1.2 Domain and Range

Student Text Pages
15–22

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)
- graphing software (optional)

Related Resources
- BLM G-1 Grid Paper
- BLM 1-4 Section 1.2 Domain and Range

Teaching Suggestions

- Continue to build on concepts and terminology of functions in this section.
- Point out to the students that Investigates A and B are different investigations. In Investigate A, domain and range are determined by equations and/or graphs. In Investigate B, domain and range are determined by the type of data (continuous versus discrete), which can be affected by real-life factors.
- Work through Investigates A and B followed by a discussion of the table on page 17 before going through the Examples. An alternative would be to work through Investigate A followed by Example 2, and then work through Investigate B followed by a discussion on the table before working through Example 1.
- It is important that students recognize and use the various ways of writing domain and range. However, begin by having students describe domain and range in words, which better reveals their understanding of the concept.

Investigate

- For **Investigate A**, you may wish to draw a table and complete the first row with the students.
- Ensure students realize that *Length in terms of Width* means that the equation is in the form $l = \blacksquare$.
- Even though **steps 6, 7, and 11** do not ask for it, suggest that students write the answers as inequalities. That is, ask students how they might write a length that is between 0 and 40.
- In **step 10**, students may choose to plot points on grid paper. Suggest that they express area in terms of width (area = length × width) and use the expression of length in terms of width in **step 3**.
- For **Investigate B**, try asking students to sort the items without first giving them the titles *Count* and *Measure* to see if they will come up with the two categories of quantities on their own.
- The key to **steps 4 to 7** is to recognize the difference between continuous and discrete data. This affects how the graphs will look (line/curve versus points) and the way the domain and range are expressed, which is the purpose of this investigation.

Investigate Responses (pages 15-16)

Investigate A

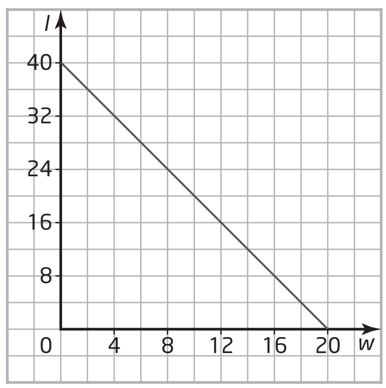
Answers to steps 1, 2, 4, 5, and 8 to 10 may vary. For example:

1.

Width (m)	Length (m)
1	38
2	36
3	34
4	32
5	30
6	28
7	26
8	24
9	22
10	20
11	18
12	16
13	14

2. To find length, subtract 2 times the width from 40 m.

3. Let w represent the width and l represent the length, both in metres. $l = 40 - 2w$

4. 

The relation is a linear function because the graph is a straight line with one x-value mapping onto only one y-value.

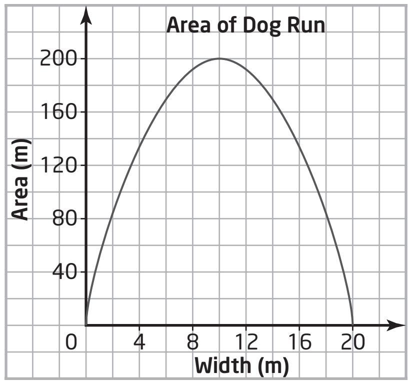
5. The width cannot be a negative value because a negative measurement is inadmissible. The width cannot be over 25 m because it will give a negative length. The width can be a decimal as lengths and widths can have decimal values.

6. The width can be any value between 0 m and 20 m.

7. The length can be any value between 0 m and 40 m.

8., 9.

Width (m)	Length (m)	Area (m^2)
1	38	38
2	36	72
3	34	102
4	32	128
5	30	150
6	28	168
7	26	182
8	24	196
9	22	198
10	20	200
11	18	198
12	16	196
13	14	182

10. 

The relation is a function because when the edge of a vertical ruler is moved across the graph, it intersects the graph at no more than one point.

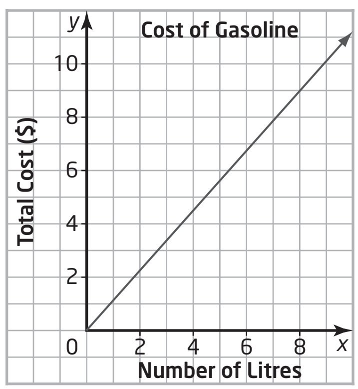
11. The area can be any value between 0 m^2 and 200 m^2.

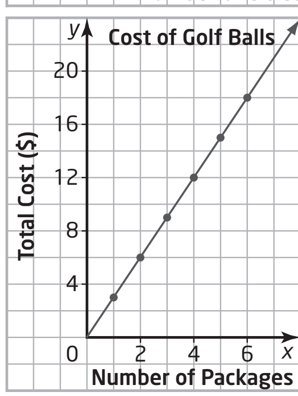
Investigate B

Answers to steps 2 to 7 may vary. For example:

1.

Count	Measure
books on a shelf	angles in a triangle
candies in a jar	arm span
CDs	distance travelled
golf balls	gasoline
water bottles	mass of a block of cheese

2. The price per kilogram is neither counted nor measured as it is not a single quantity but a ratio of two different quantities.
3. I would add "students in a class" to the *Count* column and "area of a classroom" to the *Measure* column. I cannot think of an item that can go in either column.
4. 

5. 

6. Similar: The vertical axes are both cost in dollars. The cost increases as the number along the horizontal axis increases. Different: The graph in step 4 is a continuous line whereas the graph in step 5 contains discrete points along a straight line.

7. For quantities that can be counted such as golf balls, the graph of Cost versus Number of Items are discrete points. For quantities that can be measured such as the volume of gasoline, the graph of Cost versus Volume is a continuous line.

Examples

- **Example 1** focuses on the various ways in which a function might be represented: a set of points, a graph, and an equation. Encourage students to consider the table on student text page 17 as they work through this example. Suggest to students that they should describe domain and range in words before writing them in other notations. Consider adding another point (6, 5) to part a) so that 5 appears twice as a y-value. Ask, "Should the range now be {8, 7, 6, 5, 5}? Does it matter if the values are listed from least to greatest?"
- In **Example 2**, you may mention the difference between graphing Height versus Time, and sketching the approximate path of the diver. In this situation, the two diagrams would be similar.
- As suggested in the teacher talk, it may be worth discussing whether the depth to which the diver plunges below the water surface should be considered. This would affect both the domain and the range.
- Ask students why $t \geq 0$. How would the graph look if it were extended to the left of the vertical axis? Explain that negative values for t are possible in some contexts. For example, −2 means 2 s before the instant the diver jumped.

Communicate Your Understanding

- **Question C1** is similar to Example 2. Encourage students to sketch a graph of Height versus Time before answering.
- **Question C2** is based on Investigate B. Ask students how the graphs would compare. Ask how the two situations might be approached differently when using graphing calculators: graph the equation $y = x$ to represent the cost of gas and draw a scatter plot to represent the cost of newspapers.
- In **question C3**, most students would think of a line in the form $y = mx + b$ as opposed to vertical and horizontal lines, which are the two exceptions to this statement. When addressing this, use *The Geometer's Sketchpad*® to draw a line through two points. Then drag one point to create horizontal and vertical lines.
- You may wish to use **BLM 1–4 Section 1.2 Domain and Range** for remediation or extra practice.

Communicate Your Understanding Responses (page 20)

C1 Answer **c)**, $0 \leq h \leq 100$, best describes the range.
Answers may vary. For example:
a) Real numbers include negative numbers. The ball's height cannot be a negative value.
b) The value of h cannot be more than 100, as the ball is dropped from a height of 100 m.
d) The range is the set of all values for the dependent variable. t is the independent variable.

C2 The domain for calculating the revenue from the sales of gas is the set of all real numbers between 30 and 60 inclusive. The domain for calculating the revenue from the sales of newspaper is the set all integers between 30 and 60 inclusive.

C3 The statement is false. The domain of the vertical line $x = 2$ is {2}, and the range of the horizontal line $y = 1$ is {1}.

Practise, Connect and Apply, Extend

- For **questions 1 and 2**, note that the domain and range in part a) are finite lists and the range has only 2 elements, 0 and 1. For parts c) and d), suggest sketching the graphs first.
- For **question 3**, encourage students to sketch the graph. Ensure students include the end points in both domain and range.
- **Question 4** is similar to Investigate A. Students should recognize that even though tables of values show a finite number of points, the graph may be continuous, depending on the context. Part e) would make an excellent class discussion question.
- For **question 5**, the ball is in the air for 4 s and reaches its maximum height after 2 s. Ask students how they can tell, from their table of values, that the maximum height occurs when $t = 2$ (symmetric property of a parabola). This is a good spot to have students verify their answers with a graphing calculator.
- For **question 6**, ask students to describe the relationship between Cost and Number of Kilometres. You may initiate a short discussion by asking: "Can you write the relation as an equation?" The range of the function can be addressed in different ways: any integer greater than 1 (greater than or equal to 2); {2, 3, 4, …}; $\{y \in \mathbf{I} \mid y \geq 2\}$.
- For **question 7**, suggest that students use a square sheet of paper and have them cut out increasingly larger squares.
- For **question 8**, suggest that students list ticket prices for decreasing values of x. Ask, "What is the price for $x = 3, 2, 1, 0…$?" This should help students see that a negative value of x has meaning in this context. Encourage students to sketch a graph. Ask, "From the equation, is it possible to have a negative ticket price? a negative number of people? How do these impact the domain?"
- For **question 9**, students can use their knowledge of transformations to answer the question. The domain is still the set of real numbers. The range for $y = x^2$ is any number greater than or equal to 0. This function has been stretched vertically (with no impact on the range) and translated 9 units up.
- **Question 10** is a real-life situation, so the restriction is related to practical issues such as costs and possible dimensions of carpets.
- For **question 11**, consider making a table of values for $x = 0, 1, 2, 3, 4$. Use a graphing calculator to graph the function. Then, use TRACE to identify the range. Ask, "Why is substituting $x = 0$ and $x = 4$ into the function sufficient to identify the range?"
- For **question 12**, students must first realize that 1 does not necessarily map onto 1, 2 onto 2, and so on. Note that it is possible for two values of the domain to map onto the same value of the range. However, since there are an equal number of elements in the domain and range, it must be a 1 to 1 mapping. To find how many possibilities, look for a pattern. Beginning with 1 from the domain, it might map onto any one of the five elements of the range. This leaves four possible choices for 2, three choices for 3, and so on. This gives a total of $5 \times 4 \times 3 \times 2 \times 1$ or 120 different ways for the five numbers in the domain to map onto the five numbers in the range.
- For **question 13**, a good question to ask is: "If two numbers have a product of 1, what is the restriction on either of the numbers?"

Ongoing Assessment

- You may wish to collect students' responses to the Communicate Your Understanding questions to use as a formative assessment tool.

Accommodations

Visual–provide students with a handout of the table on student text page 17.

Motor– encourage students to use technology for graphing

Literacy Connections

- Some students may have difficulty with the set notation and the inequality expression. You may use a Venn diagram to represent the set of elements and circle out the elements described by the inequality to help students understand the meaning of the symbols ∈ (is an element of) and | (such that).
- Use the domain and range written in set notation in the table for more practice in reading.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 8, 10–12
Reasoning and Proving	4, 6–12, 14
Reflecting	n/a
Selecting Tools and Computational Strategies	1, 2, 5, 6, 8, 14
Connecting	4–8, 10
Representing	3–7, 9, 12–14
Communicating	3, 4, 6–14