1.6 Sketch Graphs Using Transformations

Student Text Pages
47–53

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- CBR (optional)

Related Resources
- BLM G-1 Grid Paper
- BLM 1-11 Section 1.6 Sketch Graphs Using Transformations
- BLM 1-12 Section 1.6 Achievement Check Rubric

Teaching Suggestions

- Encourage students to build on concepts learned in Sections 1.4 and 1.5. Concepts in this section are not new, but they involve combinations of stretches and translations learned in the previous two sections.
- To complement the dragster context, use graphing calculators with a CBR. Begin by graphing the position of a student walking a certain way, and then repeat the graphing with the student walking from a different location that results in a vertical/horizontal shift of the graph. Ask students how to walk in such a way that the original graph is stretched vertically.
- If time permits, work through both methods in the Investigate. If a computer lab is not available for Method 2, demonstrate to the class using a projector or let students take turns moving the sliders.
- You might wish to begin the class with a question similar to question C1, asking students whether the coordinates of the vertex of the function $y = 2(x - 4)^2 + 5$ should be (4, 5) or (4, 10) as a result of the vertical stretch. Allow students to think alone, then share their opinions or have a discussion as a class.

Investigate

- **Steps 1 to 5** in **Method 1** examine the graph of $y = (x - 2)^2 + 1$, which is congruent to the graph of $y = x^2$ that has been shifted horizontally by 2 units to the right and vertically by 1 unit up. Ask students why the values $x = 1$ and $x = 3$ and then $x = 0$ and $x = 4$ are chosen. (It is because each pair is symmetrical about the axis of symmetry. For $x = 1$ and $x = 3$, the points can be found by moving 1 unit to the left/right and 1 unit up *from the vertex*. For $x = 0$ and $x = 4$, the points can be found by moving 2 units to the left/right and 4 units up *from the vertex*.
- **Steps 6 to 9** examine the graph of $y = 3(x - 2)^2 + 1$, where the vertex (2, 1) remains unchanged. When graphing $y = 3(x - 2)^2 + 1$, students should note that the graph is congruent to $y = 3x^2$, but with a different vertex. Again, the values $x = 1$ and $x = 3$ and then $x = 0$ and $x = 4$ are chosen because each pair is symmetrical about the axis of symmetry. For $x = 1$ and $x = 3$, the points can be found by moving 1 unit to the left/right and 1×3 units up *from the vertex*. For $x = 0$ and $x = 4$, the points can be found by moving 2 units to the left/right and 4×3 units up *from the vertex*.
- For **Method 2**, note that *Fathom*™ does not work as well as a graphing calculator for identifying points on transformations of graphs.
- Encourage students to investigate with the three sliders representing a, h, and k. Ask students to adjust the sliders to produce the graph of $y = 3(x - 2)^2 + 1$. Ask them if it matters in which order the sliders are adjusted. Also ask if it matters whether the graph of $y = x^2$ is shifted then stretched or vice versa.
- Students should be encouraged to graph transformations using whichever method they find easiest. For example, when graphing $y = -2(x + 5)^2 + 6$ in **step 10** of **Method 1**, students can sketch the graph of $y = -2x^2$, then move the vertex and other points 5 units left and 6 units up.

Alternatively, students can plot the vertex at $(-5, 6)$ and then move 1 unit to the left/right and 1×2 units down *from the vertex*, and then move 2 units to the left/right and 4×2 units down *from the vertex*, and so on to draw other points.

Investigate Responses (pages 47-49)

Method 1

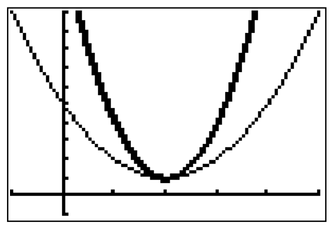
2. $y = 2$ when $x = 1$; $y = 2$ when $x = 3$

3. Answers may vary. For example, the point (1, 2) is 1 unit to the left and 1 unit up the vertex (2, 1), and the point (3, 2) is 1 unit to the right and 1 unit up the vertex. This makes sense because they are points on a parabola that results from shifting the graph of $y = x^2$.

4. Answers may vary. For example, $y = 5$ when $x = 0$; $y = 5$ when $x = 4$. The point (0, 5) is 2 units to the left and 4 units up the vertex, and the point (4, 5) is 2 units to the right and 4 units up the vertex. This makes sense because they are also points on a parabola that results from shifting the graph of $y = x^2$.

5. Answers may vary. For example, if I move 3 units to the left or to the right from the vertex, I expect to go 9 units up the vertex to meet the graph. Points on a parabola that has the same shape as the parabola for $y = x^2$ follow a pattern: 1 unit to the left/right and 1 unit up, 2 units to the left/right and 4 units up, 3 units to the left/right and 9 units up the vertex, and so on.

6.



7. Compared with the graph of $y = (x - 2)^2 + 1$, the graph of $y = 3(x - 2)^2 + 1$ is stretched vertically. It has the same vertex (2, 1) but the shape is different.

8. $y = 4$ when $x = 1$; $y = 4$ when $x = 3$. The y-coordinate of each of the points (1, 4) and (3, 4) on the graph of $y = 3(x - 2)^2 + 1$ is 3 times higher up the vertex (2, 1) than the y-coordinates of the corresponding points (1, 2) and (3, 2) on the graph of $y = (x - 2)^2 + 1$. This makes sense since the parabola for $y = 3(x - 2)^2 + 1$ is simply a vertical stretch of the parabola for $y = (x - 2)^2 + 1$ by a factor of 3.

9. Answers may vary. For example, if I move 3 units to the left or to the right from the vertex, I expect to go 3×9 units, or 27 units up the vertex to meet the graph. This follows the same pattern in step 5, with the number of units up the vertex multiplied by the vertical stretch factor of 3.

10. Answers may vary. For example, for the graph of the parabola $y = -2(x + 5)^2 + 6$, the vertex is at $(-5, 6)$. The parabola opens downward and is vertically stretched by a factor of 2. So, to locate points from the vertex, move 1 unit to the left/right and 2 units down. The points are $(-6, 4)$ and $(-4, 4)$.

Method 2

4. The slider for a does not affect the location of the vertex. The sliders for h and k do not affect the shape of the graph.

5. Answers may vary. For example, another point is (1, 3).

6. The new coordinates of the point in step 5 are $(-3, 1)$.

7. Answers may vary. For example, to plot points from the vertex $(-4, -2)$, move 1 unit to the left/right and 3 units up, 2 units to the left/right and 12 units up, 3 units to the left/right and 27 units up the vertex, and so on.

Example

- When describing transformations, encourage students to use the term *translation* as well as *shift*.
- The translation of 2 units to the right and 9 units down will determine the vertex $(2, -9)$ and the axis of symmetry $x = 2$.

- Have students pay attention to the teacher talk and arrows on the diagrams in part d): move 1 unit to the left/right and 3 × 1 units up from the vertex, 2 units to the left/right and 3 × 4 units up the vertex, and so on. Ask students how the pattern of drawing points for the transformed graph would differ if the coefficient (value of a) changes.
- An alternative method is to keep moving from one point to the next, beginning at the vertex, in this pattern: from the vertex, move 1 unit to the left/right and 3 × 1 units up, then 1 unit to the left/right and 3 × 3 units up, then 1 unit to the left/right and 3 × 5 units up, and so on.
- For part e), focus on the teacher talk for both domain and range. Ask students how the range will change if the coefficient is changed to −3. Consider giving a range and asking students to write a corresponding equation.

Communicate Your Understanding

- For **question C1**, the key is that the vertex remains unchanged even though the coefficient is changed to −4. If students respond correctly, consider challenging them with these questions: "What about the 4? Have you considered the negative sign in front of it?" This will encourage students to focus on reasoning and understand that the −4 has no bearing on the vertex.
- For **question C2**, consider the two approaches: begin with $y = 2x^2$ then translate the vertex and points on the graph 3 units left and 4 units down; begin by plotting the vertex at (−3, −4) then draw a graph congruent to $y = 2x^2$ from that point.
- For **question C3**, the key is that the order in which the transformations are applied may not matter. Encourage students to try the vertical stretch followed by the translations, then vice versa.
- You may wish to use **BLM 1–11 Section 1.6 Sketch Graphs Using Transformations** for remediation or extra practice.

Communicate Your Understanding Responses (page 51)

C1 The ordered pair in part c) will give the coordinates of the vertex of the graph of $y = -4(x + 5)^2 - 9$. The graph of $y = -4(x + 5)^2 - 9$ is a vertical stretch of the graph of $y = (x + 5)^2 - 9$ by a factor of −4. This transformation does not change the vertex.

C2 The graph of $y = 2(x + 3)^2 - 4$ is a shift of the graph of $y = 2x^2$ by 3 units to the left and 4 units down. To sketch the graph of $y = 2(x + 3)^2 - 4$, subtract 3 from the x-coordinate and subtract 4 from the y-coordinate of the points on the graph of $y = 2x^2$ to get the coordinates of the corresponding points on the graph of $y = 2(x + 4)^2 - 4$.

C3 I will perform a vertical stretch by a factor of 4 to get the graph of $y = 4x^2$. Then, I will reflect the graph of $y = 4x^2$ in the x-axis to get the graph of $y = -4x^2$. Then, I will shift the graph of $y = -4x^2$ by 7 units to the right and by 11 units up to get the graph of $y = -4(x - 7)^2 + 11$. It is easier to perform the vertical stretch and reflection in the x-axis first when the graph is still symmetrical about the x-axis.

Practise, Connect and Apply, Extend

- For **question 2**, you may wish to ask for more than one point other than the vertex (and not just the point on the opposite side of the axis of symmetry). This will ensure that students understand the effect of the vertical stretch.
- For **question 3**, suggest students refer to the Example or the table in **question 1** and work backward from the description to find the equation.

- For **question 4**, encourage students to write a generic equation in the form $y = a(x - h)^2 + k$. Use pairs of brackets as place holders for a, h and k. For each piece of information, ask, “What does this mean? How does it affect the graph?”
- For **question 5**, suggest that students use graphing technology to verify their answers to part a).
- In **question 6**, the key is that the points on a graph such as $f(x) = 2x^2$ provide the distances from the vertex for points on graphs that are the graph’s translations, such as the graph of $g(x) = 2(x - 10)^2 - 32$. For example, the point (1, 2) on the graph of $f(x) = 2x^2$ is 1 unit to the right and 2 units up the vertex. So, the corresponding point on the graph of $g(x) = 2(x - 10)^2 - 32$ is also 1 unit to the right and 2 units up the vertex (10, −32), which becomes the point (11, −30).
- For **question 7**, once students realize the vertex gives the values of h and k, they can find the value of a in a number of ways. Encourage choosing points farther away from the vertex of the parabola for more accurate calculation of the value of a. Other than substituting the coordinates of points into the equation, using graphing technology (calculator, *The Geometer’s Sketchpad*®, or *Fathom*™) with a as a slider is another good method.
- For **question 8**, ask students what effect the context has on the equation: “At what time is the ball thrown? What is its height when it lands?”
- For **question 9**, consider asking students where the axis of symmetry would be relative to the x-intercepts to answer part a). For part b), once the vertex has been determined, see notes for **question 7** for finding the value of a.
- **Question 10** is an Achievement Check question. Provide students with **BLM 1–12 Section 1.6 Achievement Check Rubric** to help them understand what is expected. Before starting, encourage students to discuss the force of gravity, how it affects motion, as well as how gravity differs on the moon and on other planets. This will activate prior knowledge and help students understand how to interpret the meaning of the various functions for heights. Suggest that students use graphing technology to find the points of intersection.
- For **question 11**, encourage students to sketch a diagram to help them see the points to determine the axis of symmetry, which gives the value of h in the equation. Students can then use different methods to find the values of a and k. For example, substitute (2, 20) and (9, 34) into the equation to produce two linear equations with two unknowns. Or, use graphing technology (sliders) to find the values of a and k that will cause the graph to pass through the required points.
- For **question 12**, encourage students to draw a diagram and label the coordinates of the point on the truck which will likely hit the bridge.
- For **question 13**, encourage students to find the coordinates of the points, the vertex in particular, on portions of the parabolas and compare the points to the corresponding points on the graph of $y = x^2$ to obtain each equation.

Common Errors

- Some students may have difficulty showing the vertical stretch accurately when graphing functions such as $y = 3(x + 1)^2 - 5$.

R_x Have students practise identifying the vertex and the points 1 unit and 2 units to the left and right. Have students perform the transformations one at a time and use a dotted curve to show the intermediate graph. For $y = 3(x + 1)^2 - 5$, graph $y = 3x^2$ and then translate points on this graph 1 unit left and 5 units down or graph $y = (x + 1)^2 - 5$ and then stretch this graph by a factor of 3.

Ongoing Assessment

- **Question 9** is a Chapter Problem question that incorporates many of the key concepts from this section and can be used as a formative assessment.
- **Question 10** is an Achievement Check question. Use **BLM 1-12 Section 1.6 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Gifted and Enrichment–assign **question 13** for a presentation

Visual–provide a copy of the graph of $y = x^2$ on tracing paper

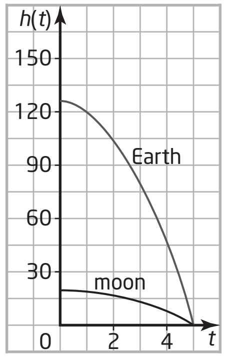
Motor–encourage students to use technology for graphing

Language–have students work in pairs

Memory–have students use index cards to remember the two strategies for plotting points other than the vertex

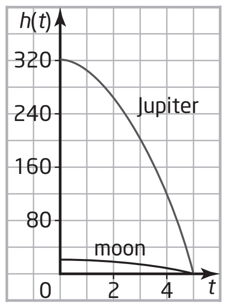
Achievement Check Sample Solution (page 53, question 10)

a)



b) Both graphs have a value of 0 when $t \doteq 5$. The common point (5, 0) indicates that each of the objects on Earth and on the moon hits the ground 5 s after it is being thrown.

c) Diagrams may vary. For example:



d) Stretch the graph of $h(t) = -0.8t^2 + 20$ vertically by a factor of 16 to get the graph of $h(t) = -12.8t^2 + 20$. Then, translate the graph by 300 units up to get the graph of $h(t) = -12.8t^2 + 320$.

e) Both graphs have a value of 0 when $t \doteq 5$. The common point (5, 0) indicates that an object dropped from 320 m on Jupiter or from 20 m on the moon hits the ground 5 s after it is being thrown.

f) For $h(t) = -0.8t^2 + 20$: domain: $\{t \in \mathbf{R} \mid 0 \le t \le 5\}$; range: $\{h \in \mathbf{R} \mid 0 \le h \le 20\}$.
For $h(t) = -12.8t^2 + 320$: domain: $\{t \in \mathbf{R} \mid 0 \le t \le 5\}$; range: $\{h \in \mathbf{R} \mid 0 \le h \le 320\}$.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	7–13
Reasoning and Proving	2, 5–7, 9, 10, 12, 13
Reflecting	7, 10, 11, 13
Selecting Tools and Computational Strategies	1, 6–8
Connecting	7–10, 12, 13
Representing	2–4, 6, 9–11, 13
Communicating	2, 5–7, 9, 10, 12, 13