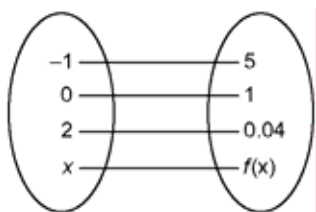# Section 6.5 Exponential Functions and Their Properties

1. a) State the domain and range of the function $f(x) = a^x$.
   b) What is the name of a straight line that the curve $y = b^x$ approaches?
   c) Consider the function $f(x) = c^x$. For what values of c is the function increasing from left to right? For what values of c is the function decreasing from left to right?

2. a) Determine two functions, $f(x)$ and $g(x)$, that have all of the properties listed in the table.

Property	$f(x)$	$g(x)$
Domain	$x \in \mathbf{R}$	$x \in \mathbf{R}$
Range	$y > 0, y \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$
x-intercept	none	none
y-intercept	1	1
Intervals of increase or decrease	always increasing	always decreasing
Asymptotes	x-axis	x-axis

   b) Draw the functions $f(x)$ and $g(x)$ on the same set of axes.

3. A mapping diagram for an exponential function is shown. Write an equation for the function.

4. The table shows several input and output values for a function machine. Find the equation of an exponential function that matches the input and output of the machine.

Input	Output
–4	0.0016
0	1
4	625

5. Each of the following situations can be modelled with an exponential function. Indicate which require a value of $a > 1$, and which require a value of $0 < a < 1$. Explain your choices.
   a) Bacteria on a contaminated counter-top double their number every 0.5 h.
   b) Half of the pizza on a counter-top is eaten every 2 h.

6. A scuba diver brought a light meter when she went diving in murky water. She noticed that the light intensity decreased by 40% for every 10 m she descended.
   a) What fraction of surface light intensity is left at a depth of 10 m? at 20 m?
   b) Assume that the meter read 1 unit at the surface. Write an exponential function to model the decrease in light intensity in 10-m intervals.
   c) At what depth will the light intensity drop to less than 10% of the light intensity at the surface?

7. The stellar magnitude scale compares the apparent brightness of stars and other celestial objects, such as the Moon. The ratio R of the apparent brightness for two celestial objects is expressed by the relation $R = \dfrac{2.512^{b_1}}{2.512^{b_2}}$,

where b_1 is the apparent brightness value of the fainter object and b_2 is the apparent brightness value of the brighter object.

a) Using a calculator, compare the apparent brightness of the Moon and Venus if the Moon has an apparent brightness value of –12.6 and Venus has an apparent brightness value of –4.7.

b) Using a calculator, compare the apparent brightness of Jupiter and Neptune if Jupiter has an apparent brightness value of –2.8 and Neptune has an apparent brightness value of 7.7.

8. Antonio used the table shown to record the temperature (T) of a bowl of soup over a 20-min period.

Time, t (min)	Temperature of Soup, T (°C)	Ratio
0	48.75	
5	39.00	
10	31.20	
15	24.34	
20	19.96	

a) Complete the table with the ratios between successive temperatures. What is the average of the ratios?

b) Use the average ratio you found in part a) and the temperature values to develop a model for the temperature of the soup over time, in 5-min intervals. Check that the model correctly predicts the data in the table.

c) Use your model to determine the temperature of the soup after 25 min.

9. Sounds create waves that exert pressure on your eardrums. Pressure is measured in pascals (Pa). The pressure level of a sound is measured in units called decibels (dB). The relation between the pressure level, d, and pressure, p, of a sound is approximated by the equation $10^{\frac{d}{20}} = 50\,000\,p$. How does the pressure of a 100 dB sound compare with the pressure of a 140 dB sound?