Overview of Functions and Applications 11

The McGraw-Hill Ryerson *Functions and Applications 11* program has six components.

Student Text

The student text introduces topics in real-world contexts. In each numbered section, **Investigate** activities encourage students to develop their own understanding of new concepts. **Examples** present solutions in a clear, step-by-step manner, and then the **Key Concepts** summarize the new principles. **Communicate Your Understanding** gives students an opportunity to reflect on the concepts of the numbered section, and helps you assess students' grasp of the new ideas and readiness to proceed with the exercises.

Practise (A) questions are single-step knowledge questions and assist students in building their understanding. **Apply (B)** questions allow students to use what they have learned to solve problems and make connections among concepts. **Extend (C)** questions are more challenging and thought-provoking. Answers to Practise, Apply, and Extend questions are provided at the back of the text. A **Chapter Problem** is introduced in the **Prerequisite Skills** section of each chapter. Students revisit different aspects of the problem in the numbered sections, leading up to the **Chapter Problem Wrap-Up** at the end of the chapter. **Chapter Tasks** are more involved problems that require students to use several concepts from the preceding chapters. Solutions to the Chapter Problem Wrap-Up and Chapter Tasks are provided in this Teacher's Resource.

A **Chapter Review** of skills and concepts is provided at the end of each chapter. Questions are organized by specific numbered sections from the chapter. **Cumulative Reviews** are provided after Chapters 3, 5, and 8 and help prepare students for the Tasks.

The **Technology Appendix** provides instructions on the use of *The Geometer's Sketchpad*®, *Fathom*™ statistical software, and TI-83 Plus/84 Plus and TI-89 graphing calculators.

The **Prerequisite Skills Appendix** provides help with any topic in the Prerequisite Skills section for each chapter. Examples and practice questions are provided. The topics are arranged in alphabetical order.

The text includes a number of items that can be used as assessment tools:

- **Communicate Your Understanding** questions assess student understanding of the concepts
- **Achievement Checks** questions provide opportunities for formative assessment using the four Achievement Chart Categories, Knowledge and Understanding, Thinking, Communication, and Application
- **Practice Tests** contain multiple choice, short response, and extended response questions to help model classroom testing practices
- **Chapter Problem Wrap-Ups** finish each chapter by providing a set of questions that involve all four Achievement Chart Categories
- **Tasks** are presented after Chapters 3, 5, and 8 and combine concepts from the preceding groups of chapters

Technology is integrated throughout the program and includes the use of scientific calculators, graphing calculators, dynamic geometry programs, statistical software, and the Internet.

Teacher's Resource

This Teacher's Resource provides the following teaching and assessment suggestions:

- **Teaching Suggestions** for all the sections
- **Literacy Link** and Career Profile
- **Practice** and chapter-specific blackline masters
- Answers to the **Investigate** questions
- Responses for the **Communicate Your Understanding** questions
- Solutions and rubrics for the **Chapter Problem Wrap-Up** and **Chapter Tasks**
- Students' **Common Errors** and suggested remedies
- Solutions and rubrics for the **Achievement Check** questions
- Suggestions for Ongoing Assessment and Summative Assessment
- **Accommodations** for students with different needs

Study Guide

The program includes a **Study Guide**, which mirrors the chapters and section organization and sequence of the student text and is cross-referenced to pages in the text.

Features include the following:

- **Key Concepts** with worked examples, and additional tips and hints for each lesson.
- **Prerequisite Skills** list which is a review of key skills.
- A set of **Practice Questions**.
- A **Chapter Review**, including **Key Terms**.
- A **Chapter Practice Exam** with tips and hints on how to succeed and do well on the Practice Exams, including preparation and study methods.
- Study Guide answers are at the end of the book.

Computerized Assessment Bank CD-ROM

The Computerized Assessment Bank CD-ROM (CAB) contains questions based on the material presented in the student text, and allows you to create and modify tests. Questions are connected to the chapters in the student text. The question types include: True/False, Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated to the corresponding Achievement Chart Category, specific curriculum expectation, and curriculum strand from the Ontario Mathematics MCF3M Curriculum.

Solutions Manual

The Solutions Manual provides worked-through solutions for all questions in the numbered sections of the student text, except for Achievement Check questions, which are in this Teacher's Resource. In addition, the Solutions Manual provides worked-through solutions for questions in the Review, Practice Test, and Cumulative Review features.

Web site

In addition to our McGraw-Hill Ryerson Web site, teachers can access the password protected site to obtain ready-made files for *The Geometer's Sketchpad®* activities in the text, information about managing TI technology, further support material for differentiated learners, and many other supplemental activities.

To access this site go to:
http://www.mcgrawhill.ca/books/functionsapplications11
username: functions11
password: teach.app

Structure of the Teacher's Resource

The teaching notes for each chapter have the following structure:

Chapter Opener

The following items are included in the Chapter Opener:

- **Specific Expectations** that apply to the chapter, listed by strand
- **Key Terms** that will be introduced in the chapter, and which are defined in the margin
- Teaching Suggestions include notes on the Chapter Opener, and Assessment
- Introduction to a **Chapter Problem** that includes questions designed to help students move toward the **Chapter Problem Wrap-Up** at the end of the chapter

Planning Chart

This table provides an overview of each chapter at a glance, and specifies:

- **Student Text Pages** references and **Suggested Timing** for numbered sections
- Related blackline masters available on the Teacher's Resource CD-ROM
- Assessment blackline masters for each section of the chapter
- Special tools and/or technology tools that may be needed

Blackline Masters Checklist

- A useful organizer, by Chapter and Section which lists relevant BLMs and their purpose

Prerequisite Skills

The following items are included in the margin:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Common Errors** and remedies to help you anticipate and deal with common errors that may occur
- **Accommodations** for students having difficulties or needing enrichment

The key items in this section include:

- **Teaching Suggestions** for how to use the **Prerequisite Skills** section
- **Assessment** ideas on how to ascertain that students are ready for this chapter

Numbered Sections

The following items are listed in the margin:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Common Errors** and remedies give you ideas on how to help students who make typical mistakes
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment

The notes in each section include the following key elements:

- **Teaching Suggestions** give insights or point out connections on how to present the material from the text
- **Investigate Answers** let you know the expected outcomes of these activities
- **Communicate Your Understanding** answers help consolidate students' understanding of the **Key Concepts** that are presented in the student text

- Notes for the **Practise**, **Apply**, and **Extend** questions in the text provide: comments on specific questions to anticipate any difficulties; ways to deal with students' questions; and hints on how to help students answer the questions
- **Achievement Check Answers** are included as are Achievement Check rubrics (as Blackline masters)
- **Ongoing Assessment** suggestions give a variety of strategies that can be used to assess the students' learning
- **Career Connections** where appropriate, link the concepts and skills learned to a career or careers.

End of Chapter Items

The **Chapter Reviews** in this Teacher's Resource include the following items:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- Insights on how to present and use the information in the **Chapter Reviews**
- **Ongoing Assessment** suggestions give a variety of strategies you can use to assess the students' learning

The **Practice Tests** in this Teacher's Resource have the following key features:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Study Guide** directs students who have difficulty with specific questions to appropriate examples to review
- **Summative Assessment** refers you to the **Chapter Test** to assess student performance
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Using the Practice Tests** gives you insights on how to present the information in the Practice Tests

The **Chapter Problem Wrap-Up** includes the following elements:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice and remediation, assessment, or enhancement
- **Using the Chapter Problem Wrap-Up** includes teaching suggestions specific to the problem
- **Summative Assessment** refers you to the **Chapter Problem Rubric** to assess student achievement
- **Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 response

A **Task** occurs at the end of each chapter and includes:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) useful for extra practice or remediation, assessment, or enhancement
- **Specific Expectations** covered in the Chapter Tasks
- **Teaching Suggestions** with steps for you to follow
- **Prompts for Getting Started** provides a list of questions you can use to help students begin the Task

- **Hints for Evaluating a Response** provides a list of questions you should consider when assessing students' responses
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Ongoing Assessment** refers you to the **Chapter Task Rubric** to assess student achievement
- **Level 3 Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 answer

Cumulative Reviews are included at the end of chapters 3, 5, and 8. The following information is provided:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed
- **Related Resources** (Blackline masters) useful for extra practice or remediation, assessment or enhancement
- **Using the Cumulative Chapter Reviews** includes specific teaching suggestions
- **Ongoing Assessment** suggestions give a variety of strategies you can use to assess student's learning

The **Teacher's Resource CD-ROM** provides various blackline masters in PDF and Word format, including:
- Generic Masters
- Technology Masters
- Practice Masters
- Assessment Masters
- Chapter-specific Masters
- This TR CD also contains all **Student Text** answers that were not included in the text itself, all Student Workbook answers, and the entire TR in PDF format.

Program Philosophy

The *Functions and Applications 11* is an exciting new resource for intermediate learners.

The *Functions and Applications 11* program is designed to:

- provide full support in teaching the Ontario MCF3M mathematics curriculum
- enable and guide students' progress from concrete to representational and then to abstract thinking
- offer a diversity of options that collectively deliver student and teacher success

> *Given the changes occurring during adolescence, school administrators and teachers need to consider how best to match instruction to … the developing capabilities and varied needs of students…*
>
> *The (Functions and Applications 11) program is based on a view that all students can be successful in mathematics… [It] reflects principles of effective practice and research on how adolescents learn, prerequisites for achieving a balanced approach to mathematics.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

During grades 7 to 10, most students progress from solely concrete thinking toward more sophisticated forms of cognition, as shown in the diagram:

Concrete Thinking

- typically work with physical objects
- focus of thinking is specific
- little or no reflection on thought processes
- able to solve very simple problems

▶

Representative Thinking

- sometimes called "semi-concrete"
- typically work with diagrams
- thinking focus becoming more general and systematic
- meta-cognitive thinking about thought processes begins to develop
- explore hypothetical or "what-if" thinking, with support
- able to solve moderately challenging problems
- use problem-solving strategies effectively, with some guidance

▶

Abstract Thinking

- able to work with or without materials or diagrams
- thinking focus instinctively general and systematic
- meta-cognitive thinking is well developed
- naturally explore hypothetical or "what-if" thinking
- able to solve problems that extend or deepen thinking
- confidently select and adapt problem-solving strategies

In *Functions and Applications 11*, students most often start with representational thinking. Concrete models are used in some sections, particularly geometry. They may be helpful to some students for other sections, for example algebra tiles can help some students understand the process of factoring quadratic expressions. Only when students are comfortable with the concrete and representative do they begin to move toward the abstract. Suggestions for alternative ways to approach some key topics provide students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Approaches to Teaching Mathematics

The following assumptions and beliefs form the foundation of the *Functions and Applications 11* program:

1. Students demonstrate a wide range of prior knowledge and experiences, and learn via various styles and different rates.
2. Learning is most effective when students are given opportunities to investigate concepts before being introduced to the abstract mathematics involved.
3. Learning is most likely when familiar, meaningful contexts are used to illustrate ideas and applications of concepts.
4. Students benefit when different learning approaches are used–independent, cooperative, hands-on and teacher guided.

> *Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.*
>
> *Ontario Ministry of Education and Training, 2004*

The concrete and abstract progression is exemplified in the following styles of mathematics teaching.

Most applied students learn best by using a concrete, discovery-oriented approach to develop concepts. Once these concepts have been developed, a connectionist approach helps students consolidate their learning.

Transmission-Oriented

- teaching involves "delivering" the curriculum
- focuses on procedures and routines
- emphasizes clear explanations and practice
- "chalk-and-talk"

▶

Connectionist-Oriented

- teaching involves helping students develop and apply their own conceptual understandings
- focuses on different models and methods and the connections among them
- emphasizes "problematic" challenges and teacher-student dialogue

▶

Discovery-Oriented

- teaching involves helping students learn by "doing"
- focuses on applying strategies to practical problems and using concrete materials
- emphasizes student-determined pacing
- "hands-on"

At this level, some transmission-oriented learning is also useful. This variety of approaches can be seen in the *Functions and Applications 11* program design.

Feature	Teaching Style(s) Supported
Chapter Problem	connectionist
Investigate	discovery, connectionist
Examples	transmission, connectionist
Key Concepts	transmission
Communicate Your Understanding	connectionist
Practise	connectionist, transmission
Apply	connectionist, transmission
Extend	connectionist, transmission
Review	transmission, connectionist
Task	discovery, connectionist

Instructional Practice

The resources available in today's classroom offer opportunities and challenges. Indeed, the principal challenge—one that many teachers of mathematics are reluctant to confront—is to teach successfully to the opportunities available.

Grouping

> *Instructional practice that incorporates a variety of grouping approaches enhances the richness of learning for students.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

At one end of the scale, individual work provides an opportunity for students to work on their own, at their own pace. At the other extreme, class discussion of problems and ideas creates a synergistic learning environment. In between, carefully selected groups bring cooperative learning into play.

Manipulatives and Materials

> *Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.*
>
> *Ontario Ministry of Education and Training, 2003*

Although many teachers feel unsure about teaching with manipulatives and other concrete materials, many students find them a powerful way to learn. The *Functions and Applications 11* program supports the use of manipulatives, helps teachers adapt to this kind of teaching. The Teaching Suggestions sections in the Teacher's Resource provide suggestions for developing student understanding using semi-concrete materials, such as diagrams and charts.

Technology

Special computer software designed for the classroom and licensed by the Ministry of Education for use in Ontario classrooms, such as *The Geometer's Sketchpad*®, provides a powerful tool for teaching and learning. The *Functions and Applications 11* program supports the use of such software as an enhancement to the classroom experience. In addition, support for Computer Algebra Systems is included. Graphing calculator instructions are provided in the Investigate activities and Technology Appendix. Multiple solutions for worked-through examples in the text allow teachers to enjoy wide flexibility in lesson planning. As a result, you can plan activities using manipulatives, pencil and paper, graphing calculators, software, or any combination of these.

The Internet provides great opportunities for enhancing learning. As with many other sources of information, students must be protected from inappropriate content. The McGraw-Hill Web site at *http://www.mcgrawhill.ca/links/functionsapplications11* (for students) has been designed to offer only safe and reliable Web site links for students to explore as an integrated part of the *Functions and Applications 11* program. The Web site also provides additional support materials for teachers.

Literacy

Effective mathematics classrooms show students that math is everywhere in their world. For example, students should see that knowledge of probability is useful when learning about the electoral process in Social Studies class. Their work in graphing can be used in Science class. Their written explanations are also a language arts product. When connections such as these are made, students begin to see that math is not an isolated subject, but rather a vital part of everyday life. Contextual examples and problems can be linked to students' everyday experiences outside the classroom, as well.

Writing and Mathematics

Being able to communicate ideas clearly is an important part of the *Functions and Applications 11* program. Students are asked to write about the mathematics they are learning, and communicate their understanding about what they are learning.

Take time to discuss the importance of being able to communicate understanding. The students' responses are meant to communicate with the teacher and are assessed as part of the mathematics work.

Literacy Connections

There are Literacy Connection suggestions in several chapters of this Teacher's Resource. These provide ideas as to how you might assist students to improve their mathematical literacy by using and extending particular questions that are in most numbered sections of the student text.

Cooperative Learning

Students learn effectively when they are actively engaged in the process of learning. Many of the sections in *Functions and Applications 11* include Investigate activities that foster this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other (Sternberg & Williams, 2002).

Teacher's Role

In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, you will need to coach them in how to learn cooperatively. This may include:

- Making sure that the materials are at hand and directions are perfectly clear so that students know what they are doing before starting group work
- Carefully structuring activities so that students can work together
- Providing coaching in how to provide peer feedback in a way that allows the listener to hear and attend
- Constantly monitoring student progress and providing assistance to groups having problems either with group cooperation or the math at hand

Types of Groups

The size of group you choose to use may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class setting (Van de Walle, 2000). Research suggests that small groups are fertile environments for developing mathematical reasoning (Artz & Yaloz-Femia, 1999).

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms (Kilpatrick, Swafford, & Findell, 2001). If the class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, you might pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. You might pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills

When coaching students about cooperative learning, you may want to consider task skills and working relationship skills, as indicated in the table below.

Task Skills	Working Relationship Skills
• Following directions • Communicating information and ideas • Seeking clarification • Ensuring that others understand • Actively listening to others • Staying on task	• Encouraging others to contribute • Acknowledging and responding to the contributions of others • Checking for agreement • Disagreeing in an agreeable way • Mediating disagreements within the group • Sharing • Showing appreciation for the efforts of others

Class discussions, modelling, peer coaching, role-playing, and drama can be used to provide positive task skills. For example, you might role-play different ways to provide feedback and have a class discussion on which ones students like and why. You might discuss common group roles and how group members can use them. Students also need to understand that the same person can play more than one role.

Role	Math Connection	Sample Comment
Leader	• Makes sure the group is on task and everyone is participating • Pushes group to come to a decision	Let's do this. Can we decide...? This is what I think we should do...
Recorder	• Manages materials • Writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• Presents the group's results and conclusions	We feel that… These are our conclusions… Our group found…
Organizer	• Watches time • Keeps on topic • Encourages getting the job done	Let's get started. Where should we start? So far we've done the following... Are we on topic? What else do we need to do?
Clarifier	• Checks that members understand and agree	Does everyone understand? So, what I hear you saying is... Do you mean that...?

Types of Strategies

A number of different types of cooperative learning strategies can be used in the mathematics classroom, and several are suggested in this Teacher's Resource. The *Functions and Applications 11* program includes selected blackline masters (BLMs) to use with some but not all of these strategies.

Think-Pair-Share

Students individually think about a concept, and then pick a partner to share their ideas. For example, students might work on the Discuss the Concepts questions, and then choose a partner to discuss the concepts with. Working together, the students could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Cooperative Task Group

Task groups of two to four students work on activities in the Investigate section. As a group, students share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw

Individual group members are responsible for researching and understanding a specific part of the information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during data management, this type of group might have "experts" in making various types of graphs using technology. Group members could then coach each other in making each kind of graph.

Another way of using the Jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching the shapes of various sports' surfaces might have a home group of four in which each member is responsible for researching one of soccer, baseball, hockey, or basketball. Individual members then move to expert groups. Expert groups include all of the students responsible for researching one of the sports. Each of the expert groups researches their particular sport. Once the information has been gathered and prepared for presentation, individual members of the expert group return to their home group and teach other members about their sport.

Placemat

In groups of four, students individually complete their section of a placemat. The group then pools their responses and completes the centre portion of the placemat with group responses. This method can be used for pre-assessment (diagnostic), review, or to summarize a topic.

Concept Attainment

Based on a list of examples and non-examples of a concept, students identify and define the concept. Then, they determine the critical attributes of the concepts and apply their defined concept to generate their own examples and non-examples.

Think Aloud

Work through a problem in front of the class, verbalizing your thinking throughout. This method can help develop process thinking in students.

Decision Tree

Students use a graphic organizer flow chart to identify key decisions and consequences.

Carousel

Students at different stations display and explain topics or concepts to other classmates who rotate through the stations, usually in order.

Timed Retell

Students sit in pairs facing each other. After some preparation time, Student A has 30 s to tell what she or he knows about the topic to Student B. Student B then retells the talk for about 30 s and adds additional information. Both students then write a summary of the talk.

Frayer Model

Students complete four quadrants for a specified topic: definition, facts/characteristics, examples, and non-examples. Variation: Give students a completed model and ask them to identify the topic/concept.

Word Wall

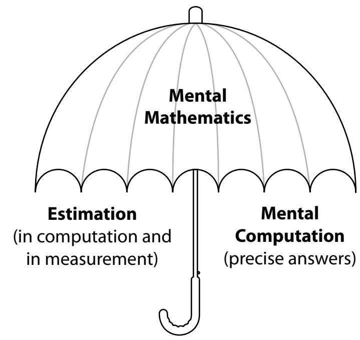
Individually or in groups, students complete cards for words or symbols, and then post the cards to use during future studies. One side of the card has the word or symbol, while the other side has four quadrants: the word, definition, picture or diagram, and an example or application.

Three-Step Interview

In triads, label students A, B, and C. Have students individually compose interview questions. Assign roles to the three groups: A = Interviewer; B = Interviewee; C = Recorder. Student A interviews Student B, while Student C records the information. Then the students rotate roles. After all the interviews are complete, students share the recorded information in a Round Robin format.

Mental Mathematics

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding.

Computational Estimation

Computational estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all key in problem solving. Over 80% of out-of-school problem-solving situations involve mental computation and estimation (Reys & Reys, 1986).

Computational estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations are a check for reasonableness and provide learners with a strategy for checking their actual calculations.

Measurement Estimation

This skill relies on awareness of the measurement attributes (e.g., metre, kilometre, litre, kilogram, hour). Just as computational estimation enhances number sense, practice in measurement estimation enhances measurement sense.

A *referent* is a personal mental tool that students can develop for use in thinking about measurement situations. Tools could include the distance from home to school, a 100-km trip, the capacity of a can of juice, the duration of 30 min, and the area of the math textbook cover. These referents develop with measurement practice, and specifically with practice that encourages students to form these frames of reference. Students can compare other measurements to these referents. By doing so, they can gain a better understanding of what may be happening in a problem-solving situation.

You can help students develop referents by doing activities such as asking students to use their fingers or hands to show such measurements as: 6 cm, 260 mm, 0.4 m, a 60° angle, or 2000 cm^3.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is these mental representations, or conceptual knowledge, that need to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Asking, *What do you see in your mind's eye* when asked to visualize, encourages students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these Mental Imaging Activities with your students.

Example 1:
Draw the mental image you have for each of the following:
- $\frac{2}{3}$
- 243 100 in relation to a million
- a 175° angle
- 0.56 m
- 36 cm
- a 6.3-kg fish
- a 6-g fish

Example 2:
Use mental imagery to answer the following:
1. How many edges does a cube have?
2. If I am facing east, what direction is to my left?
3. How many sides does a hexagonal pyramid have?
4. Imagine a 5-cm cube. What is its volume?
5. You cut off one vertex on a cube. What shape is exposed?

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with others—for finding the exact answer. These mental calculations are often referred to as Mental Math.

As with computational estimation, strategies for mental computation develop in quantity and quality over time. Students need regular practice in these strategies.

Some Points Regarding Mental Mathematics

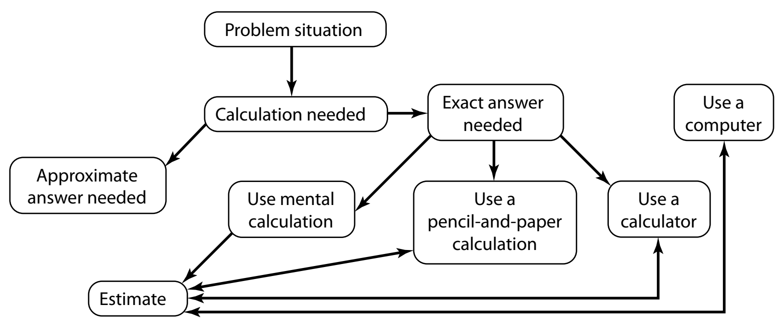
- Students must have knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- The various estimation and mental calculation strategies must be taught; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Unlike the traditional paper and pencil algorithms, there are many mental algorithms to learn. With the learning, however, comes a greater facility with numbers. Key to the development of skills in mental math is the understanding of place value and the number operations. This understanding is enhanced when students make mental math a focus when calculating.

- Mental math strategies are flexible; you need to select one that is appropriate for the numbers in the computation. Students should select appropriate strategies for a variety of computation examples, and use the strategies in problem-solving situations.
- Sometimes mental math strategies are used in conjunction with pencil and paper tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness, and to look for opportunities to calculate mentally.

Keep in Mind

Capable students of mathematics are comfortable with numbers. This comfort means that the students see patterns in numbers and intuitively know how they relate to each other and how they will behave in computational situations. Due to their comfort with numbers, these students have developed strong skills in estimation and mental math. Because of this, their understanding of numbers is further strengthened. We say they have "number sense." This sense of numbers develops gradually and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

The position of the National Council of Teachers of Mathematics (NCTM) on how to proceed when faced with a problem that requires a calculation is best explained with this chart.



The chart tells us that, given a problem requiring calculation, students should ask themselves the following questions:

- Is an approximate answer adequate or do I need the precise answer?
- If an estimate is sufficient, what estimation strategy best suits the numbers provided?
- If an exact answer is needed, can I use a mental strategy to solve it?
- If the numbers don't lend themselves to a mental strategy, can I do the calculation using a paper-and-pencil method?
- If the calculation is too complex, I will use a calculator. What is a good estimate for the answer?

NCTM's Number and Operations Standard states that, "Instructional programs from kindergarten through grade 12 should enable all students to compute fluently and make reasonable estimates" (Principles and Standards for School Mathematics, 2000). Whether the students select an estimation strategy, a mental strategy, a paper-and-pencil method, or use the calculator, they must use their estimation skills to judge the reasonableness of any answer.

Mental Math Strategies

In *Functions and Applications 11*, even though not always explicitly mentioned, students use mental math strategies throughout many parts of the text.

Problem Solving

Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

National Council of Teachers of Mathematics, 2000

Problem solving is an integral part of mathematics learning. The National Council of Teachers of Mathematics recommends that problem solving be the focus of all aspects of mathematics teaching because it encompasses skills and functions, which are an important part of everyday life.

NCTM Problem-Solving Standard
Instructional programs should enable all students to—
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill that can enhance logical reasoning. It requires students to make logical deductions, connections, and to apply their mathematical understanding to situations outside the classroom. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking, rather than just the means to an end of finding the correct answer.

In *Functions and Applications 11*, a variety of problem-solving opportunities are provided for students.

The problem-solving model involves four steps:

1. understand–indentify what the problem is asking
2. plan–choose which strategy or combination of strategies to use
3. solve–carry out the plan
4. look back–determine if the answer is reasonable

The **Examples** in the student text often provide **Solutions** using different methods. Students are encouraged to try different methods to solve problems. Common problem-solving strategies include the following: draw a diagram; make an organized list; look for a pattern; make a model; work backward; make a table or chart; act it out; use systematic trial; make an assumption; find needed information; choose a formula; solve a simpler problem.

- Each chapter includes the investigation of a specific real-life problem. The **Chapter Problem** is then revisited throughout the chapter through **Chapter Problem** questions, and ends with the **Chapter Problem Wrap-Up**.
- Questions that involve the **Mathematical Process Expectations** are embedded throughout the chapters.
- At the end of chapters 3, 5, and 8, students are presented with a **Task** where the solution path is not readily apparent and where solving the problem requires more than just applying a familiar procedure. These cross-curricular tasks require students to apply what they have learned in the current chapter and the previous chapters to solve real-life, broad-based problems.

Mathematical Processes

The seven expectations presented at the start of the mathematics curriculum in Ontario describe the mathematical processes that students need to learn and apply as they investigate mathematical concepts, solve problems, and communicate their understanding. Although the seven processes are categorized, they are interconnected and are integrated into student learning in all areas of the *Functions and Applications 11* program.

Problem Solving

Problem solving is the basis of the *Functions and Applications 11* program. Students can achieve the expectations by using this essential process, and it is an integral part of the mathematics curriculum in Ontario. Useful problem-solving strategies include: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

Reasoning and Proving

Critical thinking is an essential part of mathematics. As the students investigate mathematical concepts in *Functions and Applications 11*, they learn to: employ inductive reasoning; make generalizations based on specific findings; use counter-examples to disprove conjectures; use deductive reasoning.

Reflecting

Students are given opportunities to regularly and consciously reflect on their thought processes as they work through the **Investigates** and exercises in *Functions and Applications 11*. As they reflect, they learn to: recognize when the technique they are using is not helpful; make a conscious decision to switch to a different strategy; rethink the problem; search for related knowledge; determine the reasonableness of an answer.

Selecting Tools and Computational Strategies

Students are given many opportunities to use a variety of manipulatives, electronic tools, and computational strategies in the *Functions and Applications 11* program. The student text provides examples of and ways to use various types of technology, such as calculators, computers, and communications technology, to perform particular mathematical tasks, investigate mathematical ideas, and solve problems. These important problem-solving tools can be used to: investigate number and graphing patterns, geometric relationships, and different representations; simulate situations; collect, organize, and sort data; extend problem solving.

Connecting

Functions and Applications 11 is designed to give students many opportunities to make connections between concepts, skills, mathematical strands, and subject areas. These connections help them see that mathematics is much more than a series of isolated skills and concepts. Connecting mathematics to their everyday lives also helps students see that mathematics is useful and relevant outside the classroom.

Representing

Throughout the *Functions and Applications 11* program, students represent mathematical ideas in various forms: numeric, geometric, graphical, algebraic, pictorial, and concrete representations, as well as representation using dynamic software. Students are encouraged to use more than one representation for a single problem, seeing the connections between them.

Communicating

Students use many different ways of communicating mathematical ideas in the *Functions and Applications 11* program, including: oral, visual, writing, numbers, symbols, pictures, graphs, diagrams, and words. The process of communication helps students reflect on and clarify ideas, relationships, and mathematical arguments.

Using Mathematical Processes

You can encourage students to use the mathematical processes in their work by prompting them with questions such as the following:

- *How can you tell whether your answer is correct/reasonable?* This promotes reasoning and reflection.
- *Why did you choose this method?* This promotes reflection, reasoning, selecting tools and computational strategies, and communication.
- *Could you have solved the problem another way?* This promotes reasoning, reflection, selecting tools and computational strategies, representing, and communication.
- *In what context have you solved a problem like this before?* This promotes connecting.

You can also encourage students to use a Think-Pair-Share approach to problem solving (see the **Cooperative Learning** section in this Program Overview). They will benefit greatly from brainstorming ideas and comparing methods of approach. A useful life skill is willingness to try different methods of solving a problem, learning from methods that perhaps do not reach the final goal, and being able to change their approach to reach the solution.

Technology

The use of technology in instruction should further alter both the teaching and the learning of mathematics. Computer software can be used effectively for class demonstrations and independently by students to explore additional examples, perform independent investigations, generate and summarize data as part of a project, or complete assignments. Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings.

In this setting, the teacher encourages experimentation and provides opportunities for students to summarize ideas and establish connections with previously studied topics.

Curriculum and Evaluation Standards for School Mathematics, NCTM, 1989

Functions and Applications 11 taps the full power of today's interactive technologies to engage students in math inquiry, research, and problem solving. Technology is a major focus in several of the chapters, providing students with hands-on experience in creating graphs, and constructing and manipulating geometric figures. If at all possible, a classroom environment should be in place in which students are encouraged to reach for and apply technology whenever they feel the situation calls for it. In such an environment, the ongoing use of technology becomes another tool in the student's problem-solving tool kit, rather than a discrete event.

The *Functions and Applications 11* program includes opportunities for students to do research in the library or on the Internet. Consider having a class discussion on Internet Web sites and appropriate sources. Remind students that anyone can create a Web site on any topic on the Internet. Ask students to raise their hands if they have a personal Web site or keep an Internet journal (a *blog*). Explain that Web sites like these contain personal opinions and information contained on them should be looked at critically. This also may provide an opportunity to remind students that personal information should never be revealed over e-mail, in an on-line journal, or a chat-room, and that anything that makes them uncomfortable should be reported immediately to their parent or guardian.

Types of Programs

Several types of software programs are used in *Functions and Applications 11*.

Technology BLMs are also available, providing students with step-by-step directions on how to use technology, such as software and Computer Algebra System calculators, to explore the mathematical concepts of the lesson. These BLMs include:

- BLM T-1 Microsoft® Excel
- BLM T-2 *The Geometer's Sketchpad*® 3
- BLM T-3 *The Geometer's Sketchpad*® 4
- BLM T-4 *Fathom*™
- BLM T-5 The Computer Algebra System (CAS) on the TI-89 Calculator
- BLM T-6 Using the CBR™ (Calculator Based Ranger)

The **Technology Appendix**, on pages 436–469, of the student text provides clear step-by-step instruction in the basic functions of the TI-83 Plus Basics, TI-84 Plus Basics and TI-89 TITANIUM Basics graphing calculators and the basic features of *The Geometer's Sketchpad*® and of *Fathom*™ statistical package.

Assessment

The main purpose of assessment is to improve student learning. Assessment data helps you determine the instructional needs of your students during the learning process. Some assessment data is used to evaluate students for the purpose of reporting.

Assessment must be purposeful and inclusive for all students. It should be varied to reflect learning styles of students and be clearly communicated with students and parents. Assessment can be used diagnostically to determine prior knowledge, formatively to inform instructional planning, and in a summative manner to determine how well the students have achieved the expectations at the end of a learning cycle.

Diagnostic Assessment

Assessment for diagnostic purposes can determine where individual students will need support and will help to determine how the classroom time needs to be spent. *Functions and Applications 11* provides you with diagnostic support at the start of the text and the beginning of every chapter.

- The **Prerequisite Skills** section at the beginning of each chapter provides coaching on essential concepts and skills needed for the upcoming chapter. **Prerequisite Skills Self-Assessment** blackline masters are also provided for each chapter.
- For students needing support beyond the Prerequisite Skills, the **Practice Masters** provided in this Teacher's Resource help to develop conceptual understanding and improve procedural efficiency.

Diagnostic support is also provided at the start of every section.

- Each section begins with an introduction to facilitate open discussion in the classroom.
- Each activity starts with a question that stimulates prior knowledge and allows you to monitor students' readiness.

Formative Assessment

Formative assessment tools are provided throughout the text and Teacher's Resource. Formative assessment allows you to determine students' strengths and weaknesses and guide your class towards improvement. *Functions and Applications 11* provides blackline masters for student use that complement the text in areas where formative assessment indicates that students need support.

The **Chapter Opener**, visual, and the introduction to the **Chapter Problem** at the beginning of each chapter in the student book provide opportunities for you to do a rough formative assessment of student awareness of the chapter content.

Within each lesson:

- **Key Concepts** can be used as a focus for classroom discussion to determine the students' readiness to continue.
- **Communicate Your Understanding** questions allow you to determine if the student has developed the conceptual understanding and/or skills that were the goal of the section.
- **Practise (A)** questions allow you to determine whether students have basic knowledge skills related to the expectation(s) of the section.
- **Apply (B)** questions offers you an opportunity to determine students' understanding of concepts through conversations and written work. It also allows you to monitor students' procedural skills, their application of procedures, their ability to communicate their understanding of concepts, and their ability to solve problems related to the section's Key Concepts.

- **Achievement Check** questions allow students to demonstrate their knowledge and understanding and their ability to apply, think of, and communicate what they have learned.
- **Chapter Problem** questions provide opportunities to verify that students are developing the skills and understanding they need to complete the **Chapter Problem Wrap-Up** questions.
- **Extend (C)** questions are more challenging and thought-provoking, and are aimed at Level 3 and 4 performance.
- **Chapter Reviews** and **Cumulative Reviews** provide an opportunity to assess Knowledge/Understanding, Thinking, Communication, and Application.

Summative Assessment

Summative data is used for both planning and evaluation.

- A **Practice Test** (Text and BLM) and a **Chapter Test** (BLM only) in each chapter assess students' achievement of the expectations in the areas of Knowledge/Understanding, Thinking, Communication, and Application.
- The **Chapter Problem** provides a problem-solving opportunity using an open-ended question format that is revisited in the **Chapter Problem Wrap-Up** questions. The **Chapter Problem** can be used to evaluate students' understanding of the expectations under the categories of Knowledge and Understanding, Thinking, Communication, and Application.
- **Tasks** are open-ended investigations with rubrics provided. They are presented at the end of each chapter. The Tasks require students to use and make connections among several concepts from the preceding chapters.
- BLMs of rubrics for Chapter Problems and Tasks are provided in the Teacher's Resource CD-ROM.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios:

- Help teachers assess students' growth and mathematical understanding
- Provide insight into students' self-awareness about their own progress
- Help parents understand their child's growth

Functions and Applications 11 has many components that provide ideal portfolio items. Inclusion of all or any of these chapter items provides insight into students' progress in a non-threatening, formative manner. These items include:

- Students' responses to the **Chapter Opener**
- Students' responses to the **Chapter Problem Wrap-Up** assignments
- Responses to **Discuss the Concepts** questions, which allow students to explore their initial understanding of concepts
- Answers to **Achievement Check** questions, which are designed to show students' mastery of specific expectations
- **Task** assignments, which show students' understanding across several chapters

Assessment Masters

Functions and Applications 11 provides a variety of assessment tools with the chapter-specific blackline masters, such as Chapter Tests, Chapter Problem Wrap-Up rubrics, and Task rubrics. In addition, the program offers a wide variety of generic assessment blackline masters. These BLMs will help you to effectively monitor student progress and evaluate instructional needs.

Generic Assessment BLM	Type	Purpose
BLM A-1 Assessment Recording Sheet	Chart	Organize comments for assessment of students observations, portfolios, and presentations
BLM A-2 Attitudes Assessment Checklist	Checklist	Assess students' attitude as they work on a task
BLM A-3 Portfolio Checklist	Checklist	Assess students' portfolios
BLM A-4 Presentation Checklist	Checklist	Assess students' oral and written presentations
BLM A-5 Problem Solving Checklist	Checklist	Assess students' problem solving skills
BLM A-6 Knowledge and Understanding General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Knowledge and Understanding category
BLM A-7 Thinking General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Thinking category
BLM A-8 Application General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Application category
BLM A-9 Communication General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Communication category
BLM A-10 Observation General Scoring Rubric	Rubric	Assess students' understanding of the expectations under all four categories
BLM A-11 Group Work Assessment Recording Sheet	Worksheet	Record comments as students work on group tasks
BLM A-12 Group Work Assessment General Scoring Rubric	Rubric	Assess students' group-related work
BLM A-13 Self-Assessment Recording Sheet	Worksheet	Students self-assess their understanding of chapter material
BLM A-14 Self-Assessment Checklist	Checklist	Students self-assess their understanding of chapter material
BLM A-15 Teamwork Self Assessment	Worksheet	Students evaluate their work as part of a team
BLM A-16 Assessing Work in Progress	Worksheet	Student groups assess their progress as they work to complete a task
BLM A-17 Learning Skills Checklist	Checklist	Assess students' work habits and learning skills
BLM A-18 Opinion Piece Checklist	Checklist	Assess students' work on an opinion piece
BLM A-19 Report Checklist	Checklist	Assess students' work on a report

Intervention

Functions and Applications 11 accommodates a broad range of needs and learning styles, including those students requiring accommodations, and students with limited proficiency in English. This Teacher's Resource provides support in addressing multiple intelligences and learning styles through a variety of strategies.

- Excellent visuals and multiple representations of concepts and instructions support visual learners, ESL students, and struggling readers
- Relevant contexts, including multicultural examples, engage students and provide a purpose for the mathematics being learned
- **Extend** questions in the student text provide additional challenge for those students who can complete the Practise and Apply questions with no difficulties.
- **Accommodations** in the margin provide suggestions for students having difficulties or needing enrichment

Reaching all Students

Students may experience difficulty meeting provincial standards for a variety of reasons. General cognitive delays, social-emotional issues, behavioural difficulties, health-related factors, and extended or sporadic absences from instruction underlie the math difficulties experienced by some students. These factors do not explain the challenges other students encounter, however. For these students, math difficulties are usually related to three key areas: language, visual/perceptual/spatial/motor, or memory.

Language

Students with language learning difficulties demonstrate difficulty reading and understanding math vocabulary and math story problems, and determining saliency (e.g., picking out the most important details from irrelevant information). Processing information that is presented using oral or written language is often difficult for these students, who may be more efficient learners when information is presented in a non-verbal, visual format. Diagrams and pictorial representations of math concepts are usually more meaningful to these students than lengthy verbal or written descriptions.

Visual/Perceptual/Spatial/Motor

Some students demonstrate difficulties understanding and processing information that is presented visually and in a non-verbal format. Language support to supplement and make sense of visually presented information is often beneficial (e.g., verbal explanation of a visual chart). Visual, perceptual, spatial, and motor difficulties may be evident in students' written output, as well as in their ability to process visually inputted information. Difficulties with near and far point copying, accurately aligning numbers in columns, properly sequencing numbers, and illegible handwriting are examples of output difficulties in this area.

Memory (Short-Term, Working, and Long-Term Memory)

Students with short-term memory difficulties find it hard to remember what they have just heard or seen (e.g., auditory short-term memory, visual short-term memory). A weak working or active memory makes it difficult for students to hold information in their short-term memory and manipulate it (e.g., hold what they have just heard and then perform a mathematical operation with that information). For others, the retrieval of information from long-term memory (e.g., remembering number facts and previously taught formulae) is difficult. Students with long-term memory difficulties may also have difficulty storing information in their long-term memory, as well as retrieving it.

Modifications, Individual Education Plans (IEP), and Accommodations

A modification changes what is being taught by reaching well below or well above grade level, or by reducing the number of curriculum expectations. Students with a modified math program have an Individual Education Plan (IEP) describing how their program differs from classmates in their grade. An IEP also describes strategies, resources, and how the student will be evaluated. Modifying a student's program is a well-defined process involving the principal, teachers, parents, and student. Addressing a student's need for program modification falls outside the scope of this Teacher's Resource.

Accommodations

Accommodations do not change what is being taught. Rather, an accommodation to a student's program alters the "how," "when," or "where" the student is taught or assessed without changing curriculum expectations. This Teacher's Resource provides suggested accommodations based on the student's identified area of difficulty. Three types of accommodations are provided.

- Instructional accommodations refer to changes in teaching strategies that allow the student to access the curriculum.
- Environmental accommodations refer to changes that are required to the classroom and/or school environment.
- Assessment accommodations refer to changes that are required in order for the student to demonstrate learning.

The following three charts provide accommodations for the three key areas underlying math difficulties. Accommodations have been grouped under the headings of instructional, environmental, and assessment.

Chart I: Accommodations for Students with Language Difficulties

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Give concise, step-by-step directions • Teach students to look for cue words, highlight these words • Use visual models • Use visual representations to accompany word problems • Encourage students to look for common patterns in word problems	• Provide reference charts with operations and formulae stated simply • Post reference charts with math vocabulary • Reinforce learning with visual aids and manipulatives • Using a visual format, post strategies for problem solving • Use a peer tutor or buddy system	• Read instructions/word problems on tests to students • Extend time lines

Chart II: Accommodations for Students with Visual/Perceptual/Spatial/Motor Difficulties

Instructional	Environmental	Assessment
• Reduce copying • Provide worksheets • Provide grid paper • Provide concrete examples • Allow use of a number line • Provide a math journal • Encourage and teach self-talk strategies • Chunk learning and tasks	• visual bombardment • a work carrel or work area that is not visually distracting • rest periods and breaks	• Provide graph paper for tests • Extend time lines • Provide consumable tests • Reduce the number of questions required to indicate competency • Provide a scribe when lengthy written answers are required

Chart III: Accommodations for Students with Memory Difficulties

Instructional	Environmental	Assessment
• Regularly review concepts • Activate prior knowledge • Teach mnemonic strategies (e.g., SOHCAHTOA) • Teach visualization strategies • Allow use of multiplication tables • Colour-code steps in sequence • Teach functional math concepts related to daily living	• Provide reference charts with commonly used facts, formulae, and steps for problem-solving • Allow use of a calculator • Use games and computer programs for practice of knowledge-based skills	• Allow use of formula lists • Allow use of other reference charts as appropriate • Allow use of calculators • Extend time lines • Present one concept-type of question at a time

Accommodations for ESL Students

For ESL students, language issues are pervasive throughout all subject areas, including math. Non-math words are often more problematic for ESL students because understanding the meaning of these words is often taken for granted. Everyday language is laden with vocabulary, comparative forms, figurative speech, and complex language structures that are not explained. By contrast, key words in math are usually highlighted in the text and carefully explained by the teacher. Accommodations to the programs of ESL students do not change the curriculum expectations.

Accommodations for ESL Students

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Explain colloquial expressions and figurative speech • Review comparative forms of adjectives	• Display reference charts with mathematical terms and language • Encourage personal math dictionaries with math terms and formulae	• Allow access to personal math dictionaries • Read instructions to students and clarify terms • Allow additional time

Accommodations for Learning-Disabled Students

A student with a learning disability usually suffers from an inability to think, listen, speak, write, spell, or calculate that is not obviously caused by any mental or physical disability. There seems to be a lag in the developmental process and/or a delay in the maturation of the central nervous system. Providing simplified presentations, repetitions, more specific examples, or breaking content blocks into simpler sections may help in minor cases of learning disability.

Accommodations for At-Risk Students

Students learn in different ways. For all students to have the opportunity to succeed, we need to have alternative ways of delivering program. For example, a student whose dominant learning modality is kinesthetic/tactile needs active, hands-on investigations. A student with strong social/emotional intelligence benefits more from interpersonal interactions and needs instructional strategies like Jigsaw or Think-Pair-Share to optimize their chances of acquiring the skills and knowledge in the curriculum (see the **Cooperative Learning** section in this Teacher's Resource). These students underachieve and become at-risk not because they have acquired concepts imperfectly (and need remediation), but because they have not become engaged in their own learning, and often have failed to acquire concepts at all. At-risk students are in danger of completing their schooling without adequate skills development to function effectively in society. Risk factors include low achievement and retention, behaviour problems, poor attendance, and low socio-economic status.

By addressing topics in a new or different way, teachers can provide at-risk students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Neither failing such students nor putting them in pullout programs has produced much gain in achievement, but there are certain approaches that do help.

- Allow students to proceed at their own pace through a well-defined series of instructional objectives.
- Place students in small, mixed-ability learning groups to master the material first presented by the teacher. Reward teams based on the individual learning of all team members.
- Have students serve as peer tutors, as well as being tutored. This helps raise their self-esteem and makes them feel they have something to contribute.
- Involve students in learning about something that is relevant to them, such as money management or wise shopping.
- Get parents involved in their child's learning as much as possible.

Curriculum Correlation between McGraw-Hill Ryerson *Functions and Applications 11* and The Ontario Curriculum Functions and Applications, Grade 11, University/College Preparation (MCF3M)

This course enables students to broaden their understanding of mathematics as a problem-solving tool in the real world. Students will extend their understanding of quadratic relations; investigate situations involving exponential growth; solve problems involving compound interest; solve financial problems connected with vehicle ownership; develop their ability to reason by collecting, anlysing, and evaluating data involving one variable; connect probability and statistics; and solve problems in geometry and trigonometry. Students consolidate their mathematical skills as they solve problems and communicate their thinking.

Mathematical Process Expectations

The mathematical processes are to be integrated into student learning in all areas of this course.

	Throughout this course, students will:
Problem Solving	• develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical idea and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and realter mathematical ideas or situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; on screen dynamic representations), connect and compare them, and select and apply the appropriate representation to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

The mathematical process expectations are integrated throughout *Functions and Applications 11*.

The codes for the curriculum expectations used here are consistent with the codes used in the PDF document for Functions and Applications Expectations (MCF3M) that is available on-line from The Ontario Curriculum Unit Planner (OCUP), in the section Grade by Grade PDFs of Ontario Curriculum Expectations http://www.ocup.org.

Quadratic Functions

Overall Expectations

By the end of this course, students will:

1. simplify and evaluate numerical expressions involving exponents, and make connections between the numeric, graphical, and algebraic representations of exponential functions;
2. identify and represent exponential functions, and solve problems involving exponential functions, including problems arising from real-world applications;
3. demonstrate an understanding of compound interest and annuities, and solve related problems.

Specific Expectations

	Chapter/Section	Pages
Solving Quadratic Equations		
By the end of this course students will:		
QF1.1 – pose problems involving quadratic relations arising from real-word applications and represented by tables of values and graphs, and solve these and other problems (e.g., "From the graph of the height versus time, can you tell me how high the ball was thrown and the time when it hit the ground?")	**1.3** **2.5**	**23–30** **108–113**
QF1.2 – represent situations (e.g., the area of a picture frame of variable width) using quadratic expressions in one variable, and expand and simplify quadratic expressions in one variable [e.g., $2x(x + 4) - (x + 3)^2$]*	**2.2**	**76–85**
QF1.3 – factor quadratic expressions in one variable, including those for which $a \neq 1$, (e.g., $3x^2 + 13x - 10$), differences of squares (e.g., $4x^2 - 25$), and perfect square trinomials (e.g., $9x^2 + 24x + 16$), by selecting and applying an appropriate strategy* *Sample problem:* Factor $2x^2 - 12x + 10$.	**2.3, 2.4**	**88–107**
QF1.4 – solve quadratic equations by selecting and applying a factoring strategy	**2.4, 2.5**	**98–113**
QF1.5 – determine, through investigation, and describe the connection between the factors used in solving a quadratic equation and the x-intercepts of the graph of the corresponding quadratic relation *Sample problem:* The profit, P, of a video company, in thousands of dollars, is given by $P = -5x^2 + 550x - 500$, where x is the amount spent on advertising, in thousands of dollars. Determine, by factoring and by graphing, the amount spent on advertising that will result in a profit of $0. Describe the connection between the two strategies.	**2.5**	**108–113**
QF1.6 – explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numeric example; follow a demonstration of the algebraic development, with technology, such as computer algebra systems, or without technology [student reproduction of the development of the general case is not required]), and apply the formula to solve quadratic equations, using technology	**3.2**	**135–144**

QF1.7 – relate the real roots of a quadratic equation to the *x*-intercepts of the corresponding graph, and connect the number of real roots to the value of the discriminant (e.g., there are no real roots and no *x*-intercepts if $b^2 - 4ac < 0$)	**3.3**	**145–152**
QF1.8 – determine the real roots of a variety of quadratic equations (e.g., $100x^2 = 115x + 35$), and describe the advantages and disadvantages of each strategy (i.e., graphing; factoring; using the quadratic formula) *Sample formula:* Generate 10 quadratic equations by randomly selecting integer values for *a*, *b*, and *c* in $ax^2 + bx + c = 0$. Solve the equations using the quadratic formula. How many of the equations could you solve by factoring?	**3.3**	**145–152**
* The knowledge and skills described in this expectation may initially require the use of a variety of learning tools (e.g., computer algebra systems, algebra tiles, grid paper.		
Connecting Graphs and Equations of Quadratic Functions		
By the end of this course, students will:		
QF2.1 – explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., using the vertical-line test) *Sample problem:* Investigate, using numeric and graphical representations, whether the relation $x = y^2$ is a function, and justify your reasoning.	**1.1**	**6–14**
QF2.2 – substitute into and evaluate linear and quadratic functions represented using function notation [e.g., evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$], including functions arising from real-world applications *Sample problem:* The relationship between the selling price of a sleeping bag, *s* dollars, and the revenue at that selling price, $r(s)$ dollars, is represented by the function $r(s) = -10s^2 + 1500s$. Evaluate, interpret, and compare $r(29.95)$, $r(60.00)$, $r(75.00)$, $r(90.00)$, and $r(130.00)$.	**1.3**	**23–30**
QF2.3 – explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately (e.g., for $y = x^2 + 1$, the domain is the set of real numbers, and the range is $y \geq 1$)	**1.2, 1.3**	**15–30**
QF2.4 – explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications *Sample problem:* A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?	**1.3**	**23–30**

QF2.5 – determine, through investigation using technology, the roles of a, h, and k in quadratic functions of the form $f(x) = a(x - h)^2 + k$, and describe these roles in terms of transformations on the graph of $f(x) = x^2$ (i.e., translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis) *Sample problem:* Investigate the graph $f(x) = 3(x - h)^2 + 5$ for various values of h, using technology, and describe the effects of changing h in terms of a transformation.	**1.4, 1.5**	**31–46**
QF2.6 – sketch graphs of $g(x) = a(x - h)^2 + k$ by applying one or more transformations to the graph of $f(x) = x^2$ *Sample problem:* Transform the graph of $f(x) = x^2$ to sketch the graphs of $g(x) = x^2 - 4$ and $h(x) = -2(x + 1)^2$.	**1.6**	**47–53**
QF2.7 – express the equation of a quadratic function in the standard form $f(x) = ax^2 + bx + c$, given the vertex form $f(x) = a(x - h)^2 + k$, and verify, using graphing technology, that these forms are equivalent representations *Sample problem:* Given the vertex form $f(x) = 3(x - 1)^2 + 4$, express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.	**2.2**	**76–85**
QF2.8 – express the equation of a quadratic function in the vertex form $f(x) = a(x - h)^2 + k$, given the standard form $f(x) = ax^2 + bx + c$, by completing the square (e.g., using algebra tiles or diagrams; algebraically), including cases where $\frac{b}{a}$ is a simple rational number (e.g., $\frac{1}{2}$, 0.75), and verify, using graphing technology, that these forms are equivalent representations	**3.1**	**124–134**
QF2.9 – sketch graphs of quadratic functions in the factored form $f(x) = a(x - r)(x - s)$ by using the x-intercepts to determine the vertex	**2.1**	**64–75**
QF2.10 – describe the information (e.g., maximum, intercepts) that can be obtained by inspecting the standard form $f(x) = ax^2 + bx + c$, the vertex form $f(x) = a(x - h)^2 + k$, and the factored form $f(x) = a(x - r)(x - s)$ of a quadratic function	**2.1** **3.4**	**64–75** **153–163**
QF2.11 – sketch the graph of a quadratic function whose equation is given in the standard form $f(x) = ax^2 + bx + c$ by using a suitable strategy (e.g., completing the square and finding the vertex; factoring, if possible, to locate the x-intercepts), and identify the key features of the graph (e.g., the vertex, the x- and y-intercepts, the equation of the axis of symmetry, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing)	**3.4**	**153–163**

Solving Problems Involving Quadratic Functions		
By the end of this course, students will:		
QF3.1 – collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data *Sample problem:* When a 3 × 3 × 3 cube made up of 1 × 1 × 1 cubes is dipped into red paint, 6 of the smaller cubes will have 1 face painted. Investigate the number of smaller cubes with 1 face painted as a function of the edge length of the larger cube, and graph the function.	**1.3**	**23–30**
QF3.2 – determine, through investigation using a variety of strategies (e.g., applying properties of quadratic functions such as the *x*-intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology (e.g., graphing software, graphing calculator)	**3.5**	**164–173**
QF3.3 – solve problems arising from real-world applications, given the algebraic representation of a quadratic function (e.g., given the equation of a quadratic function representing the height of a ball over elapsed time, answer questions that involve the maximum height of the ball, the length of time needed for the ball to touch the ground, and the time interval when the ball is higher than a given measurement) *Sample problem:* In a DC electrical circuit, the relationship between the power used by a device, *P*, (in watts, W), the electric potential difference (voltage), *V* (in volts, V), the current, *I* (in amperes A), and the resistance, *R*, (in ohms, Ω), is represented by the formula $P = IV - I^2R$. Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 Ω. Determine the current needed in order for the device to use the maximum amount of power.	**1.3, 1.4, 1.5, 1.6** **3.1, 3.2, 3.3, 3.4, 3.5**	**23–53** **124–173**

Exponential Functions

Overall Expectations

By the end of this course, students will:

1. simplify and evaluate numerical expressions involving exponents, and make connections between the numeric, graphical, and algebraic representations of exponential functions;
2. identify and represent exponential functions, and solve problems involving exponential functions, including problems arising from real-world applications;
3. demonstrate an understanding of compound interest and annuities, and solve related problems.

Specific Expectations

	Chapter/Section	Pages
Connecting Graphs and Equations of Exponential Functions		
By the end of this course, students will:		
EF1.1 – determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers) *Sample problem:* The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4$. What value would you assign to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.	**6.3**	**296–304**
EF1.2 – evaluate, with and without technology, numerical expressions containing integer and rational exponents and rational bases [e.g., 2^{-3}, $(-6)^3$, 4^2, 1.01^{120}]	**6.2, 6.3**	**288–304**
EF1.3 – graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ ($a > 0$, $a \neq 1$), define this relation as the function $f(x) = a^x$, and explain why it is a function	**6.5**	**312–317**
EF1.4 – determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x$ ($a > 0$, $a \neq 1$), function machines] *Sample problem:* Graph $f(x) = 2^x$, $g(x) = 3^x$, and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the y-intercepts.	**6.5**	**312–317**

EF1.5 – determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numeric expressions involving exponents [e.g., $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3$], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g., $(5^3)^2$], and use the rules to simplify numerical expressions containing integer exponents [e.g., $(2^3)(2^5) = 2^8$]	**6.1**	**280–287**
EF1.6 – distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest, population growth) *Sample problem:* Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.	**6.6**	**318–325**
Solving Problems Involving Exponential Functions		
By the end of this course, students will:		
EF2.1 – collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data *Sample problem:* Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.	**6.4**	**305–311**
EF2.2 – identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve)	**6.6, 6.7**	**318–333**
EF2.3 – solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations *Sample problem:* The temperature of a cooling liquid over time can be modelled by the exponential function $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28°C.	**6.7**	**326–333**

Solving Financial Problems Involving Exponential Functions		
By the end of this course, students will:		
EF3.1 – compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time *Sample problem:* Compare, using tables of values and graphs, the amounts after each of the first five years for a \$1000 investment at 5% simple interest per annum and a \$1000 investment at 5% interest per annum, compounded annually.	**7.1**	**346–354**
EF3.2 – solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), and the principal, P (also referred to as present value, PV), using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$] *Sample problem:* Calculate the amount if \$1000 is invested for three years at 6% per annum, compounded quarterly.	**7.2, 7.3**	**355–366**
EF3.3 – determine, through investigation (e.g., using spreadsheets and graphs), that compound interest is an example of exponential growth [e.g., the formulas for compound interest, $A = P(1 + i)^n$, and present value, $PV = A(1 + i)^n$, are exponential functions, where the number of compounding periods, n, varies] *Sample problem:* Describe an investment that could be represented by the function $f(x) = 500(1.01)^x$.	**7.1, 7.2, 7.3**	**346–366**
EF3.4 – solve problems, using a TVM Solver on a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period, i, or the number of compounding periods, n, in the compound interest formula $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$] *Sample problem:* Use the TVM Solver in a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.	**7.4**	**367–371**
EF3.5 – explain the meaning of the term annuity, through investigation of numeric and graphical representations using technology	**8.1**	**382–389**
EF3.6 – determine, through investigation using technology (e.g., the TVM Solver on a graphing calculator, online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary simple annuities (i.e., annuities in which payments are made at the *end* of each period, and the compounding period and the payment period are the same) (e.g., long-term savings plans, loans) *Sample problem:* Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?	**8.2, 8.3**	**390–404**

EF3.7 – solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan)	**8.2, 8.4**	**390–396 405–411**

Trigonometric Functions

Overall Expectations

By the end of this course, students will:

1. solve problems involving trigonometry in acute triangles using the sine law and the cosine law, including problems arising from real-world applications;

2. demonstrate an understanding of periodic relationships and the sine function, and make connections between the numeric, graphical, and algebraic representations of sine functions;

3. identify and represent sine functions, and solve problems involving sine functions, including problems arising from real-world applications.

Specific Expectations

	Chapter/Section	Pages
Applying the Sine Law and the Cosine Law in Acute Triangles		
By the end of this course, students will:		
TF1.1 – solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios	**4.1, 4.2**	**186–196**
TF1.2 – solve problems involving two right triangles in two dimensions *Sample problem:* A helicopter hovers 500 m above a long straight road. Ahead of the helicopter on the road are two trucks. The angles of depression of the two trucks from the helicopter are 60° and 20°. How far apart are the two trucks?	**4.3**	**197–201**
TF1.3 – verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ in triangle *ABC* while dragging one of the vertices)	**4.4, 4.5**	**202–215**
TF1.4 – describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles	**4.6**	**216–221**
TF1.5 – solve problems that require the use of the sine law or the cosine law in acute triangles, including problems arising from real-world applications (e.g., surveying, navigation, building construction)	**4.6**	**216–221**

Connecting Graphs and Equations of Sine Functions		
By the end of this course, students will:		
TF2.1 – describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numeric or graphical representation	**5.1**	**232–238**
TF2.2 – predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural gas consumption in Ontario from previous consumption)	**5.1**	**232–238**
TF2.3 – make connections between the sine ratio and the sine function by graphing the relationship between angles from 0° to 360° and the corresponding sine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$, and explaining why the relationship is a function	**5.2, 5.3**	**239–253**
TF2.4 – sketch the graph of $f(x) = \sin x$ for angle measures expressed in degrees, and determine and describe its key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)	**5.3**	**248–253**
TF2.5 – make connections, through investigation with technology, between changes in a real-world situation that can be modelled using a periodic function and transformations of the corresponding graph (e.g., investigate the connection between variables for a swimmer swimming lengths of a pool and transformations of the graph of distance from the starting point versus time) *Sample problem:* Generate the graph of a periodic function by walking a circle of 2-m diameter in front of a motion sensor. Describe how the following changes in the motion change the graph: starting at a different point on the circle; starting a greater distance from the motion sensor; changing direction; increasing the radius of the circle.	**5.4, 5.5**	**254–263**
TF2.6 – determine, through investigation using technology, the roles of the parameters a, c, and d in functions in the form $f(x) = a \sin x$, $f(x) = \sin x + c$, and $f(x) = \sin(x - d)$, and describe these roles in terms of transformations on the graph of $f(x) = \sin x$ with angles expressed in degrees (i.e., translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)	**5.4, 5.5**	**254–263**
TF2.7 – sketch graphs of $f(x) = a \sin x$, $f(x) = \sin x + c$, and $f(x) = \sin(x - d)$ by applying transformations to the graph of $f(x) = \sin x$, and state the domain and range of the transformed functions *Sample problem:* Transform the graph of $f(x) = \sin x$ to sketch the graphs of $g(x) = -2 \sin x$ and $h(x) = \sin(x - 180°)$, and state the domain and range of each function.	**5.4, 5.5**	**254–263**

Connecting Graphs and Equations of Sine Functions		
By the end of this course, students will:		
TF3.1 – collect data that can be modelled as a sine function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials, measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data *Sample problem:* Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.	**5.1, 5.5**	**232–238** **264–267**
TF3.2 – identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range	**5.3, 5.5**	**248–253** **264–267**
TF3.3 – pose problems based on applications involving a sine function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation *Sample problem:* The height above the ground of a rider on a Ferris wheel can be modelled by the sine function $h(x) = 25\sin(x - 90°) + 27$, where $h(x)$ is the height, in metres, and x is the angle, in degrees, that the radius from the centre of the ferris wheel to the rider makes with the horizontal. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider and the measures of the angle when the height of the rider is 40 m.	**5.5**	**264–267**