1.5 Translations of Functions

Student Text Pages
40–46

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- CBR (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 1–9 Section 1.5 Translations of Functions
- BLM 1–10 *GSP* for Section 1.5 Investigate

Teaching Suggestions

- Have a class discussion on the javelin thrower. Ask questions such as: "How will the two throws be similar? How will they differ? How does throwing from a higher point affect the path? Does it affect the shape?"
- An alternative introduction would involve using a CBR to make position-time graphs of students walking. Ask questions such as: "How can you shift the graph up? to the right?"
- If time permits, work through both methods in each Investigate. Methods 2 of Investigate A and Investigate B are best done as demonstrations, preferably with a *Smartboard*, as opposed to taking students to a computer lab.
- Students by now have acquired a number of skills in graphing functions, including dynamic graphing techniques—the slider in *Fathom*™ or the Transfrm application of a graphing calculator. Sliders are also used in *The Geometer's Sketchpad*® for graphing transformations. After exposing students to the various tools, discuss the strengths and weaknesses of these tools and decide on which ones the class feels most comfortable using.

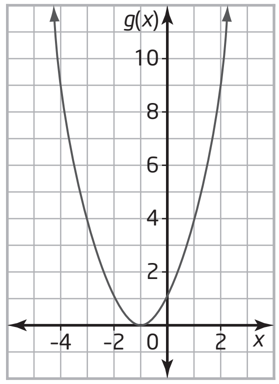
Investigate

- **Investigate A, Method** 1, suggests the use of a TI-84 Plus graphing calculator. Note that the **Transfrm** application can be transferred to a TI-83 Plus graphing calculator. As the graph is translated to the left and to the right, make sure that students note that $y = (x - 2)^2$ is a translation to the *right*, while $y = (x + 2)^2$ is a translation to the *left*. Ask students what is happening to the shape of the graph as the value of A changes. Ask them how they can find the points, other than the vertex, on the graph of a function such as $y = (x - 3)^2$.
- For **steps 7 and 8**, two strategies can be used. One is to go 1 left/right and 1 up from the vertex, then 2 left/right and 4 up from the vertex, and so on; the other starts from the vertex, go 1 left/right and 1 up, then go 1 left/right and 3 up, then 1 left/right and 5 up. Both strategies are described in the teacher talk in the **Example.**
- Students should be reminded to uninstall the **Transfrm** function every time after use. Otherwise, the program continues to run. Go to APPS, select **:Transfrm**, and then select **1:Uninstall**.
- For **Method 2,** see **BLM 1–10 *GSP* for Section 1.5 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™. If possible, consider using *Smartboard*, and let students move the slider with their hands. Ask students what is happening to the shape of the graph as they move the slider.
- For **Investigate B, Method 1**, an alternative method would be to use the Transfrm application as done in Investigate A. As with horizontal translations, the shape of the graph does not change by vertical translations. Again, after plotting the vertex, use the same two strategies described in Investigate A to find other points on the graph. Ask students why vertical translations seem to be intuitive while horizontal translations appear to be backwards. That is, $y = x^2 + 2$ translates the graph in the positive direction (up) while $y = (x - 2)^2$ translates the graph in the negative direction (left).
- For **Method 2**, see **BLM 1–10 *GSP* for Section 1.5 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™.

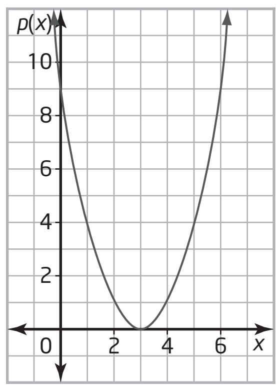
Investigate Responses (pages 40–43)

Investigate A, Method 1

4. Each time you press ►, the parabola is translated to the right 1 unit. Each time you press ◄, the parabola is translated to the left 1 unit.
5. Diagrams may vary. The vertex of the parabola in the graph of $g(x) = (x + 1)^2$ is at $(-1, 0)$.



The vertex of the parabola in the graph of $p(x) = (x - 3)^2$ is at $(3, 0)$.



6. Answers may vary. From the graphs in step 5, the graph of the quadratic function $f(x) = (x - h)^2$ is a shift of the graph of the function $f(x) = x^2$ by h units to the right. For $h < 0$, as in the function $g(x) = (x + 1)^2$ where $h = -1$, a shift by h units (-1 unit) to the right means a shift of $-h$ units (1 unit) to the left.
7. The vertex is at $(-4, 0)$. The y-coordinate of the point 1 unit left/right of the vertex is 1. The y-coordinate of the point 2 units left/right of the vertex is 4.
8. Answers may vary. For example, **o**nce you know the coordinates of the vertex, you know that the y-coordinate of the point 1 unit left/right of the vertex is 1 unit up the vertex, the y-coordinate of the point 2 units left/right of the vertex is 4 units up the vertex, the y-coordinate of the point 3 units left/right of the vertex is 9 units up the vertex, and so on. You can use this pattern to find other points on the graph.

Method 2

3. The equation is $y = (x - 3)^2$. The graph is a shift of the blue parabola for $y = x^2$ by 3 units to the right. The shape of the graph is not changed relative to the graph of $y = x^2$.
4. The vertex is at $(-6, 0)$. The value of h is -6. When the slider is moved to the value of $h = -6$, the graph is a shift of the blue parabola for $y = x^2$ by 6 units to the left, or -6 units to the right.
5. Answers may vary. For example, the y-coordinate of the point 1 unit to the left of the vertex $(-6, 0)$ is 1 unit up the vertex. So, another point on the graph of $y = (x + 6)^2$ would be a point with coordinates $(-6 - 1\ , 0 + 1)$, or $(-7, 1)$.
6. Answers may vary. For example, the graph of the quadratic function $f(x) + (x - h)^2$ is a shift of the graph of the function $f(x) = x^2$ by h units to the right. For $h < 0$, such as $h = -6$, a shift by h units (-6 units) to the right means a shift of $-h$ units (6 units) to the left.

Investigate B, Method 1

1. When $k = 0$, the coordinates of the vertex are (0, 0). When $k = 1$, the coordinates of the vertex are (0, 1). When $k = 2.5$, the coordinates of the vertex are (0, 2.5). When $k = 4$, the coordinates of the vertex are (0, 4). When $k = -3$, the coordinates of the vertex are (0, −3).
2. The vertex is at (0, 5).
3. Answers may vary. For example, the graph of the quadratic function $f(x) = x^2 + k$ is a shift of the graph of the function $f(x) = x^2$ by k units up. For $k < 0$, such as $k = -3$, a shift by k units (−3 units) up means a shift of $-k$ units (3 units) down. Since the y-coordinate of the point 1 unit to the left of the vertex (0, 5) is 1 unit up the vertex. Another point on the graph of $f(x) = x^2 + 5$ would be a point with coordinates $(0 - 1, 5 + 1)$, or (−1, 6).

Method 2

3. The equation is $y = x^2 + 3$. The graph is a shift of the blue parabola for $y = x^2$ by 3 units up. The shape of the graph is not changed relative to the graph of $y = x^2$.
4. The vertex is at (0, −10). The value of k is −10. When the slider is moved to the value of $k = -10$, the graph is a shift of the blue parabola for $y = x^2$ by 10 units down, or −10 units up.
5. Answers may vary. For example, the y-coordinate of the point 1 unit to the left of the vertex (0, −10) is 1 unit up the vertex. So, another point on the graph of $y = x^2 - 10$ would be a point with coordinates $(0 - 1, -10 + 1)$, or (−1, −9).
6. Answers may vary. For example, the graph of the quadratic function $f(x) = x^2 + k$ is a shift of the graph of the function $f(x) = x^2$ by k units up. For $k < 0$, such as $k = -10$, a shift by k units (−10 units) up means a shift of $-k$ units (10 units) down.

Example

- Ensure students can identify whether a translation is vertical or horizontal. For part a) where $y = (x - 3)^2$, ask students: "Does the 3 appear to go with the x-direction or the y-direction? Can the equation be easily rearranged so that the 3 is written with the y?" For part b), where $y = x^2 - 9$, ask: "Can the equation be easily rearranged so that the 9 is written with the y?"
- For all translations, stress that the shape of the graph will not change. Note the two strategies for finding points other than the vertex as described in the teacher talk.

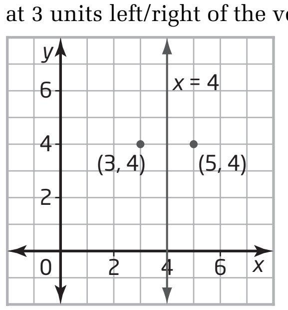
Communicate Your Understanding

- For **question C2**, a key feature of parabolas is that the axis of symmetry is halfway between any two points with the same y-coordinate.
- You may wish to use **BLM 1–9 Section 1.5 Translations of Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 45)

C1 Suppose the vertex of the parabola that has been translated is at (h, k). Two other points on this parabola would be located at 1 unit left/right of the vertex and 1 unit up the vertex. That is, they have the coordinates of $(h - 1, k + 1)$ and $(h + 1, k + 1)$. Two other points would be located at 2 units left/right of the vertex and 4 units up the vertex, and another two points would be located at 3 units left/right of the vertex and 9 units up the vertex, and so on.

C2



The two points have the same y-coordinates of 4. So, the axis of symmetry is $x = 4$ (between $x = 3$ and $x = 5$). The vertex on is on the axis of symmetry. So. the coordinates of the vertex are $(4, k)$. From the vertex, two other points would be located at 1 unit left/right of the vertex and 1 unit up the vertex. That is, they have the coordinates of $(4 - 1, k + 1)$ and $(4 + 1, k + 1)$, or $(3, k + 1)$ and $(5, k + 1)$. Since (3, 4) and (5, 4) are on the graph, $k + 1 = 4$, or $k = 3$. The vertex is at (4, 3).

Common Errors

- Some students may make errors identifying the correct sign for the h-value (the direction of horizontal translation) in functions such as $y = (x + 2)^2 + 1$ in **part c)** of the **Example**.

R_x Have students rewrite the equation in the form $y = (x - h)^2 + k$ with the h-value in brackets. So, $y = (x + 2)^2 + 1$ will become $y = [x - (-2)]^2 + 1$ and students can see that since $h = -2$, the translation is to the left.

Accommodations

Visual–provide a copy of the graph of $y = x^2$ on tracing paper

Motor–encourage students to use technology for graphing

Memory–have students use index cards to remember the two strategies for plotting points other than the vertex

Practise, Connect and Apply, Extend

- For **question 3**, encourage students to plot the vertex first. For all the functions, the graphs are congruent in shape, so plotting points relative to the vertex (go 1 left/right and 1 up from the vertex, then 2 left/right and 4 up from the vertex, and so on) will be the same in each case. If necessary, use an overhead or tracing paper with the graph of $y = x^2$ drawn on it, and simply move the vertex of the curve to the match the vertex of each function.
- For **question 4**, ask students to write a similar equation. Ensure they see the pattern of 1, 4, and 9 and note the two important features of the intercepts: both are integer values and symmetrical about the y-axis. Ask: "What vertical shift would result in x-intercepts at 10 and −10?"
- For **question 5**, encourage students to plot the points for the intercepts. If necessary, ask where the axis of symmetry would be. In part b), students should be able to see from the equation that the graph is congruent to $y = x^2$ and that the x-intercepts are points 2 units left/right from the vertex.
- For **question 6**, encourage students to sketch the position-time graph for the first dragster. Then ask them where the second one would be if the race is delayed 2 s. This would be a good opportunity to use a CBR. Ask a student to walk a certain way, then repeat, having the student wait 2 s before starting to walk. Compare the two resulting graphs.
- For **question 7**, suggest writing the area of the lawn in words before writing the equation:
  area of lawn = area of square property − area of rectangular pool
  $= x^2 - 48$.
- For **question 9**, ask students to rearrange the equations $y = x^2 + 2$, $y = 2x^2$, and $y = (x + 2)^2$ to see how the numbers that cause the transformations on $y = x^2$ are related to their variables. That is, $y - 2 = x^2$ (a shift up 2 units), $\frac{1}{2}y = x^2$ (2 times as tall), and $y = (x + 2)^2$ (a shift 2 units to the left). The numbers are −2 for y, $\frac{1}{2}$ for y, and +2 for x. The transformations do the opposites of the operations represented by the numbers −2, $\frac{1}{2}$, and +2, which are consistent with the directions defined for vertical and horizontal transformations. A good follow up question, similar to **question 10**, would be: "What are the coordinates of the centre of the circle defined by $(x + 1)^2 + (y - 3)^2 = 25$?"
- For **question 11**, encourage students to graph $f(x) = \sqrt{x}$ using graphing technology. Suggest that students describe the transformations required to obtain $g(x) = \sqrt{x - 3} + 2$ and the effect the transformations would have on the domain and range.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 7, 9–11
Reasoning and Proving	4, 6, 9–11
Reflecting	5, 6, 9, 11
Selecting Tools and Computational Strategies	1, 3, 11
Connecting	6, 7
Representing	2–5, 7, 8, 10, 11
Communicating	4, 6, 7, 9–11