Name: _____   Date: _____

Section 6.7 Exponential Growth and Decay

1. A fern was found preserved in volcanic ash believed to have been deposited 10 000 years ago. What percent of carbon-14 is expected to remain in the fern?

2. The fossil of an egg was found to contain 6% of its original carbon-14. To the nearest hundred years, estimate the age of the fossil. Use a calculator to check your estimate. How close was your estimate?

3. A quantity of the radioactive isotope polonium-209 will decay to half of its original quantity after approximately 100 years.
   a) Approximately how many years will it take a quantity of polonium-209 to decay to about 3% of its original quantity?
   b) Approximately what percent of the original quantity of polonium-209 will remain after 300 years?
   c) Using a systematic trial, estimate the time needed for a quantity of polonium-209 to decay to 75% of its original quantity.

4. The persistence of metals in the human body is often much greater than that of drugs but follows the same exponential model. Suppose that a certain metal follows the model $M = M_0(0.80)^{\frac{t}{4}}$, where M is the mass of the metal remaining in the body, M_0 is the mass of the initial ingestion of the metal, and t is the time in months since the ingestion occurred.
   a) Suppose an ingestion of 500 μg (micrograms) of this metal is non-damaging. Sketch a graph showing the mass remaining in the body up to 48 months.
   b) Use your graph to estimate the half-life of the metal in the body.
   c) Check your estimate in part b) using the equation.

d) Suppose that an ingestion of 1500 μg of this metal requires medical attention. A drug is administered immediately to decrease the persistence of the metal in the body. The metal now follows the model $M = M_0(0.68)^{\frac{h}{6}}$, where M is the mass of the metal remaining in the body, M_0 is the mass of the initial ingestion of the metal, and h is the time in hours since the ingestion occurred. After approximately how many hours will the metal decay to a safe level of 500 μg?

5. The atmospheric pressure, measured in kilopascals (kPa), is the pressure at a point in the earth's atmosphere due to the weight of air. The atmospheric pressure decreases exponentially with height and is approximated by the model $P = 101(0.87)^{\frac{h}{1100}}$, where P is the atmospheric pressure in kilopascals, and h is the height in metres above sea level. The model is considered accurate to a height of 10 000 m above sea level.
   a) Sketch a graph of the atmospheric pressure P versus height h from 0 m to 10 000 m.
   b) Use your graph to predict the height at which the atmospheric pressure decays to 50 kPa.
   c) What is the atmospheric pressure at a height of 9000 m above sea level?
   d) The cabin in a passenger aircraft must usually be pressurized to a level that is greater than the atmospheric pressure located at a point 2400 m above sea level. What is the lowest allowable cabin pressure for a passenger aircraft?

6. Radon (Rn) is a naturally occurring radioactive element that can accumulate in buildings and present a health hazard. The isotope Rn-222 has a half-life of approximately 3.8 days.
   a) Model the decay curve for 80 μg (micrograms) of Rn-222 using an exponential function.
   b) How much Rn-222 remains after 10 days?
   c) Estimate how long it would take for the amount of Rn-222 to decay to less than 10 μg.

7. The population of Ontario was approximately 12.7 million in 2006.
   a) Use a calculator and assume a growth rate of 1.4% to estimate the population of Ontario in 2016 to the nearest tenth of a million.
   b) The population of Ontario in 2001 was approximately 11.4 million. What growth rate best models Ontario's population growth from 2001 to 2006, to the nearest tenth of a percent?

8. A triangle with an area of 64 cm^2 is drawn and shaded black. A pattern is created by joining the midpoint of each side of the triangle, then joining the midpoint of each side of each upward pointing sub-triangle, and so on, as shown. In each step, the downward pointing triangles are coloured white.
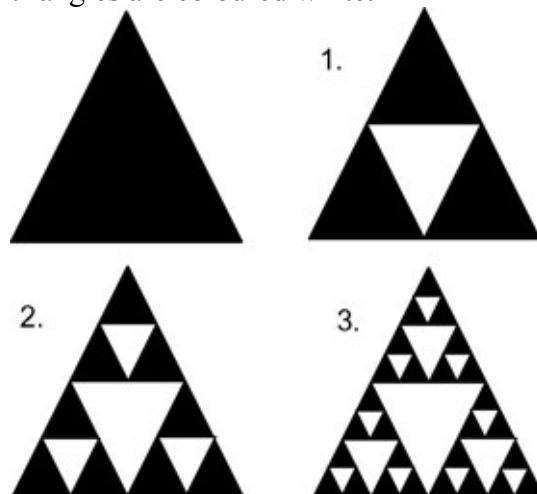
   a) In each step, 25% of the remaining black area is shaded white. Use this information to determine the total area of the black triangles in step 1.
   b) Express the total area of the black triangles A as an exponential function of the step number s.
   c) What is the total area of the black triangles after 4 steps?
   d) After how many steps will the black triangles have a total area of less than 10 cm^2?