1.1 Identify Functions

Student Text Pages
6–14

Suggested Timing
75–110 min

Materials and Technology Tools
- grid paper and rulers
- graphing software (optional)
- computers with spreadsheet software and *The Geometer's Sketchpad*® (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 1–3 Section 1.1 Identify Functions

Common Errors

- Some students may have difficulty with the $\pm$ sign in front of the square root.

R_x Have students solve the equation $y^2 = 4$. Their first response would likely be $y = 2$, and ask if there could be another number whose square is also 4.

Teaching Suggestions

- You may wish to follow this order when teaching: Complete Investigate A, work through Example 1, and then assign questions 1 to 7 to give practice on identifying functions and relations. Work through Investigate B to focus on function notation and then assign the remaining questions.
- Ask students to use a graphing calculator or other graphing software to graph a circle or $x = y^2$. This should be an indication of whether a relation is a function.
- To illustrate the concept of a function, with input/output, make students into function machines. Student A can be *times 2*, student B *squared*, and student C *times 4, then subtract 2*. You can then ask, "What is *A at 2*?" and student A should answer "4," and so on.

Investigate

- For **Investigate A, Part A**, remind students to consider the graph of $y^2 = x$ as they complete the table of values. Some students may be reluctant to write two y-values for one x-value. Ask them to note that the x-values are given as 0, 1, 4, …. and ask why these values are chosen. The key in **step 2** is that the two points lie in the same vertical line, which is an indication of a relation being a non-function.
- For **Part B**, the following is an alternative to **step 1**.
  - Draw the graphs for Relations C and D using a graphing calculator. Use the Draw command to draw a vertical line and move it from left to right across the graphs, noting the number of points of intersection.
  - To graph the circle defined by $x^2 + y^2 = 25$ using a graphing calculator, rearrange the equation to isolate y^2 ($y^2 = 25 - x^2$) then take the square root of both sides to get $y = \pm\sqrt{25 - x^2}$. There are two parts to the result, which must be graphed separately, $Y1$: $y = \sqrt{25 - x^2}$ and $Y2$: $y = -\sqrt{25 - x^2}$. The graphs of the two parts form the top and bottom half of the circle.
- Ask students why the x-values in the tables are chosen. Again, students may need to be reminded that there may be two y-values for each x-value.
- The underlying concept of the vertical line test is that for a relation to be a function, no x-coordinate can map onto more than one y-coordinate. If an x-coordinate is mapped onto more than one y-coordinate, two or more points will lie on a vertical line. The edge of the ruler simulates a vertical line moving from left to right across a graph, and if it intersects the graph in more than one point at a certain x-value, then the graph does not represent a function. This test can also be done by moving a vertical line along a graph using a graphing calculator.
- In the introduction to **Investigate B**, relate $f(2) = 5$ to the point (2, 5), where $x = 2$ is the input, and $y = 5$ is the result.
- For **Part A,** ensure students are able to see that $(2, f(2))$ or $(x, f(x))$ is an ordered pair like (x, y) that they are familiar with.
- If students cannot recognize the pattern in **step 3**, ask them to plot the points (0, 3), (1, 5), and (2, 7). Once they see the linear relation, they should recognize the $2x + 3$ pattern.
- At the end of **step 5**, ask students to write the output $f(m)$, $f(b)$, and $f(a + b)$. Tell them that writing $f(x)$ as $f(\ ) = 2(\ ) + 3$ might help.

- For **Part B,** encourage students to recognize that the area function is not something different from what they have learned in the past. Now $A = \pi r^2$ is written in function notation $A(r) = \pi r^2$.

Investigate Responses (pages 6-10)

Investigate A, Part A

1. Relation A: $y = x^2$ Relation B: $y^2 = x$

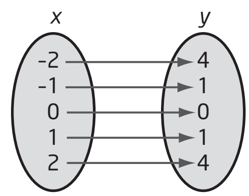
x	y
−2	4
−1	1
0	0
1	1
2	4

x	y
0	0
1	1 or −1
4	2 or −2
9	3 or −3
16	4 or −4

2. The points (4, 2) and (4, −2) are on the line $x = 4$ and on the opposite sides of the x-axis.

3. Answers may vary. For example:

Relation A



4. In the mapping diagram for Relation A, there is only one arrow starting from each x-value. This result is different from that of Relation B where there is more than one arrow starting from some x-values.

5. Answers may vary. For example:

	Relation A	Relation B
Equation	The power of y is 1 while the power of x is 2.	The power of y is 2 while the power of x is 1.
Graph	There is only one point on the curve for each x-value.	There can be two points on the curve for some x-values.
Table of values	Each x-value gives only one y-value.	An x-value can give more than one y-value.
Mapping diagram	Only one arrow starting from each x-value.	Some x-values have two arrows starting from them.

6. Answers may vary. For example, for a relation to be a function, each x-value is mapped onto only one y-value.

Part B

1. Answers may vary. For example: The edge of the ruler intersects the graph for Relation C at no more than one point. The edge of the ruler intersects the graph for Relation D at more than one point.

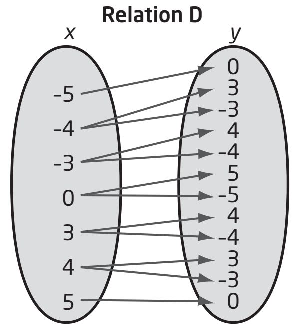
2. Relation C: $x + y = 5$ Relation D: $x^2 + y^2 = 25$

x	y
−3	8
−2	7
−1	6
0	5
1	4
2	3
3	2

x	y
−5	0
−4	3 or −3
−3	4 or −4
0	5 or −5
3	4 or −4
4	3 or −3
5	0

3. When $x = 0$, $y = 5$ or −5. The y-intercepts are 5 and −5.

4. I think there will be two arrows starting from $x = 0$.



5. There is only one arrow starting from each x-value in Relation C, and there are two arrows starting from each x-value in Relation D.

Part C

Answers may vary. For example:

1.

	Functions		Non-Functions	
	Relation A	**Relation C**	**Relation B**	**Relation D**
Equation	The power of y is 1 while the power of x is 2.	The power of y is 1 while the power of x is 1.	The power of y is 2 while the power of x is 1.	The power of y is 2 while the power of x is 2.
Graph	non-linear	linear	non-linear	non-linear
Table of values	Each x-value gives only one y-value.	Each x-value gives only one y-value.	An x-value can give more than one y-value.	An x-value can give more than one y-value.
Mapping diagram	Only one arrow starting from each x-value.	Only one arrow starting from each x-value.	Some x-values have two arrows starting from them.	Each x-value has two arrows starting from it.
Vertical line test	A vertical line intersects the graph at only one point.	A vertical line intersects the graph at only one point.	A vertical line intersects the graph at more than one point.	A vertical line intersects the graph at more than one point.

2. A function has each x-value mapped onto exactly one y-value, whereas a non-function can have an x-value mapped onto more than one y-value. A vertical line intersects the graph of a function at no more than one point, whereas a vertical line can intersect the graph of a non-function at more than one point.
3. The edge of a ruler can be used as a vertical line to test if the graph of a relation is a function. The edge of the ruler, when moved across the graph of a function, intersects the graph at no more than one point.

Investigate B, Part A

2. $f(0) = 3$; $f(1) = 5$; $f(2) = 7$
3. $f(3) = 9$; $f(4) = 11$; $f(5) = 13$; $f(6) = 15$; $f(7) = 17$; $f(8) = 19$; $f(9) = 21$; $f(10) = 23$
4. Answers may vary. For example, multiply each input by 2 and add 3.
5. **a)** $f(x) = 2x + 3$ **b)** $f(n) = 2n + 3$

Part B

1. $A(1) = \pi(1)^2 = \pi$
2. $A(3) = \pi(3)^2 = 9\pi$
3. Answers may vary. For example, the number 3 is squared and multiplied by π to find the area.
4. The area of the circle with radius b is: $A(b) = \pi(b)^2 = \pi b^2$
5. The area of the circle with radius r is: $A(r) = \pi r^2$

Example

- The Example is fairly straight forward. Even though it already says it in teacher talk, remind students that two x-values can map onto the same y-value. In a mapping diagram, two arrows can go to the same y-value, but two arrows cannot come from the same x-value. On a graph, the points can lie on the same horizontal line but not the same vertical line.

Communicate Your Understanding

- In **question C2**, you may use *The Geometer's Sketchpad*® or other graphing software to draw a line (or ray), rotate it, and ask students to tell you when the line is not a function.
- In **question C3**, before answering this question, open a spreadsheet and ask students to write a formula for the area of a circle, or some other formula that they are familiar with. Maybe ask students to make themselves into a function machine used in the Investigate. The cell reference, A2, takes the place of the variable. This is an excellent way to allow students to see something like the function machine—A2 is the input and B2 is the output.
- You may wish to use **BLM 1–3 Section 1.1 Identify Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 12)

C1 If a vertical line intersects the graph of a relation at more than one point, that means one x-value is mapped onto more than one y-value. Then, the relation is not a function.

C2 The statement is false. The vertical line with the equation $x = 2$ is not a function. For each x-value, $x = 2$, there is an infinite number of y-values.

C3 The formula in cell B2, area = $3.14 \times (\text{radius})^2$, is a function. For each radius (x-value), there is only one corresponding area (y-value).

Practise, Connect and Apply, Extend

- In **question 1**, some students may not recognize that $x = 5$ and $x = 10$ both appear twice in part b). In tables of values, non-functions may have two y-values beside the same x-value, or the same x-value may appear twice for different y-values.
- For **question 5**, encourage students to plot the points. In part c), students may think that $y = 1$ is a non-function. Again, ask them to plot the points to find out.
- **Question 7** can be approached in several ways: by rearranging the equation into the form $y = mx + b$ or by plotting points to show that the graph is a straight line that is not vertical. Note that many non-functions have an exponent on the y-term.
- **Questions 8 and 9** belong to the type of questions addressed in the Investigate, but not in the Example. Work with students a few examples before they answer these two questions. For example, given $f(x) = 2x - 9$, ask students to find $f(2)$ and $f(0)$, and find b when $f(b) = 21$. This is also a good spot for an extension. Ask students to find expressions for $f(m)$, $f(\pi)$, $f(a + b)$, etc.
- In **question 10**, encourage the use of graphing calculators, if available. Students may need a prompt for the meaning of *instant it is thrown* and that when the ball lands, $h(t) = 0$.
- In **question 14**, students should begin to recognize that any equation with an exponent on the y-term are possible non-functions, In particular, equations of even degree in y (y_2, y_4, ...) are assured of being non-functions.
- In **question 16**, students may be confused about which function to substitute into first. For part a), ask, "What is the input for the function (machine) g?" The response should be $f(2)$, which is the result when $x = 2$ is the input of function (machine) f.
- **Question 17** is exactly the same question as finding the equation of the line through the points (0, 3) and (1, 5), except that the points are given using function notation.

Common Errors

- Some students may think that repeated y-coordinates of points in a list (points lying on the same horizontal line) indicate that the points represent a non-function.

R_x Have students plot the points to see that the points lie on a horizontal line, and use the vertical line test to confirm that the points represent a function.

- Some students may not realize that function notation, like $f(x) = x^2$, shows what happens to a value substituted into the function.

R_x Encourage the use of spreadsheets, images of function machines, different variables and/or different brackets: $f(x) = x^2$ can be written $f(a) = a^2$, $f(\ ) = (\ )^2$, $f[\ ] = [\ ]^2$. Maybe mention that x is somewhat a place-holder—the place where the value goes.

Ongoing Assessment

- While students are working, circulate to observe how well each works. This is an opportunity to observe and record individual student's learning skills.

Accommodations

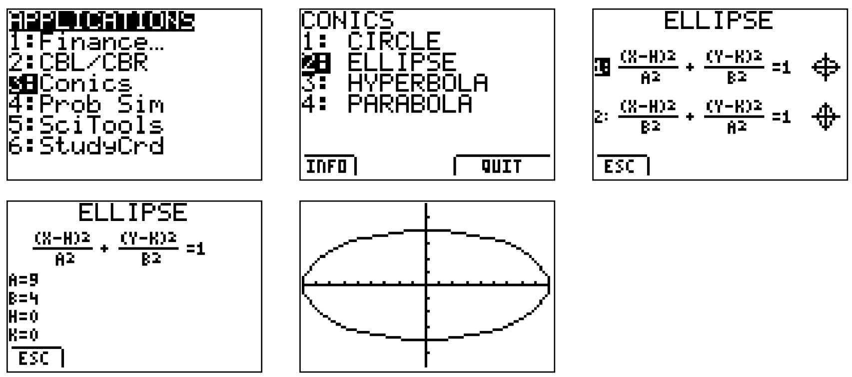
Spatial—give students a handout of the graphs and charts from **Investigate A**

Motor—encourage students to use technology for graphing

Language—encourage students to work in pairs for reading

- In **question 18**, students should be able to see that the point (2, 8) is on the graph. Substitute this point into the equation to find a. The value of a can also be found using trial and error or by recognizing the graph as a vertical stretch of factor 2.
- In **question 19**, part a), ask students what other type of equation looks similar to this one. Suggest finding the x- and y-intercepts. In part b), ask what happens when you try to isolate y and why this poses a problem when using a graphing calculator. See the following notes on using the Conics application on a TI-84 graphing calculator.
  - Be sure to download the application into your calculator first.
  - To plot ellipses such as $4x^2 + 9y^2 = 36$, students should be asked to Zoom-Square (Zoom-5) so that the shape looks more accurate. In this case, the stretch in the x-direction is bigger than the stretch in the y-direction.
  - Students should be asked to isolate y for data entry into the calculator or use the Conics application as described below:
  - Press APPS and cursor down to **Conics**, then press ENTER.
  - Select **2: ELLIPSE**, then press ENTER for the first equation.
  - Enter 3 for A, 2 for B, and leave H and K as 0.

  Here are some screenshots:



Literacy Connections

- Encourage students to practise reading functions written in function notation as they work through this chapter.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12, 13, 15, 17–19
Reasoning and Proving	2, 4–7, 10, 12, 14, 15, 19
Reflecting	n/a
Selecting Tools and Computational Strategies	1, 3, 5, 8–11, 15, 16, 18
Connecting	10, 12, 15
Representing	6, 12–15, 17
Communicating	2, 4, 6, 10, 12, 14, 15, 19