first day of each month for one year.

Month	Temperature (°C)
Jan	−1
Feb	−1
Mar	3
Apr	12
May	19
Jun	25
Jul	27
Aug	26
Sep	22
Oct	14
Nov	6
Dec	0

   a) Sketch a graph of the data.

   b) Estimate the high temperature on April 15th and September 15th.

   c) Estimate the mean high temperature for the year.

5.2 Circles and the Sine Ratio

Textbook pp. 239–247

3. An angle is in standard position with its terminal point, P, given. Find the radius of the circle in exact form and the measure of the angle to the nearest tenth of a degree.

   a) $P(2, 6)$ b) $P(-3, 4)$

   c) $P(-1, -4)$ d) $P(4, -5)$

4. A terminal point P(x, y), on the unit circle forms the given angle in standard position. Find the coordinates of point P, to three decimal places.

a) 80°   **b)** 115°

c) 205°   **d)** 320°

5.3 Investigate the Sine Function
Textbook pp. 248–253

5. A crank handle is used to raise and lower a patio umbrella. The radius of the crank handle is 1 unit and it rotates counterclockwise starting from a horizontal position. Round answers to the nearest tenth of a unit when necessary.

a) What is the height of the handle after it rotates 30°?

b) What is the height of the handle after it rotates 100°?

c) What is the height of the handle after it rotates 375°?

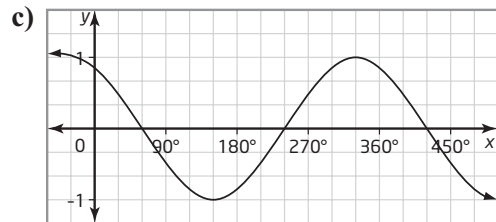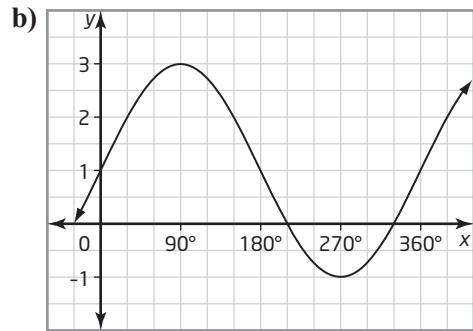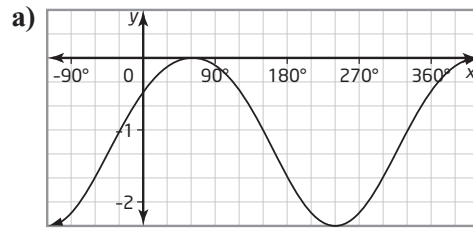**d)** Find two rotational angles at which the handle has a height of –0.5 units.

5.4 Investigate Transformations of Sine Functions
textbook pages 254– 263

6. Sketch a graph of each function for one cycle. Determine the period, amplitude, phase shift, domain, range, and the equation of the horizontal axis.

a) $y = -0.5 \sin x + 1$

b) $y = \sin(x - 45°) - 2$

c) $y = -3 \sin(x + 30°)$

7. Write an equation for each sine function.

a)

b)

**c)**

5.5 Make Connections With Sine Functions
textbook pages 264–267

8. The function $p(t) = 20 \sin 360t + 100$ models a person's blood pressure while resting, where $p(t)$ represents the blood pressure, in millimetres of mercury, and t is the time, in seconds.

a) Use a graphing calculator to graph the function for $0 < t < 3$.

b) Determine the period and the amplitude of the function, and describe what each represents.

c) How would the period and amplitude change if the person were exercising instead?

Chapter 5 Practice Exam

For questions 1 to 4, select the best answer.

1. What is the period of the graph shown?
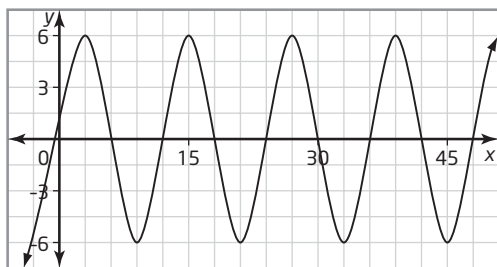
A 6   **B** 45   **C** 12   **D** 1

2. The value of c in $y = \sin x + c$ refers to

  A the phase shift

  B the amplitude

  C the vertical translation

  D the period

3. An angle θ is in standard position with terminal arm in the fourth quadrant. Which statement is true?

  A Sin θ is positive and tan θ is positive.

  B Cos θ is positive and sin θ is negative.

  C Cos θ is negative and tan θ is positive.

  D Cos θ is negative and sin θ is positive.

4. Which angle is not coterminal with 78°?

  A 438°   **B** 1158°   **C** 798°   **D** 1088°

5. Determine the measure of angle θ in standard position, to one decimal place.

  a) $\sin \theta = -0.9652$, if θ is in the third quadrant.

  b) $\cos \theta = 0.7123$, if θ is in the fourth quadrant.

  c) $\tan \theta = -1.8521$, if θ is in the second quadrant.

6. Sketch each angle in standard position. Then, find two coterminal angles for each given angle.

  a) 73°   **b)** 160°   **c)** 260°   **d)** 340°

7. Draw a sketch of $y = \sin x$ for one period.

  a) Locate all the points where $y = -1$ and give the values of x.

  b) Locate all the points where $y = 0.8660$ and give the values of x.

  c) Locate all the points where $y = 0.5$ and give the values of x.

8. Sketch a graph of each sine function, showing two cycles.

  a) $y = 2 \sin x - 4$

  b) $y = -\sin(x - 135°)$

9. Write an equation for each sine function. Indicate the intervals in which the function is increasing and decreasing over one period.

  a) amplitude $= 2$, horizontal axis along $y = -0.5$

  b) amplitude $= 0.5$, horizontal axis along the x-axis, phase shift of 60° to the left.

  c) amplitude $= 1$, horizontal axis along $y = 2$, phase shift of 45° to the right.

10. The height, h, in metres, of the tip of a fan blade, relative to the floor is defined by $h(\theta) = -0.26 \sin(2\theta) + 0.28$, where θ is the rotational angle, in degrees.

  a) With the help of a graphing calculator, sketch a graph of the function for three rotations of the fan.

  b) What would be the maximum and minimum heights of the fan blade?

  c) Determine the height of the tip of the fan blade after rotations of 190° and 550°. Explain the results.