Chapter 6 Exponential Functions

Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 278–279 of the *Functions and Applications 11* textbook.

Work with Powers

1. Write the base and the exponent of each power. Then, write each power in expanded form.

a) 9^2 **b)** 4^6 **c)** 3.7^8 **d)** $\left(-\frac{2}{7}\right)^4$ **e)** $2a^2$ **f)** $\left(\frac{1}{5}\right)^5$

2. Write each product as a single power.

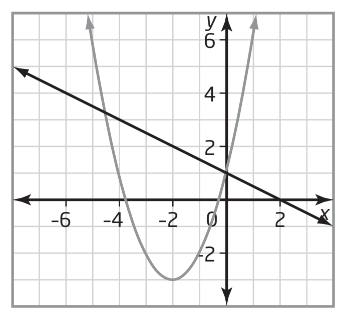
a) $6 \times 6 \times 6 \times 6$ **b)** $(-2)(-2)(-2)(-2)$ **c)** $1.8 \times 1.8 \times 1.8$

Graph Linear and Quadratic Relations

3. Graph each relation.

a) $y = x + 5$ **b)** $y - 4x = 0$ **c)** $y = x^2 + 2$ **d)** $y = -x^2 - 6$

4. Refer to the graph shown. Copy and complete the equations.



a) Write an equation for the line.

$y = mx + b$

$= \square$

b) Write an equation for the parabola.

$y = a(x - h)^2 + k$

$= \square$

Use the vertex and the *y*-intercept.

5. The graph of $y = x^2$ is translated right 4 units and up 2 units.

a) Write the equation of the translated relation.

Use the vertex form of the equation. What is the new vertex?

b) Expand and simplify the equation you found in part a).

Model Data

6. Tamara opened an ice cream stand on the beach. She kept track of her sales over the first six days of business. The data are shown.

Day	1	2	3	4	5	6
Cones Sold	5	16	27	40	52	64

a) Graph the data.

b) Which model, linear or quadratic, seems to fit the data better?

c) Sketch a line or curve of best fit. Then, predict the sales for the seventh day of business.

Evaluate Formulas

7. Evaluate each formula for the given values.

a) $P = 4s$ for $s = 5$ cm **b)** $A = 0.5bh$ for $b = 8$ m and $h = 2$ m

c) $F = 9h + 1.3t$ for $h = 1$ h and $t = 0.4$ h **d)** $V = \pi r^2 h$ for $r = 9$ cm and $h = 12$ cm

6.1 The Exponent Rules

Textbook pp. 280–287

Prerequisite Skills

1. Write the base and the exponent of each power.

a) 6^2 **b)** 7^5 **c)** 8.12^8 **d)** $\left(\frac{1}{4}\right)^9$ **e)** $3x^7$ **f)** $\left(-\frac{3}{8}\right)^4$

2. Write each product as a single power.

a) $8 \times 8 \times 8 \times 8 \times 8$ **b)** $(-5)(-5)(-5)$ **c)** $11 \times 11 \times 11 \times 11$

d) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ **e)** $4.1 \times 4.1 \times 4.1$ **f)** $6.7 \times 6.7 \times ... \times 6.7$ (40 terms)

3. Write each power in expanded form.

a) 3^7 **b)** $(-2.5)^3$ **c)** $\left(\frac{3}{5}\right)^6$ **d)** 4.8^2 **e)** $\left(-\frac{1}{6}\right)^3$ **f)** $(-1.75)^4$

A

1. Use the exponent rules to express each of the following as a single power.

a) $21^2 \times 21^5$ **b)** $15^4 \times 15^7$

c) $\left(\frac{3}{8}\right)^4 \times \left(\frac{3}{8}\right)^3$ **d)** $(-10)^2(-10)^9$

2. Use the exponent rules to express each of the following as a single power.

a) $17^8 \div 17^4$ **b)** $\frac{12^8}{12^5}$

c) $5^{15} \div 5^4$ **d)** $\frac{(-9)^{11}}{(-9)^4}$

3. Use the exponent rules to express each of the following as a single power.

a) $(8^3)^5$ **b)** $(4^4)^4$

c) $(19^9)^6$ **d)** $(-7^4)^6$

4. Use the exponent rules to express each of the following as a single power.

a) $2^7 \times 2^3$ **b)** $6^2 \times 6^{10}$

c) $\frac{3^8}{3^2}$ **d)** $\frac{7^9}{7^6}$

5. Use the exponent rules to express each of the following as a single power.

a) $\frac{9^2 \times 9^5}{9^6}$ **b)** $\frac{10^{20}}{(10^7)^2}$

c) $\frac{(-2)^3 \times (-2)^9}{[(-2)^4]^5}$ **d)** $(4^5)^2$

6. Use the exponent rules to express each of the following as a single power.

a) $\frac{\left(-\frac{2}{3}\right)^9}{\left(\frac{2}{3}\right)^4}$ **b)** $\frac{(-1.25)^{10}}{(-1.25)^7 \times (-1.25)}$

c) $\left[\left(\frac{1}{5}\right)^6\right]^2$ **d)** $(-12)^4(-12)^2$

7. Use the exponent rules to simplify each algebraic expression.

a) $(x^4)^8$

b) $(x^2y^3)^2$

c) $(4xy^2)(3x^3y^3)$

d) $\frac{6x^6y^3}{3xy}$

e) $\frac{(5x^2y^7)^3}{25x^2y}$

8. Consider the expression $\dfrac{8^9 \times 8^3}{8^5 \times 8^4}$.

a) Evaluate each power using a calculator and then evaluate the whole expression.

b) Simplify the expression first using the exponent rules and then evaluate the simplified form.

c) Which method, a) or b), do you prefer? Why?

9. Consider the power 3^{36}.

a) Write this power as the product of two powers in three different ways.

b) Write this power as the quotient of two powers in three different ways.

c) Write this power as the power of a power in three different ways.

d) Write this power as a product of the greatest possible number of powers of 3, each of which has an exponent greater than 1.

10. Evaluate and simplify each of the following without using a calculator.

a) $\left[\left(\dfrac{1}{3}\right)^2\right]^2$

b) $\dfrac{\left(\dfrac{2}{7}\right)^7}{\left(\dfrac{2}{7}\right)^5}$

c) $[(10)^3]^5$

B

11. **a)** Determine two positive integers x and y such that $9^x = 3^y$.

b) Are these the only values of x and y that will work? If so, explain why. If not, find another set of values that will work.

12. Computers use the binary number system, which represents numbers as sums of powers of 2. For example, $12 = 2^3 + 2^2$.

a) Express 36 as a sum of powers of 2.

b) Express 144 as a sum of powers of 2.

c) What number is represented as $2^8 + 2^6 + 2^4 + 2^2$?

13. Sound intensity levels are recorded in decibels (dB). The faintest sound that can be heard by the human ear is assigned an intensity level of 0 dB. A sound that is 10 times more intense is assigned a sound level of 10 dB. A sound that is 10^2 or 100 times more intense is assigned a sound level of 20 dB. To calculate how much more intense sound A is than sound B, divide their intensity levels by 10 to get a and b, then use the ratio $\dfrac{10^a}{10^b}$.

Source	Intensity Level
Threshold of hearing	0 dB
Rustling leaves	10 dB
Whisper	20 dB
Normal conversation	60 dB
Busy street traffic	70 dB
Vacuum cleaner	80 dB
MP3 player at maximum volume	100 dB
Front row at a rock concert	110 dB
Threshold of pain	130 dB

a) How much more intense is the sound from normal conversation compared to the sound of rustling leaves?

b) How much more intense is the sound from an MP3 player at maximum volume compared to the sound of busy traffic?

C

14. Use the exponent rules to show that $[(2^2)^6]^3$ is equal to $[(2^6)^3]^2$. Do not evaluate.

15. Use the exponent rules to simplify each expression. Then, evaluate.

a) $\dfrac{6^{4.2}}{6^{1.2}}$ **b)** $2^{0.7} \times 2^{4.3}$ **c)** $(3^5)^{0.4}$

6.2 Evaluate Powers with Integer Exponents

Textbook pp. 288–295

Prerequisite Skills

1. Use the exponent rules to express each of the following as a single power.

a) $9^3 \times 9^5$ **b)** $25^6 \times 25^4$ **c)** $6^4 \times 6^5$

d) $(-8)^{10}(-8)^8$ **e)** $\left(\frac{1}{4}\right)^7 \times \left(\frac{1}{4}\right)^5$ **f)** $\left(-\frac{5}{3}\right)^9 \times \left(-\frac{5}{3}\right)^8$

2. Use the exponent rules to express each of the following as a single power.

a) $11^9 \div 11^4$ **b)** $\frac{7^{12}}{7^7}$ **c)** $16^{15} \div 16^9$

d) $\frac{(-3)^8}{(-3)^5}$ **e)** $10^7 \div 10^3$ **f)** $\frac{(-2)^{10}}{(-2)^2}$

3. Use the exponent rules to express each of the following as a single power.

a) $(5^6)^3$ **b)** $(15^2)^8$ **c)** $(20^5)^9$

d) $(-12^7)^6$ **e)** $(-4^5)^8$ **f)** $(-17^4)^7$

4. Use the exponent rules to express each of the following as a single power.

a) $\frac{14^{11} \times 14^4}{14^6}$ **b)** $\frac{33^{25}}{(33^4)^4}$ **c)** $\frac{(-5)^6 \times (-5)^7}{[(-5)^3]^5}$

d) $\frac{[(-8)^9]^3}{(-8)^{10}}$ **e)** $\frac{(-0.75)^{15}}{(-0.75)^8}$ **f)** $\left[\left(\frac{2}{3}\right)^8\right]^9$

A

1. Evaluate each power without a calculator. Express each answer as a fraction.

a) 7^{-2} **b)** 2^{-4}

c) 6^{-1} **d)** 4^{-3}

e) 18^{-1} **f)** 9^{-2}

2. Evaluate each power using a calculator. Express each answer as a fraction.

a) 12^{-5} **b)** 10^{-8}

c) 3^{-9} **d)** 15^{-6}

3. Express each fraction or decimal as a power with a negative exponent. For the base, use the smallest positive integer possible.

a) $\frac{1}{81}$ **b)** $\frac{1}{64}$

c) 0.000 1 **d)** $\frac{1}{1\,000\,000}$

4. Evaluate. Simplify answers as much as possible while leaving them as fractions.

a) $6^1 + 6^{-2}$ **b)** $5^{-3} + 5^0$

c) $2^0 - 2^{-4}$ **d)** $3^{-1} + (-3)^{-3}$

5. Evaluate. Where fractions are used, leave answers in fraction form.

a) $\left(\frac{1}{2}\right)^{-3}$ **b)** $\left(\frac{3}{4}\right)^{-4}$

c) $(0.4)^{-2}$ **d)** $(0.005)^{-3}$

6. Use two different positive bases to express each fraction as a power with a negative exponent.

a) $\frac{1}{256}$ **b)** $\frac{1}{512}$

7. Use the exponent rules to express each of the following as a single power.

a) $\dfrac{5^4 + 5^{-3}}{5^4}$ b) $\dfrac{6^8}{(6^2)^5}$

c) $\dfrac{3^6 \times 3^{-2}}{(3^3)^4}$ d) $(-10^5)^{-1}$

8. Use the exponent rules to express each of the following as a single power.

a) $\dfrac{\left(\frac{2}{3}\right)^6}{\left(\frac{2}{3}\right)^7}$ b) $\dfrac{(0.8)^7}{(0.8)^2 \times (0.8)^3}$

c) $\left[\left(-\frac{1}{5}\right)^4\right]^2$ d) $(-4)^3(-4)^{-9}$

9. The diameter of an E. coli bacterium is approximately 0.000 000 8 m. Express this as 8 multiplied by a power of base 10 with a negative exponent.

10. Express 9^{-4} as a power with base 3.

B

11. Kim has an upright piano. B is tuned to a standard 493.88 Hz. What is the frequency of a note 3 octaves below this B on Kim's piano?

The frequency change from one octave to the next is 2^{-1}.

12. Sound pressure, p, which is measured in pascals (Pa), is inversely proportional to the distance r from the sound source, $p = \dfrac{k}{r}$.

a) Dave is 6 m away from his stereo, which is playing in another room. At this distance, the sound pressure is 2×10^{-2} Pa. If he sits down 2 m from the speakers, what is the sound pressure?

b) Jessie and Ed are 1 m away from the speakers at a rock concert. At this distance, the sound pressure is 2×10^0 Pa. If they walk 20 m to the back of the concert hall, what is the sound pressure?

13. The acceleration due to an object's gravity follows an inverse-square law. The acceleration due to a planet's gravity at a particular distance r from the centre of the planet can be calculated from the formula $g = G\dfrac{m_1}{r^2}$, where G is the universal gravitational constant (approximately 6.7×10^{-11} $m^3 kg^{-1} s^{-2}$) and m_1 is the mass of the planet.

a) The radius of Mercury is approximately 2.4×10^6 m and its mass is approximately 3.3×10^{23} kg. Calculate the acceleration due to gravity on Mercury's surface.

b) The radius of Mars is approximately 3.4×10^6 m and its mass is approximately 6.4×10^{23} kg. Calculate the acceleration due to gravity on Mars's surface.

c) The radius of Jupiter is approximately 7.1×10^7 m and its mass is approximately 1.9×10^{27} kg. Calculate the acceleration due to gravity on Jupiter's surface.

C

14. Assign the values 2, 3, −2, −3 to u, v, x, and y, such that the expression $u^v - x^y$ has a minimum and maximum value.

15. Find three pairs of integers x and y such that $x^y - y^x = -1$.

6.3 Investigate Rational Exponents

Textbook pp. 296–304

Prerequisite Skills

1. Evaluate each power without a calculator. Express each answer as a fraction.

a) 3^{-3} **b)** 8^{-2} **c)** 4^{-4} **d)** 5^{-2} **e)** 1^{-20} **f)** 10^{-4}

2. Evaluate each power using a calculator. Express each answer as a fraction.

a) 5^{-6} **b)** 17^{-4} **c)** 6^{-5} **d)** 9^{-4}

3. Express each fraction or decimal as a power with a negative exponent. For the base, use the smallest positive integer possible.

a) $\frac{1}{27}$ **b)** $\frac{1}{32}$ **c)** $\frac{1}{216}$ **d)** 0.04

4. Evaluate. Simplify answers as much as possible while leaving them as fractions.

a) $7^0 + 7^0$ **b)** $4^{-3} + 4^3$ **c)** $6^0 - 6^{-2}$ **d)** $2^{-3} - (-2)^{-4}$

5. Evaluate. Leave answers in fraction form.

a) $\left(\frac{1}{5}\right)^{-2}$ **b)** $\left(\frac{3}{2}\right)^{-4}$ **c)** $\left(\frac{1}{9}\right)^{-2}$ **d)** $\left(\frac{1}{3}\right)^{-3}$

6. Evaluate each formula for the given values.

a) $P = 4s$ for $s = 9$ cm **b)** $A = 0.5bh$ for $b = 7$ m and $h = 4$ m

c) $F = 3h + 24t$ for $h = 2$ h and $t = 0.5$ h **d)** $V = \pi r^2 h$ for $r = 4$ cm and $h = 15$ cm

A

1. Evaluate, without using a calculator.

a) $16^{\frac{1}{4}}$ **b)** $36^{\frac{1}{2}}$ **c)** $81^{\frac{1}{2}}$

d) $25^{\frac{1}{2}}$ **e)** $(-64)^{\frac{1}{3}}$ **f)** $900^{\frac{1}{2}}$

g) $\sqrt{1600}$ **h)** $\sqrt[3]{1000}$ **i)** $\sqrt[3]{-8000}$

2. Evaluate, without using a calculator.

a) $\left(\frac{1}{25}\right)^{\frac{1}{2}}$ **b)** $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}}$

$= \frac{\square}{\square}$

c) $\left(\frac{256}{625}\right)^{\frac{1}{4}}$ **d)** $\left(-\frac{216}{1000}\right)^{\frac{1}{3}}$

e) $\sqrt[3]{128}$ **f)** $\left(\frac{9}{64}\right)^{\frac{1}{2}}$

3. Evaluate, without using a calculator.

a) $81^{\frac{3}{4}} = (\sqrt[4]{81})^3$

$= (\square)^3$

$= \square$

b) $(-216)^{\frac{2}{3}}$

c) $8^{\frac{2}{3}}$ **d)** $16^{\frac{3}{4}}$

e) $64^{\frac{5}{6}}$ **f)** $0.027^{\frac{2}{3}}$

4. Evaluate, without using a calculator.

a) $\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{9}}{\sqrt{16}}\right)^3$

$= \left(\frac{\square}{\square}\right)^3$

$= \square$

b) $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

c) $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **d)** $\left(\frac{27}{125}\right)^{\frac{4}{3}}$

e) $0.0004^{\frac{3}{2}}$ **f)** $0.0081^{\frac{3}{4}}$

5. Evaluate, without using a calculator.

a) $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$

b) $\left(\frac{125}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{125}\right)^{\frac{2}{3}}$

$= \left(\frac{\sqrt[3]{8}}{\sqrt[3]{125}}\right)^2$

$= \left(\frac{\square}{\square}\right)^2$

$= \boxed{}$

c) $\left(\frac{16}{9}\right)^{-\frac{3}{2}}$

d) $\left(\frac{81}{625}\right)^{-\frac{3}{4}}$

6. Use a calculator to evaluate each of the following. Round answers to 3 decimal places.

a) $17^{\frac{1}{3}}$

b) $26^{\frac{2}{5}}$

c) $(-3)^{-\frac{1}{4}}$

d) $22^{-\frac{2}{3}}$

7. Write each expression as a power. Do not evaluate.

a) $\sqrt[3]{14}$

b) $\left(\sqrt[3]{14}\right)^5$

B

8. Due to the "square-cube" law, larger animals have thicker limbs in proportion to their bodies than smaller animals. This can be represented using the relation $s = \left(\frac{h_2}{h_1}\right)^{\frac{3}{2}}$, where s is the strength factor (or thickness factor) required for the limbs, h_1 is the original height, and h_2 is the new height. Both h_1 and h_2 are measured in units of length, so the ratio is a number without any units, and so is s.

a) A giant kangaroo is 5 times the height of the 1.6-m tall kangaroo from which it is modelled. To comply with the square-cube law, how much thicker must the legs be compared with the kangaroo's?

b) A snowshoe hare is 45 cm tall. An artist wants to paint a snowshoe hare that is 1.5 m tall. How much thicker must the legs of the painted hare be compared with the actual hare's?

9. The scientist Johannes Kepler (1571–1630) developed three laws that describe how the planets orbit the sun. The third law relates the average radius r of the orbit to the period T of the orbit (that is, the length of time in years it takes the planet to complete one orbit): $T = kr^{\frac{3}{2}}$. The radius of Earth's orbit is 1 astronomical unit (AU) and the period of the orbit is 1 year, so $k = 1$.

a) The radius of the orbit of Saturn is about 9.54 AU. How many years does it take Saturn to circle the sun once?

b) The radius of the orbit of Neptune is about 30.06 AU. How many years does it take Neptune to circle the sun once?

c) The radius of the orbit of Venus is about 0.72 AU. How long does it take Venus to circle the sun once? Give your answer in days.

10. In the 1930s, the biologist Max Kleiber studied animals' metabolic rates. He expressed the ratio of the rates for two animals by the relation $\frac{r_1}{r_2} = \left(\frac{m_1}{m_2}\right)^{\frac{3}{4}}$, where r_1 and r_2 are the animals' metabolic rates and m_1 and m_2 are the animals' masses.

a) Compare the metabolic rates of a 820-kg moose and a 62-kg cougar.

b) Compare the metabolic rate of a 27-kg boa constrictor with that of a 2.3-kg rattlesnake.

C

11. Find values of x and y such that $\left(\frac{8}{125}\right)^x = \left(\frac{25}{4}\right)^y$

12. Find a value of x such that $\frac{1}{4}x = 200^x$. If this is not possible, explain why.

Use a spreadsheet to test values of x.

6.4 Model Data with Exponential Functions

Textbook pp. 305–311

Prerequisite Skills

1. Aldo opened a hotdog stand beside the sports stadium. He kept track of his sales over the first six days of business. The data are shown.

Day	1	2	3	4	5	6
Hotdogs Sold	12	43	93	154	231	319

a) Graph the data.

b) Which model, linear or quadratic, seems to fit the data better?

c) Sketch a line or curve of best fit. Then, predict the sales for the seventh day of business.

2. a) Copy and complete the table by calculating the first and second finite differences.

x	y	**First Difference**	**Second Difference**
−2	0		
−1	−1		
0	0		
1	3		
2	8		
3	15		
4	24		

b) Is the relation linear or quadratic? Justify your answer.

A

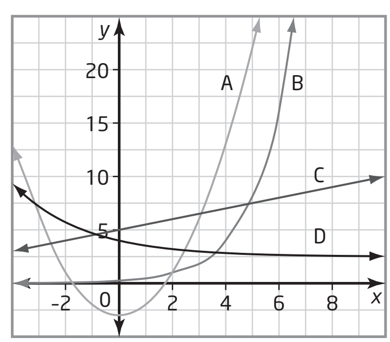
1. Classify each of the following situations as linear, quadratic, or exponential. Explain your choices.

a) Cora's father gave her an allowance of \$20 and increased her allowance by the square of the month number, starting with 1.

b) Sonya's father gave her an allowance of \$20 and increased her allowance by \$3 every month.

c) Martin's father gave him an allowance of \$20 and increased his allowance by 5% every month.

2. Classify each of the graphs shown as linear, quadratic, or exponential. Explain your choices.



B

3. A pond is being covered with algae. The table shows the surface area of the pond that is not covered with algae each month.

Month	Uncovered Surface Area of Pond (m^2)
0	2560
1	1280
2	640

a) Copy the table of values and extend the pattern for three more months.

b) Plot the data.

c) Draw a smooth curve through all of the data points.

d) Describe your graph.

e) Use a graphing calculator to determine if the data models an exponential relation. Was your description in part d) correct? Explain.

f) When will the pond be completely covered with algae? Explain how you found your answer.

4. Louis decided to take a bicycle trip to visit his brother in Chalk River, a distance of 300 km. On the first day, he rode half the distance. On the second day, he rode half of the remaining distance. He continued this pattern each day, riding half the distance he had ridden the day before.

a) Make a table of values to show the distance Louis has travelled each day for 5 days.

b) Plot the data on a graph. Draw a smooth curve through the data points.

c) How long will it take before Louis is within 1 km of Chalk River? Explain how you found your answer.

d) When will he reach Chalk River? Explain.

5. The table shows the yearly average price for a barrel of crude oil for the past 11 years.

Year	Average Price (US$)
1998	12.28
1999	17.48
2000	27.60
2001	23.12
2002	24.36
2003	28.10
2004	36.05
2005	50.64
2006	61.08
2007	69.08
2008*	109.30

*From January to July

a) Graph the data in the table. Draw a curve of best fit through the data points.

b) Describe the graph. Use words such as increasing, decreasing, linear, quadratic, and exponential.

6. The temperature of the water in an outdoor hot tub was 40°C when the owner turned the heater off over night. Seven hours later, the owner checked the hot tub and found that the temperature had dropped to 32°C. If the hot tub were left to cool for another 7 h, is it possible for its temperature to drop to 22°C? Explain.

C

7. Kristine bought a flower that was 4 cm tall. The next day, she found that the flower had grown half of its height, 2 cm. If the flower keeps growing in this pattern, how long will it take for the flower to be taller than 7.5 cm?

6.5 Exponential Functions and Their Properties

Textbook pp. 312–318

Prerequisite Skills

1. Classify each of the following situations as linear, quadratic, or exponential. Explain your choices.

a) Wilson has a job that pays \$12/h. He gets a 5% raise each year.

b) Rani has a job that pays \$13/h. She gets a raise of \$0.50/h each year.

c) Enrico has a job that pays \$7/h. He gets a raise equal to \$0.10/h times the square of the year number, starting with 1.

2. Refer to question 1. Use a spreadsheet or a graphing calculator and table to determine who will be earning more per hour after

a) 5 years **b)** 10 years **c)** 15 years

If using a graphing calculator, use a table to organize your work.

A

1. Consider the relation $y = 4^x$.

a) Sketch the graph of this relation.

b) Explain why this relation is a function.

c) Express the function using function notation.

d) What are the domain and range of this function?

e) Find the intercepts.

f) Find the intervals of increase and decrease.

g) Find any asymptotes.

2. Consider the exponential functions.

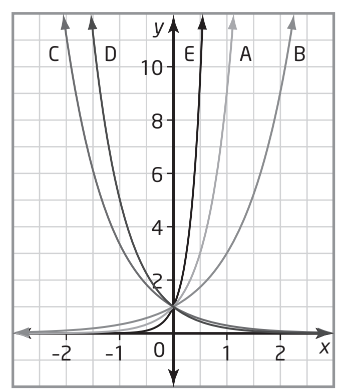
- $f(x) = 9^x$
- $g(x) = 3^x$
- $h(x) = (0.3)^x$

a) Which is greatest when $x = 2$?

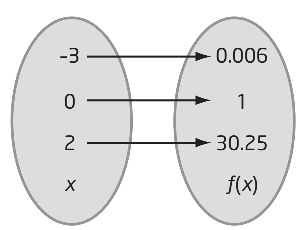
b) Which is greatest when $x = -2$?

c) For what value of x do all three functions have the same value? What is this value?

3. Match the graphs shown to the functions in question 2.



4. A mapping diagram for an exponential function is shown. Write an equation for the function.



5. Raminder has constructed a function machine. The table shows several input and output values. Find the equation of an exponential function that matches the input and output of the machine.

Input	Output
−2	0.08163
0	1
2	12.25
3	42.875

6. Each of the following situations can be modelled with an exponential function. Indicate which require a value of $a > 1$, and which require a value of $0 < a < 1$. Explain your choices.

a) Water in a pond absorbs 5% of the light passing through it for every metre of depth.

b) A spider population doubles in number every 40 days.

c) Half of a radioactive isotope in a test tube decays every 2 s.

B

7. Water absorbs light, as scuba divers observe. When Noreen went diving, she took a light meter with her. She noticed that the amount of light available decreased by 5% for every 8 m that she descended.

a) What fraction of the surface intensity of light is left at a depth of 8 m? at 16 m?

b) Assuming that the meter read 1 unit at the surface, model the decrease in light intensity with an exponential function, in 8-m intervals.

c) At what depth will the light intensity drop to half the intensity at the surface?

8. Patrick went diving with a light meter. When he was at a depth of 20 m, only $\frac{1}{70}$ of the surface intensity of the light remained. Use your calculator or other technology to find an exponential function that models the decrease in light intensity.

9. The Palermo Hazard Impact scale represents the potential hazard of the impact of an asteroid or other object hitting the Earth. In simplified form, the scale follows the relation $H = 10^P$, where H is the hazard and P is the Palermo scale value.

a) How does the hazard posed by an asteroid with a Palermo scale value of 4 compare with the hazard of an asteroid with a Palermo scale value of 1?

b) A newspaper story claims that an asteroid with a Palermo scale value of 8 is four times as hazardous as one with a scale value of 2. Explain how the story needs to be corrected. Include numbers.

C

10. A computer program will take any integer as input, and output a square with side lengths of 10^x units, where x is the input number. Ajavit generated squares using −2 and 5. How many of the first square can fit inside the second square?

6.6 Compare Linear, Quadratic, and Exponential Functions

Textbook pp. 319–325

Prerequisite Skills

1. Classify each of the following situations as linear, quadratic, or exponential. Explain your choices.

a) Paula has a stock that has increased by $4 every week for the past two months.

b) Sophia has a stock that has increased every week by the square of the week number for the past two months.

c) Daniel has a stock that has increased by 3% every week for the past two months.

A

1. Classify each of the following relations as linear, quadratic, or exponential.

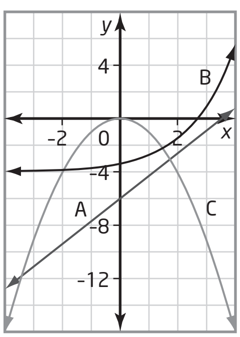
a) $y = -6x$ **b)** $y = 5x - 1$

c) $y = (x + 3)^2$ **d)** $y = -x$

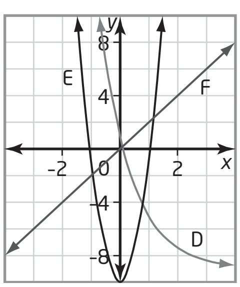
e) $y = (2.2)^x$ **f)** $y = 4x^2$

2. Classify each of the relations shown as linear, quadratic, or exponential.

a)



b)



3. Predict whether a linear, quadratic, or exponential model would best fit each situation.

a) the number of bacteria in a petri dish, given that the bacteria double every 4 h

b) the height of a diver in the first few seconds after jumping from a diving board

c) the value of shares that have been decreasing at a rate of 3% every year

d) the position of a car driving down a straight road at a constant speed of 50 km/h

4. Use first differences, second differences, and/or ratios to classify each relation as linear, quadratic, exponential, or none of these.

a)

x	*y*
−3	4.629
−2	2.778
−1	1.667
0	1.000
1	0.600
2	0.360
3	0.213

b)

x	*y*
−3	9.2
−2	8.8
−1	8.4
0	8.0
1	7.6
2	7.2
3	6.8

c)

x	y
−3	5.208
−2	4.167
−1	1.667
0	1.000
1	0.400
2	0.160
3	0.128

d)

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

B

5. Reena purchased shares in Nickel Mining Corp. for $16.75 per share. She kept track of the share value each week for 10 weeks, as shown. Is its growth best modelled by a linear, a quadratic, or an exponential function? Justify your answer.

Time (weeks)	Share Value ($)
0	16.75
1	17.76
2	18.82
3	19.95
4	21.15
5	22.42
6	23.76
7	25.19
8	26.70
9	28.30
10	30.00

6. There might be a leak in Scott's scuba gear. If the leak is in the tank itself, the pressure in the tank should decrease exponentially. If the leak is in the regulator, the pressure should decrease linearly. Scott filled the tank to its maximum rating of 22 500 KPa and kept a record of time and pressure, as shown.

Time (min)	Pressure (KPa)
0	22 500
5	21 264
10	20 028
15	18 792
20	17 556
25	16 320
30	15 084

a) Is the leak linear, exponential, or neither? Justify your answer.

b) Should Julian repair the tank or the regulator? Explain.

7. The cost of the paint to cover a cylindrical tank with a height of 4 m is shown. Is the relation between cost and height best modelled using a linear, quadratic, or exponential relation? Justify your answer.

Radius (m)	Cost of Paint ($)
1.4	198.60
2.0	561.47
2.6	1156.45
3.2	2012.31

C

8. How does the graph of $y = 40(2^{x-1})$ compare with the graph of $y = 10(2^{x+1})$?

9. Classify each of the following relations as linear, quadratic, or exponential.

a) $y = -\dfrac{6x^7}{11(x^3)^2}$

b) $y = 3x^{-18}\left(\dfrac{x^{-4}}{(x^2)^3}\right)^{-2}$

c) $y = (2^x + 4)^2 - (2^x - 4)^2 + 2^{x+3}$

6.7 Exponential Growth and Decay

Textbook pp. 326–333

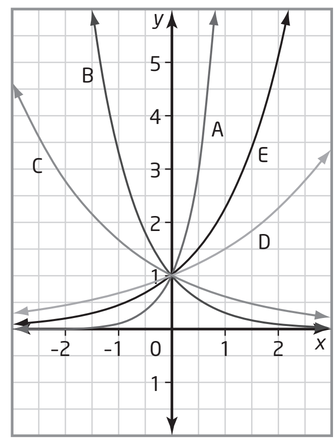
Prerequisite Skills

1. Consider the exponential functions.

- $f(x) = 2.25^x$
- $g(x) = 1.5^x$
- $h(x) = (0.6)^x$

a) Which is greatest when $x = 2$?
b) Which is greatest when $x = -2$?
c) For what value of x do all three functions have the same value? What is this value?

2. Match the graphs shown to the functions in question 1.



A

1. A fossil was found in a rock layer believed to have been deposited 20 000 years ago. What percent of carbon-14 is expected to remain in the fossil?

Use the function $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

2. The fossil of a tree was found to contain 18% of its original carbon-14. To the nearest hundred years, estimate the age of the fossil. Use a calculator to check your estimate. How close was your estimate?

If you have a graphing calculator, you can use the intersect method from Example 2 in your textbook to check your estimate.

3. Suppose that a bacterial culture was known to double every 2 days. After 46 days, it covered the entire surface area of the agar in the petri dish. When did it cover half the area? Explain.

You could graph the function and interpolate.

4. The persistence of drugs in the human body often follows an exponential model. Suppose that a new drug follows the model $M = M_0(0.79)^{\frac{h}{3}}$, where M is the mass of drug remaining in the body, M_0 is the mass of the dose taken, in milligrams, and h is the time in hours since the dose was taken.

a) A standard dose is 60 mg. Sketch a graph showing the mass remaining in the body up to 12 h.

b) Use your graph to estimate the half-life of the drug in the body.

c) Check your estimate in part b) using the equation.

d) Once the mass remaining drops is less than 1 mg, the standard test can no longer detect the drug. How long will it take before the drug is no longer detectable? Use a graphing calculator.

5. The half-life of iridium-192, a radioactive isotope, can be modelled using the function $P = 100\left(\frac{1}{2}\right)^{\frac{t}{74}}$, where P represents the percent of iridium-192 remaining and t represents the time, in days.

   a) Sketch a graph of the percent of iridium-192 remaining versus time for 0 days to 50 days.

   b) Use your graph to predict the time when 75% of the sample remains.

   c) Use the function to determine the percent remaining after 100 days.

   d) What is the half-life of iridium-192? How did you find your answer?

6. Portia grew a colony of bacteria as a science project. She measured the bacterial growth for two weeks. She found that the growth could be modelled by the exponential function $N(t) = 7(1.15)^{\frac{1}{2}}$, where $N(t)$ represents the number of bacteria present, in thousands, and t represents the time, in days.

   a) Calculate the number of bacteria present at the start of the experiment.

   b) How many bacteria were present after 1 day? 2 days? 7 days?

   c) Sketch a graph showing the number of bacteria versus time for the two-week experiment.

   d) Use the graph to estimate the time needed for the number of bacteria to double from the number present at the start of the experiment. Round your estimate to the nearest day.

   e) Write any restrictions on the domain and range of the function. Explain the reason for these restrictions.

7. The radioactive isotope cesium-137 has a half-life of approximately 30 years.

   a) Model the decay curve for 500 μg of cesium-137 using an exponential function.

   b) How much cesium-137 remains after 5 years?

   c) Estimate how long it would take for the amount of cesium-137 to decay to less than 20 μg.

8. Iodine-131 is a radioactive isotope of iodine. It has a half-life of about 8 days.

   a) If a scientist has a 40 g sample of iodine-131, how much would be left after 10 days? 20 days? 30 days?

   b) Write an exponential decay model that predicts the mass of iodine-131 left after d days.

   c) Use your model to predict how long it would take for the amount of iodine-131 to be reduced to less than 0.5 g.

9. Brigit has a rectangular shawl that is 150 cm long and 50 cm wide. It is made of wool, which shrinks when washed. Each time she washes it, the length of the shawl decreases by $\frac{1}{7}$ and the width decreases by $\frac{1}{5}$.

   a) Express the area of the shawl A as an exponential function of the number of washes n. Simplify the expression so that it contains only one power.

   b) How many times can Brigit wash the shawl before its area decreases to less than 1000 cm^2?

C

10. After 40 h, only 25% of the original sample of a radioactive isotope remained. How much of the original sample will remain after one week?

11. The rabbit population in a provincial park tripled over 10 years.

    a) Assume that the growth is exponential. Determine a function that models the growth.

    b) What is the annual growth rate?

Chapter 6 Review

Work with a classmate to verify your answers. Use technology where appropriate.

6.1 The Exponent Rules

Textbook pp. 280–287

1. Simplify. Leave your answers in exponential form.

a) $\dfrac{9^2 \times 9^6}{9^4}$ **b)** $((4)^4)^7$

c) $\dfrac{x^{20}}{x^3 \times x^5}$ **d)** $(n^3)^4 \div n^2$

6.2 Evaluate Powers with Integer Exponents

Textbook 288–295

2. Evaluate each of the following. When appropriate, express answers as fractions.

a) 6^{-2} **b)** $\left(\dfrac{1}{2}\right)^{-3}$ **c)** $5^3 + 5^0$

6.3 Investigate Rational Exponents

Textbook pp. 296–304

3. Evaluate each of the following. Where appropriate, use a calculator.

a) $372^{\frac{1}{6}}$ **b)** $0.002^{\frac{1}{2}}$ **c)** $-5^{\frac{6}{8}}$

4. The period of orbit of a planet to its distance from the sun is $T = r^{\frac{3}{2}}$, if T is measured in years and r is measured in astronomical units (AU). The radius of orbit for the planet Mars is 1.52 AU. What is its period in years?

6.4 Model Data with Exponential Functions

Textbook pp. 305–311

5. The growth rate of an urban population of raccoons is estimated at 1.76% per year.

a) In 2005, the population was estimated at 436 raccoons. What is the estimated population for 2012?

b) What was the estimated raccoon population in 2002?

6.5 Exponential Functions and Their Properties

Textbook pp. 312–317

6. a) Make a table of values and sketch a graph of the function $y = 1.4^x$.

b) Find the domain, range, intercepts, and intervals of increase and decrease, as well as any asymptotes.

6.6 Compare Linear, Quadratic, and Exponential Functions

Textbook pp. 318–325

7. Use first or second differences, and/or ratios to classify each relation as linear, quadratic, exponential, or none of these.

a)

x	y
−2	0.278
−1	1.667
0	10
1	60
2	360

b)

x	−2	−1	0	1	2
y	10	7	6	7	10

c)

x	−2	−1	0	1	2
y	6.54	6.21	5.88	5.55	5.22

6.7 Exponential Growth and Decay

Textbook pp. 326–333

8. A drug persists in the human body following the model $M = M_0(0.62)^{\frac{h}{2}}$. M is the mass of drug remaining in the body, in milligrams; M_0 is the mass of the dose taken, also in milligrams; and h is the time in hours since the dose was taken.

a) A standard dose is 100 mg. Sketch a graph showing the mass remaining in the body for up to 20 h.

b) Use your graph to estimate the half-life of the drug in the body.

c) Check your estimate in part b) using the equation.

Chapter 6 Practice Exam

For questions 1 to 5, select the best answer.

1. Simplify $\left(\frac{1}{2}\right)^{-3} - 6^{-1}$.

A $-5\frac{7}{8}$

B $7\frac{5}{6}$

C 2

D $5\frac{5}{6}$

2. A house was found to double in value every 30 years. If the current value is \$500 000, what was the value of the house 60 years ago?

A \$50 000

B \$62 500

C \$125 000

D \$250 000

3. Evaluate $625^{\frac{3}{5}}$.

A 5

B 312.5

C 25

D 125

4. Simplify $\frac{(3^2)^4}{3^6}$.

A 3^1

B 3^2

C 3^{-2}

D 3^{-1}

5. Consider the exponential functions $y = 0.2^x - 1$, $y = 2.5^x - 1$, and $y = 5^x - 1$. What value of x results in the same y-value for each?

A -1

B 0

C 1

D There is none.

6. A cubical tank at an aquarium has a capacity of 216 m^3. How long are its sides?

7. The probability of rolling a sum of 3 or rolling a sum of a 11 with two number cubes is $\frac{2}{36}$ or $\frac{1}{18}$ for each sum.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Write an expression for the probability of rolling three sums of 11 followed by four sums of 3 followed by one sum of 11. Use a calculator to evaluate the expression.

8. To simulate the growth rate of a particular tropical fish, a science teacher took a large, empty fish tank and set up some mousetraps and ping-pong balls in it so that each trap would launch a ball when sprung. Each trap sprung represents 6 fish hatched. When everything was ready, one of the students sprung the first trap.

a) Suppose that each launched ball sprung an average of 2.5 other traps. Write an exponential relation to model the number of fish hatched at each successive generation.

b) Approximately how many fish would be hatched at the sixth generation?

9. The input and output for a function machine is shown.

Input	Output
−3	0.125
0	1
2	64

Find the equation of an exponential function that matches the input to the output.

10. A large rubber sac is used to maintain the pressure in a jet pump that provides water for a barn. Compton suspects a leak in the pump assembly. If the leak is in the sac, it should follow an exponential relation. If the leak is in the water tank, it should follow a linear relation. He pumps it to a pressure of 175 KPa and then measures the pressure every minute. The table shows his results.

Time (min)	Pressure (KPa)
0	175.0
1	162.8
2	151.4
3	140.8
4	130.9
5	121.8
6	113.2

a) Is the leak best modelled by a linear or exponential function? Justify your answer.

b) Should Compton replace the sac? Explain.

11. A doctor at a medical convention wants to model the spread of an infectious disease. She writes "infected" on three of her business cards and gives them to three other attendees. She asks each person to also write "infected" on three business cards and give them to three new people with her instructions. She estimates it will take 15 min for each new group to hand out three cards. There are 7000 people at the convention.

a) Make a table of values of time, in 15-min intervals, versus the number of people "Infected".

b) Use your table from part a) to sketch a graph of the Number of People "Infected" versus Time.

c) Show that the relationship between the number of people "infected" and the time can be modelled using an exponential function.

d) Write an exponential function that models this situation.

e) Verify that your model correctly predicts the initial number of people "infected" and the number "infected" after 1 h.

f) Use your model to predict how long it will take for at least 90% of the people at the conference to be "infected".