1.4 Stretches of Functions

Student Text Pages
31–39

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- Computer-Based Ranger (CBR) (optional)
- tennis balls (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 1–6 Section 1.4 Stretches of Functions
- BLM 1–7 Section 1.4 Achievement Check Rubric
- BLM 1–8 *GSP* for Section 1.4 Investigate

Teaching Suggestions

- Discuss as a class answers to these questions: "Why would the path of the high jumper be different on the moon? Will the high jumper's path on the moon simply be taller than that on Earth? Will it have the same landing point?"
- An alternative approach is to show the position-time graphs of two people who start walking from the same point at the same time at different speeds. Or, ask students how the position-time graphs of two cars would compare if they left the same starting point with different accelerations. A CBR could be used to explore these two contexts.
- If time permits and your class has access to graphing calculators and a computer with *Fathom*™ work through both methods.
- In the Investigation, Method 1 is preferable for examining how the y-values compare for a given value of x. Method 2 is more dynamic, with the graph changing instantaneously as the slider is moved.
- Sections 1.4 to 1.6 deal with graphing transformations. Students may also use *The Geometer's Sketchpad*® (including the use of sliders), *TI-Interactive*, *Winplot*, or other graphing software for the Investigates.

Investigate

- For **Method 1**, **steps 1 to 4** address different vertical stretches on the graph of $y = x^2$.
- When completing in the table in **step 3**, consider using the TRACE function. For each given value of x, use ► to move from the graph of $y = x^2$, to the graph of $y = 0.5x^2$ and to the graph of $y = 3x^2$. Each time, note and record the y-coordinates.
- **Steps 5 to 8** address reflections of the graph of $y = x^2$ in the x-axis. Again, consider using TRACE to compare the y-values for each given value of x.
- As an alternative to graphing three graphs on the same set of axes, using the Transfrm application (in Section 1.5) to graph $y = \text{A}x^2$. Change the value of A and see what effect it has on the graph of $y = x^2$.
- When a parabola is stretched vertically, the y-values are increased while the x-values remain the same. Tell students that describing the graph as narrower or wider may not be appropriate. **Question 14** in the Extend section will examine this in more detail.
- In **Method 2**, the use of the slider in *Fathom*™ compares the changes made to the graph of $y = x^2$ when $a > 1$, $a < 0$, and $0 < a < 1$. See **BLM 1–8 GSP for Section 1.4 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™.
- Ask students what happens when $a = 0$. Why does this make sense?
- Consider adding the vertical line $x = 1$ to the graph. Label its intersection point with the graph of $y = x^2$. Note the change on this y-coordinate as the value of a changes. How is the change related to the value of a?
- If technology is not available, an alternative would be to create a table of values for $y = x^2$, $y = 0.5x^2$ and $y = 3x^2$. Once complete, compare the y-values for the corresponding values of x.

Investigate Responses (pages 31-33)

Method 1

2. b) Answers may vary. For example, compared to the graph of $y = x^2$, the graphs of $y = 3x^2$ and $y = 0.5x^2$ are similar in shape. However, the parabola in the graph of $y = 3x^2$ appears to be stretched vertically and the parabola in the graph of $y = 0.5x^2$ appears to be compressed vertically.

3.

x	$y = x^2$	$y = 3x^2$	$y = 0.5x^2$
−3	9	27	4.5
−2	4	12	2
−1	1	3	0.5
0	0	0	0
1	1	3	0.5
2	4	12	2
3	9	27	4.5

4. Each of the y-values for $y = 3x^2$ is 3 times the corresponding y-value for $y = x^2$. Each of the y-values for $y = 0.5x^2$ is 0.5 times the corresponding y-value for $y = x^2$.

6. Answers may vary. For example:

b) The graph of $y = -x^2$ is a parabola in the same shape as the graph of $y = x^2$ but the parabola appears to be reflected in the x-axis as it opens downward instead of upward.

c) The graph of $y = -2x^2$ is a parabola that opens in the same direction (downward) as the graph of $y = -x^2$ but the parabola appears to be stretched vertically.

7.

x	$y = x^2$	$y = -x^2$	$y = -2x^2$
−3	9	−9	−18
−2	4	−4	−8
−1	1	−1	−2
0	0	0	0
1	1	−1	−2
2	4	−4	−8
3	9	−9	−18

8. Each of the y-values for $y = -x^2$ is the negative of the corresponding y-value for $y = x^2$. Each of the y-values for $y = -2x^2$ is 2 times the negative of the corresponding y-value for $y = x^2$.

9. Answers may vary. For example, for a quadratic function of the form $y = ax^2$, the value of a determines whether the parabola represents a stretch, a compression, or a reflection in the x-axis. When $a > 1$, the parabola appears to be stretched vertically. When $0 < a < 1$, the parabola appears to be compressed vertically. When $a < 1$, the parabola appears to be reflected in the x-axis.

Method 2

4. Compared to the blue parabola representing $y = x^2$, the red parabola appears to be narrower when $a > 1$.

6. Compared to the blue parabola representing $y = x^2$, the red parabola appears to be wider when $0 < a < 1$.

8. As a becomes negative (when the slider goes past 0), the red parabola starts to open downward.

10. Answers may vary. For example, for a quadratic function of the form $y = ax^2$, the value of a determines whether the parabola represents a stretch, a compression, or a reflection in the x-axis. When $a > 1$, the parabola appears to be stretched vertically (narrower). When $0 < a < 1$, the parabola appears to be compressed vertically (wider). When $a < 1$, the parabola appears to be reflected in the x-axis (open downward).

Examples

- **Example 1** suggests that a vertical stretch of 2 will double the distance from the x-axis, that is, the y-coordinate. Mention to student that this distance can actually be thought of as the vertical distance relative to the vertex, which is a key concept for stretches done in conjunction with translations.
- In the solution, note the reference to the key points (1, 1), (2, 4), and so on. An alternate approach is to note how to get from the vertex to successive points. For example, from the vertex, go 1 right and 1 up to (1, 1); then 1 right and 3 up to (2, 4); and 1 right and 5 up to (3, 9). This approach is revisited in Sections 1.5 and 1.6.
- For part b), note the different intermediate steps for the two methods that result in the graph of $y = -3x^2$: a stretch by a factor of 3 to give $y = 3x^2$ followed by a reflection in the x-axis to give $y = -3x^2$; a reflection in the x-axis to give $y = -x^2$ followed by a stretch by a factor of 3 to give $y = -3x^2$. In fact, you can simply multiply the y-coordinates of the graph of $y = x^2$ to get the points for the graph of $y = -3x^2$.
- The use of both methods in **Example 2** is strongly recommended. In similar questions, encourage students to compare the given point, in this case (3, 12), to the corresponding point on the graph of $y = x^2$, which would be (3, 9). Students should be able to approximate the value of a in $y = ax^2$ as a value between 1 and 2.

Communicate Your Understanding

- For **question C1**, encourage students to sketch the graph of $y = x^2$ and plot the point (4, 4) before answering the question. Ask students for different methods of explanation such as using technology and comparing y-values.
- For **question C2**, the key is that the order may not matter. Encourage students to do transformations one at a time, using a thin or dotted curve to show the intermediate graph.
- You may wish to use **BLM 1–6 Section 1.4 Stretches of Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 37)

C1 The graph of $y = x^2$ passes through the point (4, 16). The parabola passes through the point (4, 4). This y-coordinate equals the corresponding y-coordinate on the graph of $y = x^2$ multiplied by 0.25. Since 0.25 is positive, the parabola represents a vertical compression of the graph of $y = x^2$ by a factor of 0.25.

C2 All the three methods work. I prefer to use method a) to draw a dotted compressed graph first and then reflect the compressed graph in the x-axis. It is easier to see errors by doing transformations one at a time and using a dotted graph to show the intermediate step.

Practise, Connect and Apply, Extend

- For **question 3**, part c), ask students if it matters whether the graph is stretched then reflected or vice versa and why. For a reflection in the x-axis, the y-coordinate is multiplied by -1. For a vertical stretch, the y-coordinate is multiplied by 4. The result is that the y-coordinate is multiplied by -4. The result would be the same if a stretch (multiply by 4) is done before a reflection in the x-axis (then multiply by -1).

- For **question 5**, encourage different approaches. Algebraically, substitute 10 for y and 5 for x into the equation $y = \mathrm{a}x^2$, and then solve for a. Or, use technology to graph $y = ax^2$. Use the slider in *Fathom*™ or the Transfrm application of a graphing calculator. Adjust a until the graph passes through the point (5, 10). Compare the y-coordinates when $x = 5$. On the graph of $y = x^2$, the y-value is 25. Since $\frac{10}{25} = 0.4$, the value of a is 0.4.
- **Question 6** is similar to **question 5**, with the transformed graph shown. The methods used in **question 5** apply.
- For **question 7**, stress that for the same x-coordinate, the y-coordinate is multiplied by 4.
- For **question 8**, have students graph a simple function such as $y = 2\mathrm{x} + 2$. Ask students to graph $y = 4f(x)$ on the same set of axes.
- For **question 9**, part a), the answer can be obtained by simply evaluating $d(1)$. Ask students how far the object falls during the next second, which is $d(2) - d(1)$, and not simply $d(2)$. Then, ask students think of the difference between how far the object has fallen after 2 s and during the first 2 s.
- For **question 10**, encourage different approaches. Students may recognize that the coefficient 4.9 is roughly 6 times the coefficient 0.81, meaning that $h(t) = 4.9t^2$ can be obtained by performing a vertical stretch by a factor of 6 (more accurately $4.9 \div 0.81$) on the graph of $h(t) = 0.81t^2$. Another approach would be to sketch the graphs of both functions on the same set of axes using graphing technology. For a given x-value, compare the corresponding y-values to find the vertical stretch factor.
- For **question 11**, it is important that students have the origin marked at the vertex of the parabola if the equation of the bridge is in the form $b(x) = ax^2$. Students can then use the coordinates of any point, other than the vertex, and substitute the x- and y-values into the equation $b(x) = ax^2$ to solve for a.
- **Question 12** is an Achievement Check question. Provide students with **BLM 1–7 Section 1.4 Achievement Check Rubric** to help them understand what is expected. Encourage students to discuss what they expect to happen to the stopping distance when the pavement is dry, wet, and icy.
- For **question 13**, part a), ask students as a class whether they think the path of the second arrow would represent a vertical stretch of the path of the first arrow. After some feedback, use a tennis ball and a ramp or tilted table top to model the situation. For part b), go to the gym or playground, and ask students to throw a ball to a partner standing at a fixed distance away. Discuss whether the paths of the two balls being launched from the same point and landing at the same point are vertical stretches of each other. This may be an opportunity to show the equation $h(t) = -4.9t^2 + v_0t + h_0$, where v_0 and $h0$ are respectively the initial velocity and height. If time permits, use sliders in *Fathom*™ or *The Geometers' Sketchpad*® to experiment with v_0 and h_0.
- For **question 14**, begin by asking students to graph $y = x$ and $y = 2x$ on the same set of axes. Ask whether $y = 2x$ is twice as tall or half as wide (twice as narrow). Repeat with $y = x^2$ and $y = 2x^2$, and then with $y = x^2$ and $y = 4x^2$. Each time, for a given x-coordinate, compare the two y-coordinates. Then, compare the x- and y-coordinates the other way. That is, for a given y-coordinate, compare the two x-coordinates. For strong students, ask how writing $y = 4x^2$ as $y = (2x)^2$ may help explain what is happening.

Common Errors

- Some students may forget that the y-value for the point they choose is a negative value, which will result in a negative value for a.

R_x Remind students that the value of a is positive when a parabola opens up, and negative when the parabola opens down.

Ongoing Assessment

- **Question 12** is an Achievement Check question. Use **BLM 1-7 Section 1.4 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Visual–have students experiment with balls to produce parabolic paths that model real-life situation

Spatial–provide tables on a handout

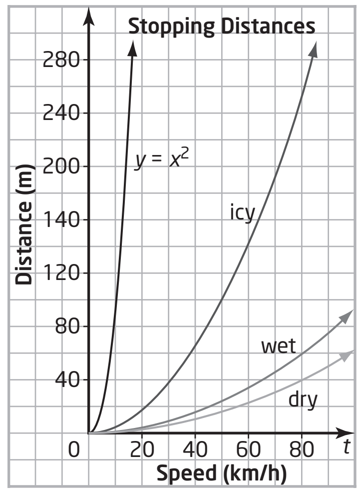
Motor–encourage students to use technology for graphing

Achievement Check Sample Solution (page 39, question 12)

a) On dry asphalt: $d(80) = 0.006(80)^2 = 38.4$. The stopping distance for a car travelling at 80 km/h is 38.4 m. On wet asphalt: $d(80) = 0.009(80)^2 = 57.6$. The stopping distance for a car travelling at 80 km/h is 57.6 m. On ice: $d(80) = 0.04(80)^2 = 256$. The stopping distance for a car travelling at 80 km/h is 256 m.

b) Answers may vary. For example, for a maximum stopping distance of around 60 m:
For $d(s) = 0.006s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 100\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 60\}$.
For $d(s) = 0.009s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 80\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 57.6\}$.
For $d(s) = 0.04s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 40\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 64\}$

c)



The graph of the function $d(s) = 0.006s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.006. The graph of the function $d(s) = 0.009s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.009. The graph of the function $d(s) = 0.04s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.04.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	9–14
Reasoning and Proving	7, 8, 12–14
Reflecting	11, 12, 14
Selecting Tools and Computational Strategies	1, 5, 6, 8–12
Connecting	9–13
Representing	2–4, 7, 12–14
Communicating	2, 3, 7, 12–14