1.3 Analyse Quadratic Functions

Student Text Pages
23–30

Suggested Timing
120–150 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computer with spreadsheet software (optional)
- paper and scissors (optional)
- linking cubes (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM G–5 Second Differences Tables
- BLM 1–5 Section 1.3 Analyse Quadratic Functions

Teaching Suggestions

- Before beginning this section, suggest that students look at the photographs in the introduction. Ask students if they can think of where they have seen this shape in their neighbourhood.
- Throughout this section, focus on first and second differences and how they are used to identify quadratic functions.
- You may wish to introduce other examples that are modelled using quadratics: area of a rectangle, with a fixed perimeter, in terms of its length, area of a circle in terms of its radius, and revenue in terms of ticket price in situations similar to the chapter problem.
- At the end of the Investigates, revisit Sections 1.1 and 1.2 to see if students can identify other quadratic functions. Refer students to the methods and techniques used in those Investigates to help them decide whether or not a function is quadratic.
- Encourage students to work on more questions involving the use of the QuadReg operation on the graphing calculator. The dog run problem in Section 1.2, Investigate A would be suitable.

Investigate

- For **Investigate A**, initiate a discussion on the domain and range in the context of the problem. Ask, "Is it possible to have a negative value for x? Can a profit be negative?"
- The intention of **steps 4 to 6** is to let students recognize that the graph for the profit function is symmetrical.
- In **steps 7 and 8**, students should tell that the first differences form a linear pattern. If necessary, calculate the second differences, which are constant. Students must grasp this concept as it is a key to identifying quadratic relations.
- An alternative to Investigate A, using pencil and paper, is:
  1. Create a table of values showing the profit, P, in terms of the advertising budget, x. As you do so, consider what values of x you will use.
  2. Use paper and pencil to sketch a graph of Profit versus Advertising Budget.
  3. What amount of advertising appears to produce the maximum profit? What is the maximum profit?
  4. Write the domain and range of this function. Be sure to consider the context.
  5. Circle the maximum value in your table of values. What would be the profit for $1000 more in advertising? for $1000 less? How do these values compare to each other?
  6. Repeat step 5 for $2000 more and then $2000 less. What do you notice about the profit?
  7. What property about this graph is suggested by your response to steps 5 and 6? List other properties of the graph.
  8. Add a column to the table of values and calculate the first differences.
  9. What do you notice in the first differences? What is their significance?
- As an introduction to **Investigate B**, have two students toss a ball back and forth in front of the class. Ask students to watch and describe the path of the ball.

- For **steps 1 and 2**, have students refer to the definitions of first and second differences which are given in Investigate A.
- Before students use the QuadReg operation on the graphing calculator, remind students of their linear regression work in grade 9 and/or grade 10. Perhaps, try one question with a graphing calculator to refresh students' memory before they attempt the investigation.
- After finding the values of a, b, and c in $y = ax^2 + bx + c$, consider having students change some of the values in the table slightly to note the effect on the values of a, b, and c.

Investigate Responses (pages 23-25)

Investigate A

2. An amount of advertising of $5000 will produce the maximum profit. The maximum profit is $100 000.

3. domain: $\{x \in \mathbf{R} \mid 0 \le x \le 12\}$; range: $\{P(x) \in \mathbf{R} \mid 0 \le P(x) \le 100\}$

4. The profit will be $98 000 for $1000 more in advertising. The profit will also be $98 000 for $1000 less in advertising. These values are the same.

5. The profit will be $92 000 for $1000 more in advertising. The profit will also be $92 000 for $1000 less in advertising. The profit is the same in both cases.

6. Answers may vary. For example, the graph is symmetrical about the maximum value when $x = 5$.

7.

x	$P(x)$	First Differences	Second Differences
0	50	18	
1	68	14	−4
2	82	10	−4
3	92	6	−4
4	98	2	−4
5	100	−2	−4
6	98	−6	−4
7	92	−10	−4
8	82	−14	−4
9	68	−18	−4
10	50	−22	−4
11	28	−26	−4
12	2		

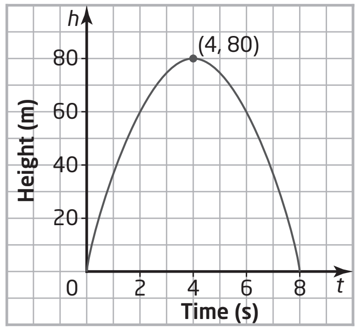
8. Answers may vary. For example, the first differences form a linear pattern and the second differences are constant. That means the function $P(x) = -2x^2 + 20x + 50$ is quadratic.

Investigate B

1.

Time (s)	Height (m)	First Differences	Second Differences
0	0	35	
1	35	25	−10
2	60	15	−10
3	75	5	−10
4	80	−5	−10
5	75	−15	−10
6	60	−25	−10
7	35	−35	−10
8	0		

2. Answers may vary. For example, the first differences form a linear pattern and the second differences are constant.
3. Diagrams may vary. For example:



domain: $\{t \in \mathbf{R} \mid 0 \leq t \leq 8\}$; range: $\{h \in \mathbf{R} \mid 0 \leq h \leq 80\}$

4. When the parabola opens downward, there is a maximum value. When the parabola opens upward, there is a minimum value.
6. $y = -5x^2 + 40x$. Substitute $x = 0$ into the equation: $y = -5(0)^2 + 40(0) = 0$. (0, 0) is on the graph. Substitute $x = 2$ into the equation: $y = -5(2)^2 + 40(2) = 60$. (2, 60) is on the graph. The results make sense as they match the data in the table and the corresponding points on the graph.
7. A continuous curve in the shape of a parabola that opens downward appears. It seems to pass through all the points.
8. Answers may vary. For example, this process helps to find the quadratic equation that best models a set of data that appears to be quadratic. The QuadReg operation is a tool that determines if the data should be modelled by a quadratic function.

Examples

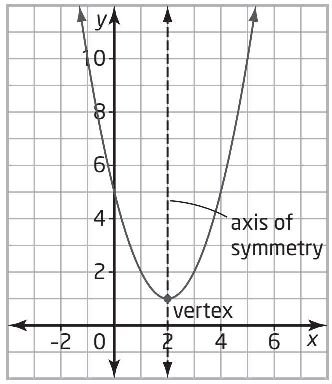
- Consider working with students on the area calculations for one or two rows in the table of values in **Example 1**. Point out that the teacher talk has a formula that relates length and width.
- Discuss with students whether it is possible to have a length or width of zero. Ask why zero might be included in the table of values.
- Use the terms *domain* and *range*, where appropriate to reinforce this concept learned in Section 1.2.
- Ask students whether they are certain that the maximum occurs when $x = 30$ and how they found this answer. They should use the symmetric property of parabolas in their response.
- **Example 2** addresses the key concept of the second differences of quadratic relations being constant.
- Try some more relations with students in which the difference between x-values is not 1, the y-values are decimals, or the relation is not quadratic but appears very close to being quadratic and can be modelled by a curve of best fit.

Communicate Your Understanding

- For **question C1**, encourage students to refer to the vocabulary introduced in this section.
- For **question C2**, students should find it easier to see the second differences that are constant rather than the first differences that form a linear pattern.
- For **question C3**, encourage students to refer to the real-life situations used throughout the section. They should notice that each situation involves a maximum, or potentially a minimum, value.
- You may wish to use **BLM 1–5 Section 1.3 Analyse Quadratic Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 28)

C1



A parabola is a symmetrical U-shaped curve. When the parabola opens upward, there is a minimum value. When the parabola opens downward, there is a maximum value. The maximum or minimum point is called the vertex. The axis of symmetry is a vertical line that divides the parabola into two congruent halves.

C2 If a function is quadratic, the first differences between consecutive *y*-values in a table with evenly spaced *x*-values shows a linear pattern and the second differences are constant.

C3 The vertex of a parabola, which indicates a maximum or minimum value, can be used to solve problems such as the maximum profit or minimum area in real-life situations.

Practise, Connect and Apply, Extend

- For **question 1**, encourage students to identify and recognize the features of quadratic equations. Alternatively, use the equation to produce a table of values, and then calculate the first differences.
- For **questions 3 and 4**, ask students how the axis of symmetry, the vertex, and the direction of opening are related to a quadratic equation.
- For **question 5,** part e), encourage students to give more than one reason, such as the linear pattern in the first differences and the side lengths of 6 different size large cubes that produce perfect square numbers of small cubes with one face painted. Some students may benefit from the use of linking cubes.
- For **question 6**, note the reference to domain in part a). In part b), encourage students to find an expression for the total area in terms of *x*; students may need some scaffolding. Provide paper and scissors for those students who might benefit from building the box. The table of values and the first and second differences can be done easily using spreadsheet software. This question is addressed again in **question 11**. Consider doing the two questions together.
- **Question 7** is a good discussion question. Ask students to look for ways of counting the number of games required. Find out if they can explain using this logic: for 5 teams, each would play 4 games. Since each game involves 2 teams, there would be 10 games required. Or, the first team plays 4 others, the second team has 3 others to play, and so on. This will result in $4 + 3 + 2 + 1$ games. Encourage students to discover how the values in the first column of the table are related to the values in the second column. For 5 teams: $10 = \frac{5 \times 4}{2}$; for 6 teams: $15 = \frac{6 \times 5}{2}$, and so on. This will lead to the relation: number of games $= \frac{n(n-1)}{2}$, or $\frac{n^2 - n}{2}$. For part d), students may need to be reminded how to do a regression.
- For **question 9**, it may be helpful to ask students to write the dimensions of the square on each face: $n - 2$ by $n - 2$.

Accommodations

Motor–provide students with copies of **BLM G-5 Second Differences Tables**; use technology for graphing

Language–simplify instructions and provide additional scaffolding for problems in Connect and Apply.

Memory–use index cards with graphing calculator key stroke sequences; provide a graphic organizer for key terms

Student Success

- Throughout this chapter, the use of technology for graphing is strongly recommended in many places for speed and accuracy. Ensure that students are familiar with the calculator sequences required to produce an appropriate graph when needed.
- Encourage students to write on index cards or in their notebooks the calculator sequences that they find useful but are difficult to remember.

- **Question 10** is virtually identical to the situation in **question 7** if students start finding the number of line segments required to connect 3 points, 4 points, and so on.

Literacy Connections

- There is no definite answer to the question. The equation of a quadratic function has the variable raised to the power 2, or squared. Since a square has four sides, the word *quadratic* is used probably for that reason. Students may suggest variable answers. As long as they relate quadratic to the square of the variable, they are on track.

Career Connections

- Draw students' attention to the Career Connection on student text page 30. Students who are interested in the video game developer career described might also be interested in the following related careers:
  - game producer
  - game designer
  - game artist
  - game programmer
  - game tester
- Ask students to choose one of the related careers that interest them most. Research on the qualification and skills required for the job, such as university/college education or training.
- You may also ask students to list problems that they might have to solve or difficulties they expect to overcome for the job.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–11
Reasoning and Proving	3, 5–8, 10, 11
Reflecting	4
Selecting Tools and Computational Strategies	1–3, 5–7, 10, 11
Connecting	3, 4, 6–8
Representing	3–5, 7–9, 11
Communicating	3, 5–8, 10, 11