2.2 Quadratic Functions: Comparing Forms

Student Text Pages
76–85

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- algebra tiles
- computers with graphing software (optional)
- overhead projector (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 2–4 Section 2.2 Quadratic Functions: Comparing Forms
- BLM 2–5 Section 2.2 Achievement Check Rubric

Teaching Suggestions

- Review as a class the different forms of a quadratic function and the information provided by each form about the corresponding graph.
- If technology is available, have students work through the section using algebraic methods as well as technology.
- Graphing technology is recommended to expedite the graphing steps.

Investigate

- Algebra tiles provide students with a concrete/visual representation of a quadratic function in factored form.
- In **Part A**, provide discussion and explanations to help students explore the connection between the length and width of a rectangular area model and the x-intercepts of a quadratic function. Students should see that a quadratic function written as a product of two binomial factors can be represented by a rectangle whose length and width give the zeros of the function. This is a building block for further work in the chapter on factoring methods.
- In **Part B**, provide discussion and explanations to help students explore the connection between the length of a square area model (plus residual unit tiles) and the vertex of a quadratic function. Students should see that a quadratic function written in vertex form can be represented by a square plus some “left over” unit tiles, and that the side length of the square gives the value of h and the number of “left over” unit tiles gives the value of k.
- Using area models to convert quadratic functions from one form to another is easy for positive values. Models for quadratic functions involving negative values are much more challenging to construct and interpret.

Investigate Responses (pages 76-78)

Part A

1. a) To find the x-intercepts of a function, substitute $y = 0$ into the function and solve for x.

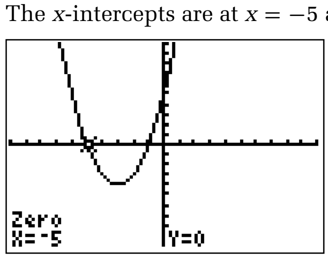
$0 = (x + 5)(x + 1)$

$0 = x + 5$ or $0 = x + 1$

$x = -5$ or $x = -1$

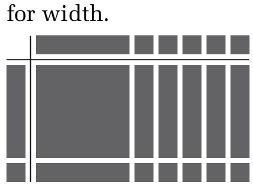
The x-intercepts are at $x = -5$ and $x = -1$.

b)



2. a) Count out 1 x-tile and 5 unit tiles for length. Count out 1 x-tile and 1 unit tile for width.

c)



3. a) I need 1 x^2-tile, 6 x-tiles, and 5 unit tiles to fill the rectangle.

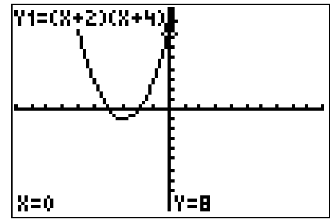
b) The area can be represented as $x^2 + 6x + 5$.

c) Area = length × width; $x^2 + 6x + 5 = (x + 5)(x + 1)$. The standard form of the function $f(x) = (x + 5)(x + 1)$ is $f(x) = x^2 + 6x + 5$.

d) The factored form of the function has already been input as Y1. The standard form of the function should be input as Y2.

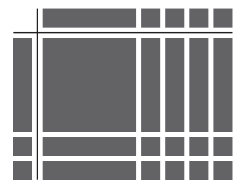
e) The two forms represent the same function because the graphs and tables produced by the calculator are identical.

4. a) For the function $f(x) = (x + 2)(x + 4)$, the x-intercepts are at $x = -2$ and $x = -4$. The answers can be verified using a graphing calculator.



Next, count out and arrange 1 x-tile and 4 unit tiles for the length. Count out and arrange 1 x-tile and 2 unit tiles for the width.

The rectangle requires 1 x^2-tile, 6 x-tiles, and 8 unit tiles to fill. The area can be represented algebraically as $x^2 + 6x + 8$. The standard form of the function $f(x) = (x + 2)(x + 4)$ is $f(x) = x^2 + 6x + 8$.

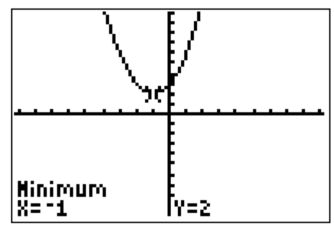


b) Answers may vary.

Part B

5. a) The vertex of the parabola defined by $f(x) = (x + 1)^2 + 2$ is located at $(-1, 2)$. I know this because the vertex form of the equation of a quadratic function with the vertex at (h, k) is $f(x) = a(x - h)^2 + k$.

b) Input $f(x) = (x + 1)^2 + 2$ into a graphing calculator and follow the steps in the technology tip to locate the vertex as shown.



6. b)

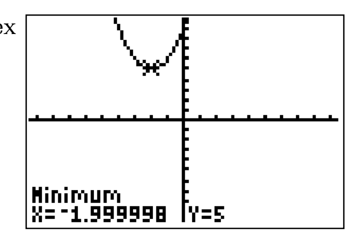
7. a) The total area represented by the algebra tile model is $x^2 + 2x + 3$.

b) The standard form of $f(x) = (x + 1)^2 + 2$ is $f(x) = x^2 + 2x + 3$.

c) The vertex form of the function has already been input as Y1. The standard form of the function should be input as Y2.

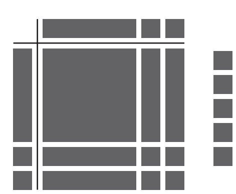
d) The two forms represent the same function because the graphs and tables produced by the calculator are identical.

8. a) For the function $f(x) = (x + 2)^2 + 5$, the vertex is located at $(-2, 5)$. The answer can be verified using a graphing calculator.



Next, build a square having a side length of 1 x-tile and 2 unit tiles. Add 5 unit tiles to the right to complete the model.

The total area represented by the algebra tile model is $x^2 + 4x + 9$. The standard form of the function $f(x) = (x + 2)^2 + 5$ is $f(x) = x^2 + 4x + 9$.



b) Answers may vary.

9. For expressions in factored form, represent each factor as the side length of a rectangle. By constructing the rectangle using algebra tiles, you can illustrate the expansion of the product of the two factors. This expansion will give the standard form of the function which is equivalent to the original factored form.

For expressions in vertex form, represent the repeated factor as the side length of a square. By constructing the square using algebra tiles and adding unit tiles for the constant term, you can illustrate the expansion of the binomial squared. The simplified form of the expansion plus the constant term will give the standard form of the function which is equivalent to the original vertex form.

Examples

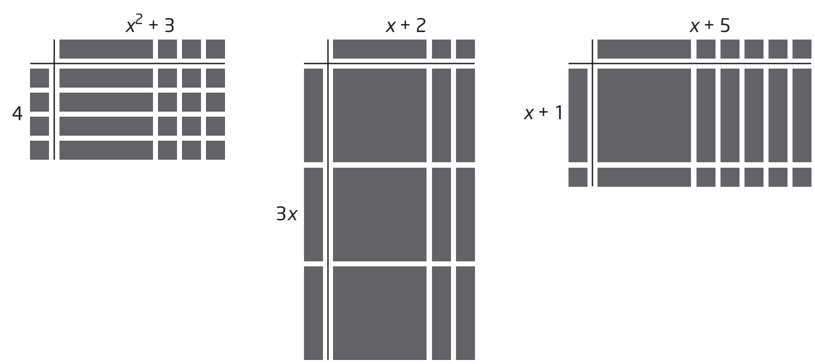
- **Example 1** focuses on the mechanical processes of expanding and simplifying algebraic expressions (including the square of binomials). Method 2 requires the use of the distributive property and collecting like terms. Use algebra tiles to introduce these skills early, with the goal of moving students toward abstract algebraic reasoning. The use of a CAS is recommended to check the results of difficult questions, such as part c).
- **Examples 2 and 3** apply the algebraic skills from Example 1 to convert a quadratic function given in factored form (Example 2) or vertex form (Example 3) to standard form. Examining the key information in the various forms helps to sketch the graph of the function. Graphing technology is recommended to check the results.

Communicate Your Understanding

- Have algebra tiles available for students to use.
- For **questions C1**, have students work in small groups and assign an expression to each group. The groups can take turns sharing their results with the rest of the class using an overhead projector.
- For **questions C2 and C3**, have students work in pairs. Consider using the think-pair-share strategy.
- You may wish to use **BLM 2–4 Section 2.2 Quadratic Functions: Comparing Forms** for remediation or extra practice.

Communicate Your Understanding Responses (pages 82-83)

C1 **a)**



b) For each model, count out the tiles used to construct the rectangle to give the expanded form of the expression: $4x + 12$; $3x^2 + 6x$; $x^2 + 6x + 5$.

c) Using the distributive property: $4(x + 3) = 4x + 12$; $3x(x + 4) = 3x^2 + 6x$; $(x + 5)(x + 1) = x(x + 1) + 5(x + 1) = x^2 + x + 5x + 5 = x^2 + 6x + 5$. The result is the same as in part b).

d) Answers may vary.

C2 **a)** Sahar's method is not correct. When squaring a binomial, there are four individual products which must be calculated. Sahar's method only accounts for two of the four products.

b) Answers may vary. I tried $x = 0$, in which case Sahar's method appeared to be correct. When I tried $x = 5$, $(x + 4)^2 = 81$ and $x^2 + 16 = 41$. After I tried several values of x, I found that the difference between the two expressions

c) The expanded form of $(x + 4)^2$ is $x^2 + 8x + 16$.

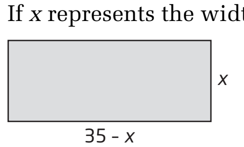
C3 Suki's claim is correct. For any product, $a \times b = b \times a$. If you construct an area model for each product, you will get the same rectangle in each case. The orientation is different but the tiles needed to construct its interior are the same. The result by applying the distributive property also shows that the two expressions $5(x - 3)$ and $(x - 3)5$ are equivalent.

Practise, Connect and Apply, Extend

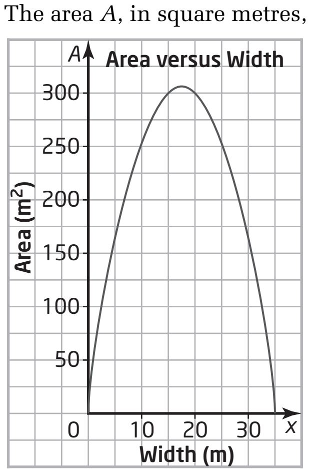
- For **questions 1 and 2**, students may need some review on the operations with integers.
- For **questions 1 to 4**, the mnemonic FOIL may be helpful when applying the distributive property involving two binomials:
  F First (multiply the first terms in each set of brackets)
  O Outside (multiply the outside terms in each set of brackets)
  I Inside (multiply the inside terms in each set of brackets)
  L Last (multiply the last terms in each set of brackets)
- For **questions 9 and 10**, students here explore how algebra tiles can be used to simplify more complex expressions.
- By **question 11**, many students should be moving from the concrete to the abstract.
- For **questions 12 and 13**, it is important for students to realize that the graphs of the functions represent the vertical height versus time. They do not have to trace the actual two-dimensional path of the projectile. For example, in part b) of **question 13**, the vertex gives the maximum height and the time when the maximum height occurs, but does not give the horizontal position at which the maximum height occurs. This is to be found in **question 14**.
- Note that the pattern in **question 15** will be revisited later in the chapter when student learn to factor perfect square trinomials.
- For **questions 17 and 18**, encourage the use of a variety of methods including graphing the function and locating the vertex from the graph, finding the vertex algebraically, and systematic trial.
- **Question 18** is an Achievement Check question. Provide students with **BLM 2–5 Section 2.2 Achievement Check Rubric** to help them understand what is expected.

Achievement Check Sample Solution (page 85, question 18)

a) If x represents the width, then the length is $\frac{70 - 2x}{2}$ or $35 - x$.



The area A, in square metres, would be $A = x(35 - x)$ or $A = 35x - x^2$.



Common Errors

- Some students may mix up signs when expanding and simplifying expressions.

R_x Have students review integer operations. Use colour tiles, if needed, to develop conceptual understanding.

- Some students may incorrectly square a binomial, missing the linear term.

R_x Have students write out the square binomial as a product of two identical binomials before they apply the distributive property.

- Some students may get confused with the signs when identifying the zeroes from the factored form.

R_x Have students solve for each intercept by setting each factor equal to zero and solving the linear equation.

- Some students may get confused with the signs when identifying the coordinates of the vertex from the vertex form.

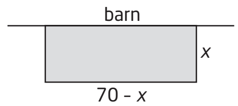
R_x Have students review horizontal and vertical translations and draw connections between the graphical and algebraic effects. Consider using graphing technology to expedite this.

Ongoing Assessment

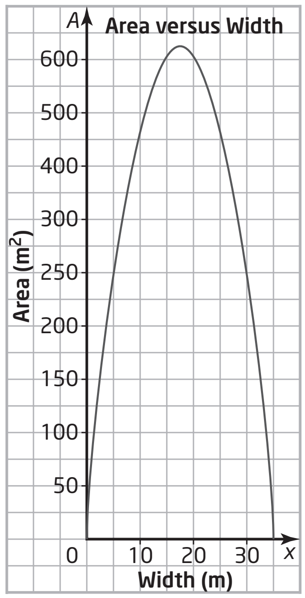
- **Question 18** is an Achievement Check question. Use **BLM 2-5 Section 2.2 Achievement Check Rubric** as a summative assessment tool.

From the graph, the maximum possible area occurs at $x = 17.5$. The dog run with maximum area has a width of 17.5 m and a length of 17.5 m. The maximum area would be 306.25 m².

b) If x represents the width, then the length is $70 - 2x$.



The area A, in square metres, would be $x(70 - 2x)$ or $A = 70x - 2x^2$.



From the graph, the maximum possible area occurs at $x = 17.5$. The dog run with maximum area has a width of 17.5 m and a length of 35 m. The maximum area would be 612.5 m².

c) The second scenario gives the largest enclosed area. You can enclose a much larger area if you do not need to use fencing on one side of the enclosure.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12–15, 17–19
Reasoning and Proving	5, 7, 12–17, 19
Reflecting	13–16
Selecting Tools and Computational Strategies	1, 2, 9–19
Connecting	12–15
Representing	1–4, 6, 8, 9, 12, 13, 15–18
Communicating	3, 4, 6, 12–19

Student Text Pages
86–87

Suggested Timing
75 min

Materials and Technology Tools
- video camera, or digital camera with motion-picture playback capability
- tennis balls
- computers with *The Geometer's Sketchpad*® and video playback software
- pre-made videos of projectiles (optional)

Model the Motion of a Projectile

Specific Expectations

1.2, 2.5

Teaching Suggestions

- Digital video cameras work best; however, a regular digital camera with short video capabilities is sufficient for this activity.
- Better results are obtained if a light coloured ball, such as a tennis ball, is filmed against a dark background, a dark red brick wall, for example.
- Have students perform a couple of trial runs before actually filming. Then, have them take at least 3 takes until they feel confident that they have adequately captured the trajectory of the ball.
- It is probably wise to have some pre-made videos available in case something goes wrong with the filming.