

Chapter 8 Annuities

Prerequisite Skills

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 380–381 of the *Functions and Applications 11* textbook.

Proportional Thinking

1. Ben saves \$90 per month. How much will he save for each time period?
a) 3 months b) 1 year c) 5 years d) 10 years e) 30 years f) 40 years

Exponents and Exponential Functions

2. Without using a calculator, decide whether each product will give a value greater than or less than the initial value. Then, determine the value of each expression. Round your answers to two decimal places.

	Initial Value (\$)	Product (\$)
a)	5000	$5000(1.065)^{-1}$
b)	10 000	$10\ 000(0.73)^5$
c)	3600	$3600(1.0422)^2$
d)	820	$820(1.013)^{-10}$

3. a) On the same set of axes, graph the following exponential functions for $0 \leq x \leq 5$.
i) $f(x) = 2.5^x$ ii) $g(x) = 0.45^x$ iii) $h(x) = 1.6^x$ iv) $k(x) = -1.03^x$
b) Explain why all of the functions pass through the point (0, 1).
4. Make a table of values to each function for $0 \leq x \leq 5$. Round your answers to two decimal places. Then use a graphing calculator to graph each function.
a) $f(x) = 100 - 100(0.07)^x$ b) $g(x) = 100(1.07)^x$

Simple and Compound Interest

5. Determine the simple interest earned in each situation. Round your answers to two decimal places.
a) A \$1500 GIC earns 3.9% interest per year, for 1 year.
b) A \$600 investment earns 6.45% annual interest, for 3 years.
c) A \$380 deposit into a savings account earns 1.12% interest per year, for 72 days.
d) A \$400 term deposit earns 2.6% annual interest for 42 months.
6. Determine i , the interest rate per compounding period, and n , the number of compounding periods, for the each compound interest situation. Round your answers to six decimal places. Then determine the future value of each investment.
a) \$1000 invested at 7.6% interest per year, compounded semi-annually for 4 years
b) \$2500 invested at 8.75% annual interest, compounded monthly for 10 years
c) \$200 deposited into an account paying 0.75% annual interest, compounded daily for 6 months
d) \$10 000 invested at 6.8% annual interest, compounded quarterly, for 5 years

8.1 Future Value of an Ordinary Simple Annuity

Textbook pp. 382–389

Prerequisite Skills

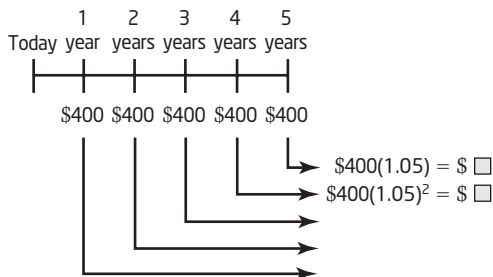
- Tina saves \$120 per month. How much will she save for each time period?
 a) 1 year b) 4 years c) 10 years d) 25 years e) 30 years
- Determine i and n for the each compound interest situation. Round your answers to six decimal places.
 - \$800 invested at 2.956% interest per year, compounded semi-annually for 2 years
 - \$9000 invested at 13.5% annual interest, compounded monthly for 6 years
 - \$50 deposited into an account paying 1.2% annual interest, compounded daily for 8 months
 - \$6500 invested at 5.9% annual interest, compounded quarterly, for 7 years
- Use the compound interest formula or a TVM Solver to determine the future value of each investment in question 2.

A

- Draw a time line representing the future value of each of the following ordinary annuities.

Copy and complete the following.

- \$400 invested annually for 5 years into a fund that pays 5% per year, compounded annually



- \$650 invested monthly for 2 years into an account that pays 3.9% per year, compounded monthly
- \$5000 invested semi-annually for 6 years into a mutual fund that historically has paid 12% per year, compounded semi-annually
- \$2000 invested quarterly for 1 year into a fund that pays 2.25% per year, compounded quarterly

- Express the future value of each annuity in question 1 as the sum of a series of compound interest investments.
 - Calculate the future value of each of the annuities by adding each series.
- Use the TVM Solver on a graphing calculator to determine the future values of each of the annuities in question 1.
- Determine the variables PMT, i , and n for each of the annuities in question 1.
 - Use the future value of an annuity formula, $FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$, to calculate the future value of each annuity in question 1.

5. a) Use a method of your choice to determine the future value of the following ordinary annuities.
- monthly payments of \$100 at 4.5% per year, compounded monthly for 3 years
 - \$2000 deposited annually for 5 years into a fund that pays 6.75% per year, compounded annually
 - semi-annual payments of \$1500 for 1.5 years at 2.9% annual interest, compounded semi-annually
- b) Check one of your answers using a different method.
- B**
6. Margo is 19 and earned \$20 000 last year after taxes. She decides to invest \$2500 in a mutual fund. which, after management fees were paid, averaged 11.5% net growth per year, with interest compounded annually.
- Assuming the rate of growth remains the same, determine the future value of Margo's \$2500 investment after one year. (Note that a mutual fund's past performance does not necessarily indicate its future rate of growth.)
 - If Margo continues to invest \$2500 annually, calculate the total value of her investments when she makes her 6th investment, assuming the same growth rate.
7. Tran is in grade 11 and has worked at a garden supply store for just over 9 months. At the end of each month, he deposited \$300 into an account that paid 3.5% interest per year, compounded monthly.
- How much is in Tran's account after 9 months?
 - If he continues, will the amount in his account after 1.5 years be double the answer to part a)? Without using a calculator, explain why or why not.
 - Calculate the amount that will be in Tran's account after 1.5 years.
8. When their son was a month old, and at the end of every month thereafter, Joan and Quenton invested a total of \$75 into an RESP (Registered Education Savings Plan) that paid 6.45% annual interest, compounded monthly.
- Use a TVM Solver or an on-line calculator to determine the amount in the fund on the child's 7th birthday, 10th birthday, and 14th birthday.
 - By the time their son turned 17, how much of their own money had Joan and Quenton invested?
 - How much interest did their RESP earn?
 - Their son decided to work and travel for a few years before starting college. If they continued with their monthly contributions after their son turned 17, how much more money would be in the fund after 28 months?
9. When Maria was 16, she got a summer job as a swim instructor for 4 months. At the end of each month, she deposited \$650 into an account that paid 3.75% per year, compounded monthly. She decided not to touch the account for 2 years and then use the money as a down payment for a car. Determine how much money she will have for the down payment.
- C**
10. Suppose you can save \$7 per week and you deposit the money in an account that pays 3.5% interest per year, compounded monthly.
- How much money would you have in the account after 1 year?
 - How long would it take to have at least \$500 in the account?
 - How much more would you have to save each week in order to have at least \$500 in the account after 1 year?
11. Use an Internet search engine to search for a "time value of money calculator." Repeat part a) of question 11 using at least two different on-line calculators.

8.2 Present Value of an Ordinary Simple Annuity

Textbook pp. 390–396

Prerequisite Skills

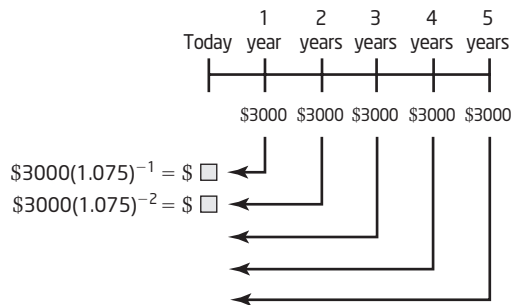
- Determine i and n for each compound interest situation. Round your answers to six decimal places.
 - \$2200 in 3 years at 4.75% per year, compounded annually
 - \$6800 in 2 years at 9% per year, compounded quarterly
 - \$15 000 in 30 months at 8.5% per year, compounded semi-annually
 - \$540 in 7 months at 5.4% per year, compounded monthly
- Use the method of your choice to determine the present value of each amount in question 1. Round your answers to two decimal places.

A

- Draw a time line or use a table to represent the present value of each of the following ordinary simple annuities.

Copy and complete the following.

- \$3000 paid annually for 5 years at 7.5% per year, compounded annually



- 8 monthly payments of \$420 at 13.7% per year, compounded monthly

Copy and complete the table.

Month	Amount (\$)
1	$420(1.0114)^{-1} = 415.27$
2	$420(1.0114)^{-2} = \square$
3	$420(1.0114)^{-3} = \square$
4	$420(1.0114)^{-4} = \square$
5	$420(1.0114)^{-5} = \square$
6	$420(1.0114)^{-6} = \square$
7	$420(1.0114)^{-7} = \square$
8	$420(1.0114)^{-8} = \square$
	Total = \square

- \$600 payments every 3 months for 2 years at 6.4% annual interest, compounded quarterly
- payments of \$130 every 6 months for 2.5 years at 5.2% annual interest, compounded semi-annually

- Express the present value of each annuity in question 1 as the sum of a series of individual present value calculations.
 - Calculate the present value of each series of payments.
- Use a TVM Solver to determine the present value of each of the annuities in question 1.
- Determine the values of the variables PMT, i , and n for each of the annuities in question 1.
 - Use the present value of an annuity formula to calculate the present values in question 1.
- Use a method of your choice to determine the present value of the following ordinary simple annuities.
 - monthly payments of \$575 at 10.4% per year, compounded monthly, for 3 years

- b) quarterly income of \$1930 for 2 years, generated by a fund that earns 9.5% annual interest, compounded quarterly
- c) payments of \$7600 every 6 months, for 4 years at 5.75% annual interest, compounded semi-annually

B

6. A motorcycle retailer offers a payment plan at 9.95% annual interest, compounded monthly. Customers can pay for a moped by monthly payments of \$329.95 for 2.5 years.
- a) Determine the total cost for a customer who accepts the payment plan.
 - b) Determine the cost of the moped for a customer paying cash at the time of purchase.
 - c) How much total interest would be paid by a customer who accepts the payment plan?
7. Determine the lump-sum amount needed to generate a retirement income of \$40 000 per year for 20 years, assuming interest at 4.5% per year, compounded annually.
8. Cynthia is 27 and has 3 years' worth of payments remaining on her student loan. She pays \$312.75 per month.
- a) If Cynthia's bank charges 8.95% interest per year, compounded monthly, determine the present value of the remainder of the loan.
 - b) If the bank is willing to accept early payment, how much interest would Cynthia save by paying back the rest of the loan today?
9. A local business has created a scholarship, starting next year. The scholarship will give \$2500 to one eligible high school student for each of the following 12 years. If the business can invest money at 10.5% per year, compounded annually, how much money must it invest today in order to provide the scholarship fund?

10. Ethan is planning to set up a retirement income fund that will let him draw a monthly income when he turns 70. He wants to be able to withdraw \$2000 at the end of each month from this fund. His investments currently earn 6.75% per year, compounded monthly.

- a) If he wants his savings to last for 20 years (until he turns 90), determine the amount he will need to have in the fund by the time he reaches age 70.
- b) Calculate how much total income his fund will generate over the 20 years.
- c) Calculate the interest earned by the fund.

C

11. a) Rearrange the present value of an annuity formula to solve for the payment.
- b) Philip has a \$6000 loan payable over 12 months at 12% annual interest, compounded monthly. Use the formula from part a) to determine the monthly payment due at the end of each month.
 - c) What is the total amount that Philip will repay?
12. Lucretia purchased a \$3500 computer gaming system, and has been making payments of \$195.25 per month for 14 months on a loan used to buy the system. Lucretia has decided that it is in her best interest to pay off the rest of her debt in one payment at the end of the 15th month. If Lucretia's loan is compounded monthly at a rate of 9.65% per year, how much will she need to fully repay the loan at the end of the 15th month?

How much of the \$3500 loan has been repaid by the 14 monthly payments? What amount remains, 15 months after she bought the system?

8.3 Payments and Total Interest

Textbook pp. 397–402

Prerequisite Skills

- Without using a calculator, decide whether each product will give a value greater than or less than the initial value. Then, determine the value of each expression. Round your answers to two decimal places.

	Initial Value (\$)	Product (\$)
a)	300	$300(1.002)^6$
b)	2500	$2500(0.73)^{-12}$
c)	16 000	$16\ 000(1.045)^2$

	Initial Value (\$)	Product (\$)
d)	55 000	$55\ 000(1.075)^{20}$
e)	4000	$4000(0.03)^{-3}$
f)	38 000	$38\ 000(1.09)^{-15}$

- For each compound interest situation, identify if you need to determine the present value or the future value.
 - \$900 in 1 year at 3.25% per year, compounded annually
 - \$5500 invested at 8.3% interest per year, compounded semi-annually for 2 years
 - \$1600 deposited into an account paying 1.25% annual interest, compounded daily for 3 months
 - \$4000 in 3 years at 5.2% per year, compounded quarterly
- Use the method of your choice to determine the value of each amount in question 2. Round your answers to two decimal places.

A

- Determine the value of PV, i , and n in each of the following situations.
 - \$16 000 is to be repaid quarterly for 4 years with interest at 9% per year, compounded quarterly
 - a \$21 000 car loan at 6.25% annual interest, compounded monthly, is to be repaid with monthly payments for 5 years
 - a \$35 000 business start-up loan is to be repaid with annual payments for 7 years, and the interest rate on the loan is 8.95% per year, compounded annually
 - \$1900 was borrowed to purchase a mountain bike, and the loan is to be repaid in monthly installments over 1.5 years, with interest at 7.3% per year, compounded monthly
- Substitute the values for PV, i , and n from each part of question 1 into the formula

$$\text{PMT} = \text{PV} \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$
 to solve for the payment.
- Solve for the payment for each part of question 1 using a TVM Solver.
 - Rearrange the future value of an ordinary simple annuity formula

$$\text{FV} = \text{PMT} \left[\frac{(1 + i)^n - 1}{i} \right]$$
 to solve for the payment.
 - Use the formula determined in part a) to calculate the annual payment needed to generate \$8500 in 2 years if interest is earned at 7.7% per year, compounded annually.

5. Calculate the total amount paid for the duration of each loan.
 - a) quarterly payments of \$940 for 2 years
 - b) \$165 per month for 40 months
 - c) weekly payments of \$98.50 for 8 years
 - d) \$132 per month for 4 years

6. Calculate the total amount of interest paid on each of the following loans.
 - a) weekly payments of \$384.62 for 30 years on a \$300 000 home mortgage
 - b) \$79 per month for 1.5 years to pay for photography equipment with a pre-tax price of \$899 (taxes paid at time of purchase)
 - c) monthly payments of \$54.95 for 36 months on a computer with a pre-tax price of \$1200 (taxes paid at time of purchase)
 - d) quarterly payments of \$992 for 4 years to repay a \$14 000 small business loan

7.
 - a) Determine the value of the home in question 6, part a) after 30 years if it appreciates 3.5% per year.
 - b) Determine the value of the photography equipment question 6, part b) 3 years after purchase if it depreciates 25% per year.
 - c) Determine the value of the computer in question 6, part c) at the time the loan is fully repaid, assuming the equipment depreciates 40% per year.

- B**
8. Gus has 3 years to repay a \$8500 personal loan at 7.35% per year, compounded monthly.
 - a) Calculate his monthly payment using the appropriate formula.
 - b) Calculate the total amount Gus ends up paying.
 - c) Calculate the amount of interest Gus will pay over the life of the loan.

9.
 - a) Use a TVM Solver or an on-line calculator to determine the monthly payment required to pay off a \$4000 loan in 2 years with a 8% annual interest rate, compounded monthly.
 - b) Calculate the total interest paid over the life of the loan.

10. Rebecca is deciding between two new compact sedans. One has a pre-tax cost of \$17 100 and is financed at 0.9% per year, compounded monthly, for 36 months. The other has a pre-tax cost of \$13 500 with financing available at a bank for 7.5% per year, also compounded monthly for 36 months. Taxes will be paid at the time of purchase in either case.
 - a) Determine the monthly payment for each sedan.
 - b) Determine the total cost of each loan. If a loan is used to pay for the sedan, which one ends up costing less?
 - c) Determine the total interest that would be paid for each loan.
 - d) What factors other than cost should influence Rebecca's decision?

11. Basir wants to replace the parquet tiles in his living room and dining room with hardwood flooring in 2 years. He received a quote from a local flooring company for \$3200; however, he suspects the cost will increase by 5% per year. He currently has \$900 in a bank account that pays 3.75% annual interest, compounded monthly, to be used for the new floor.
 - a) How much money does Basir need to deposit each month in order to pay cash for the new floor in 2 years?
 - b) Basir decides to pay for the future renovation now, by making a single deposit today. It will accumulate interest until the floor is replaced. How much must he deposit?

- 12.** Trevor is purchasing a car for \$29 625, including taxes. He hopes to replace it in 3 years with a similar car. He estimates that in 3 years, the price will have increased by 20% and his present car will have lost 50% of its value. GST of 5% is charged on the difference between the trade-in value and the new car price. He will start saving in 2 months by making a payment every month into an account paying 7.2% interest per year, compounded monthly.
- How much should each payment be so that he can pay cash for the new car in 3 years?
 - Explain any assumptions you made when finding this payment amount.
- 13.** Determine the monthly investment needed to generate \$7000 in 3 years if interest is earned at 2.5% per year, compounded monthly, by rearranging the formula for calculating the future value of an ordinary simple annuity.
- 14.** Olivia has just purchased a one-bedroom condominium for \$179 900. Her mortgage is at 7.2% per year, amortized over 25 years. All mortgages in Canada have interest compounded semi-annually.
- Use a TVM Solver or an on-line calculator to determine her monthly payment.
 - Assuming that interest rates remain the same, how much will she end up paying before the condominium is truly hers?
 - How much interest will she pay over the life of the mortgage?
 - Olivia's condominium is located in a neighbourhood where prices are expected to rise by 12% per year. Express the growth in the selling price of the condominium as an exponential function.
 - Calculate the estimated selling price of her condominium 3 years from now.
 - How much will the condominium have gained in value by then?

C

- 15.** Elana needs to commute to her new job. She is looking at a 4-year-old sedan selling at a local Ontario dealership for a pre-tax price of \$8954. As the dealership only provides financing on the sale of new cars, Gunter needs to approach financial institutions.
- Research the interest rates for a 4-year personal loan from a bank, a credit union, and an Internet bank.
 - Select the institution with the best rate and use an on-line calculator or a TVM Solver to determine Elana's monthly payment, assuming that she will borrow the entire after-tax cost of the vehicle.
 - Calculate the total amount of the loan.
 - Calculate the total interest paid over the life of the loan.
 - This type of vehicle depreciates at a rate of about 15% per year. Express the depreciation of Elana's vehicle as an exponential function.
 - Determine the value of Elana's sedan after she has finished paying the loan.
 - Research the definitions of an asset and a liability. Explain whether you think a car is an asset or a liability.

8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity

Textbook pp. 405–411

Prerequisite Skills

- For each compound interest situation, identify if you need to determine the present value or the future value.
 - \$700 in 6 months at 5.4% per year, compounded monthly
 - \$54 000 invested at 6.5% interest per year, compounded semi-annually for 10 years
 - \$3000 deposited into an account paying 4.5% annual interest, compounded daily for 9 months
 - \$6000 in 2 years at 3.75% per year, compounded quarterly
 - \$20 000 in 18 months at 11.4% per year, compounded semi-annually
 - \$7700 invested at 6.5% annual interest, compounded monthly for 5 years
 - \$30 000 invested at 7.95% annual interest, compounded quarterly, for 15 years
 - \$1300 in 4 months at 6.9% per year, compounded monthly
- Use the method of your choice to determine the value of each amount in question 1. Round your answers to two decimal places.

A

- Determine the monthly payment on \$10 000 borrowed at 9.5% annual interest, compounded monthly, for
 - 1 year
 - 2 years
 - 3 years
 - 4 years
 - 5 years
 - 8 years
- Calculate the total interest paid for each loan period in question 1.
- Determine the number of months (rounded up to the nearest month) required to pay back a \$30 000 loan borrowed at 8% per year, compounded monthly, given monthly payments of
 - \$300
 - \$500
 - \$750
 - \$900
- Determine the monthly payment on a \$12 000 loan for 3 years, when interest is compounded monthly at an annual rate of
 - 6.5%
 - 7%
 - 8.25%
 - 9%
- Calculate the total interest paid for each part of question 4.

Use the formula $PMT = PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$.

Use a TVM Solver.

B

6. Sean wants to purchase better camera equipment after he graduates from college. His week-end job will allow him to save \$90 each month. He is offered two savings plans.

Plan 1: 3.25% annual interest, compounded monthly, with monthly deposits of \$90

Plan 2: 3.75% annual interest, compounded semi-annually, with semi-annual deposits of \$540

- a) Sean expects to graduate from college in 2.5 years. Which option should he choose in order to save the most money?
- b) How much more interest will Sean earn with the better savings plan?
7. Nicola is purchasing new furniture. The after-tax cost of the furniture is \$3965.99. The interest rate on a personal loan from her bank is 6.25% per year, compounded monthly. Nicola can manage a monthly payment of \$200.

Use a TVM Solver.

- a) Determine the number of months it will take for Nicola to repay the loan.
- b) Round N to the nearest multiple of 12 and solve for PMT.
- c) Determine the total amount that Nicola will pay for the furniture when making the payments she can afford.
- d) Another bank is offering personal loans at 5.5% annual interest, compounded monthly. Repeat parts a) and b) with these changes. Then, calculate the amount Nicola will save if she chooses the 5.5% loan.

8. a) Compare the amounts at age 65 that would result from making annual contributions to a retirement savings fund that earns 8% annual interest, compounded annually, of
- i) \$900, starting at age 19
 - ii) \$2000, starting at age 34
 - iii) \$3000, starting at age 45
- b) What is the total amount deposited in each situation?
- c) How much interest would each investment earn?
9. Olivia earned about \$72 850 when she sold her condominium. She purchased a second condominium farther away from the city centre and took a mortgage of \$120 000. The best available mortgage rate she could find was 6.25% per year, compounded semi-annually.

Use a TVM Solver.

- a) Determine the monthly payment based on an amortization of 15 years. Use this monthly payment to determine the total amount Olivia will pay over the life of the mortgage.
- b) Change the amortization period to 10 years. Determine the monthly payment and the total amount that will be paid over the life of the mortgage.
- c) Keep the amortization at 10 years and change the payment frequency to semi-monthly. Determine the semi-monthly payment and the total amount that will be paid over the life of the mortgage.
- d) Olivia decided on a 7-year mortgage with weekly payments. Determine the amount of her payment and the total amount she will pay over the life of the mortgage.
- e) How much did she save by choosing a 7-year mortgage with weekly payments over a 15-year mortgage with monthly payments?

- 10.** Wilson needs to repay a \$8000 loan. His bank offers personal loans from 1 to 5 years at 7.65% per year, compounded monthly.
- Determine his monthly payment if he repays the loan over a 4-year period.
 - Calculate the total interest he will pay on this loan.
 - Use a TVM Solver to determine Wilson's payment if he chooses to make weekly payments.
 - Calculate the total interest he will pay on the loan if he makes weekly payments.
 - Explain why changing the frequency of the payments results in a reduction in the total interest paid on the loan.
- 11.** Ing has several debts. She owes \$10 000 on her student loan, approximately \$2500 on each of two credit cards, and \$2000 to her sister. Ing is approaching her bank for a consolidation loan, a loan that is used to combine and finance payments on other loans.
- Calculate Ing's monthly payment on a 6-year consolidation loan at 8.4% annual interest, compounded monthly.
 - Determine the total amount Ing would pay on the loan.
 - How much interest would she have to pay in order to eliminate her debt this way?
 - How much could Ing save by taking out a 5-year loan instead?
 - Determine the monthly payment on an 8-year loan.

C

- 12.** Fatima purchased a new car 6 months ago. She borrowed \$20 000 over 3 years at 6.35% annual interest, compounded monthly.
- Determine Fatima's monthly payment.
 - Calculate the total amount she has paid so far.
 - Fatima wishes to pay the rest of the loan in full now. Determine the amount remaining on her loan.
 - How much interest did Fatima save by deciding to eliminate her debt today?
- 13.** Patsy has signed a loan contract that requires a down payment of \$2000 and payments of \$300 a month for 4 years. The interest rate is 5.3% per year, compounded monthly.
- What is the present value of the contract?
 - If Patsy misses the first 6 payments, how much will be due on her 7th payment to catch up?
 - If Patsy makes the first 6 payments, how much would her 7th payment need to be to pay off the contract completely?

Chapter 8 Review

Work with a classmate to verify your answers. Use technology where appropriate.

8.1 Future Value of an Ordinary Simple Annuity

Textbook pp. 382–389

1. Consider an annuity with payments of \$4500 invested annually for 5 years into a fund that pays 5.5% per year, compounded annually.
 - a) Draw a time line representing the future value of the annuity.
 - b) Express the future value of the annuity as the sum of a series of compound interest investments.
 - c) Use a TVM Solver or an on-line calculator to determine the future value of the annuity.
 - d) Use the future value of an annuity formula to calculate the value of the annuity.
2. Use any two methods of your choice to determine the future value of the following ordinary simple annuities.
 - a) monthly payments of \$95 at 4.2% per year, compounded monthly for 1.5 years
 - b) \$1000 deposited every 6 months for 5 years into a fund that earns 10.5% per year, compounded semi-annually

8.2 Present Value of an Ordinary Simple Annuity

Textbook pp. 390–396

3. \$700 is paid monthly for 6 months at 7.2% annual interest, compounded monthly.
 - a) Draw a time line representing the present value of this ordinary annuity.
 - b) Express the present value of the annuity as the sum of a series of individual present value calculations.
 - c) Use a TVM Solver or an on-line calculator to determine the present value of the annuity.
 - d) Use the relevant formula to calculate the present value of the annuity.
4. Use the method of your choice to determine the present value of the following ordinary simple annuities.
 - a) monthly payments of \$160 at 7.75% per year, compounded monthly, for 4 years
 - b) quarterly income of \$9000 for 3 years, generated by a fund that earns 12.3% annual interest, compounded quarterly

8.3 Payments and Total Interest

Textbook pp. 397–404

5. Determine the values of i and n in each of the following situations.
 - a) a \$23 000 loan on a new sedan at 7.6% per year, with interest compounded monthly, is to be repaid with monthly payments over 5 years
 - b) a \$60 000 business loan is due in semi-annual payments over 15 years; the interest rate on the loan is 6.5% per year, compounded semi-annually
6. Substitute the values for PV , i , and n from each part of question 5 into the formula
$$PMT = PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$
to solve for the payment.
7. Calculate the total interest paid for each of the loans in question 5.
8. A homeowner is planning to have new windows installed in 2 years. He receives a quote of \$5500 from a local company. The homeowner has a savings account that pays 4.25% annual interest, compounded monthly.
 - a) How much money per month does he need to deposit in his account in order to be able to pay in cash for the windows in 2 years?
 - b) How much of the money he uses to pay for the driveway will come from
 - i) money deposited into the account?
 - ii) interest?

8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity

Textbook pp. 405–411

9. Kumiko wants to convert her \$650 000 retirement savings fund into an income fund. She looks at two income fund options. With the first option, Kumiko would receive semi-annual payments, with interest calculated at 6.3% per year, compounded semi-annually. With the second option, she would receive monthly payments, with interest calculated at 6.35% per year, compounded monthly. On either option, Kumiko wants the income fund to provide income for 25 years.
 - a) Calculate the total value of the payments for each income fund option.
 - b) Which payment option yields the highest annual income?
10. A retirement savings fund earns 7.3% annual interest, compounded annually.
 - a) Calculate the amounts at age 65 that would result from
 - i) making annual deposits of \$1500 starting at age 18
 - ii) making annual deposits of \$3000 starting at age 36
 - b) What is the total of the deposits in each situation?
 - c) How much did each investment earn?
 - d) Explain the effect of time on the future value of an investment.

Chapter 8 Practice Exam

For questions 1 to 4, select the best answer.

1. Which ordinary simple annuity formula is not correct?

A $FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$

B $PMT = PV \left[\frac{i}{1 - (1 + i)^n} \right]$

C $PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$

D $PMT = FV \left[\frac{i}{1 - (1 + i)^n} \right]$

2. Which statement is false?

A Paying off a loan in a shorter term has a greater impact on the total interest paid than increasing the payment frequency.

B Increasing the interest rate for a loan will decrease the monthly payments.

C Buying a vehicle is an expense not an investment.

D Increasing the term of a loan will decrease the monthly payments.

3. Without calculating, choose the approximate future value for monthly deposits of \$500 for 6 years earning 7% annual interest, compounded monthly

A \$2050 B \$44 500

C \$550 500 D \$15 500

4. Without calculating, choose the approximate present value needed for a scholarship fund that will give out a scholarship of \$1500 every year for 10 years if the fund earns 9% per year, compounded annually.

A \$1000 B \$3000

C \$10 000 D \$100 000

5. Determine the monthly investment needed to generate \$8000 in 4 years if interest is compounded monthly at 5.7% per year.
6. A charity lottery is offering a grand prize of \$1000 at the end of each week for 10 years.
- a) How much will the winner collect in total?
- b) A fund must be set up to create an annuity for the winner of the lottery. Determine the present value of the fund on the day the winner is announced if interest is earned at 7% per year, compounded weekly.
7. Four years from now, Dexter plans to buy a new car. He deposits \$500 at the end of each month into an account that earns 4.25% annual interest, compounded monthly. If he wants to pay no more than \$25 000 for the car, will he have enough money saved to pay cash for the car in 4 years?
8. Midori and Raymond want to start saving for their son's education by investing in an RESP. They can afford to invest \$300 each month, and their bank has offered them two choices of RESP. With the first option, they invest \$300 each month, at 3.5% interest per year, compounded monthly. With the second option, they invest \$1800 semi-annually at 4.25% interest per year, compounded semi-annually.
- a) Use a TVM Solver to determine which option is the better investment.
- b) If they choose the better investment, how much will they have saved in 17 years?

9. A couple obtains a 5-year mortgage for \$250 000 on the purchase of a two-bedroom condominium. The mortgage is amortized over 25 years at 5.85% per year, compounded semi-annually.
- Use a TVM Solver to determine their monthly payment.
 - How much will they pay in total over the first 5 years of ownership?
 - If in 5 years, when they renew their mortgage, the outstanding balance on the condominium is approximately \$224 100 and the interest rate has changed to 6.35% per year, compounded semi-annually, determine their monthly payment for an amortization period of 20 years.
 - If the interest rate remains constant, determine the total amount that the couple will have paid for the condominium over 25 years.
 - The condominium will likely appreciate by 9% per year. Express the value of the condominium as an exponential function.
 - Calculate the approximate value of the condominium in
 - 15 years
 - 25 years
10. When Sophia bought her house 10 years ago, it included all the kitchen appliances. She has paid down her mortgage and can now afford to replace the old appliances with new, energy efficient models. She paid \$9000 for a new refrigerator, stove, and dishwasher, and does not have to pay the balance for 15 months. Sophia must pay the balance in full on the due date or enter into a financing plan consisting of 36 monthly payments of \$285.
- Determine the present value of \$9000 15 months before the balance is due, if money can be invested at 4.25% annual interest, compounded monthly.
 - If Sophia makes deposits at the end of each month into an account that earns 4.25% annual interest, compounded monthly, how much must she deposit each month to pay the balance by the due date?
 - Calculate the total amount deposited if Sophia makes monthly deposits.
 - If Sophia does not pay the debt on the due date, how much will she end up paying for the appliances?
 - How much more will she pay by making 36 payments rather than 15 deposits?