2.4 Select and Apply Factoring Strategies

Student Text Pages
98–107

Suggested Timing
75–150 min

Materials and Technology Tools
- algebra tiles
- calculators or computers with Computer Algebra System (CAS)

Related Resources
- BLM 2-7 Section 2.4 Select and Apply Factoring Strategies

Teaching Suggestions

- Depending on the needs and strength of the class, this material may require one to two 75-minute periods.
- You may wish to follow this order of teaching. Work through the Investigate, spending about 15 to 20 min. Go through Examples 1 and 2 with students, spending another 15 min. Discuss Communicate Your Understanding questions C1 and C2. Then, assign questions 1 to 3 in the Practise section. If students are struggling, provide additional practise using **BLM 2–7 Section 2.4 Select and Apply Factoring Strategies** and leave the remaining content for the next day. If students are ready and time permits, go through Examples 3 to 5, work on questions C3 and C4, and assign the remaining exercise questions.

Investigate

- This investigation provides a concrete/visual representation of a perfect square trinomial using algebra tiles. Students have the opportunity to explore the connection between the algebraic and geometric representations of a perfect square trinomial.
- Some students may benefit from exploring patterns using a CAS.

Investigate Responses (pages 98-99)

1. a) The polynomial is $x^2 + 2x + 1$.

b) The length and width are equal and are represented by the expression $x + 1$.

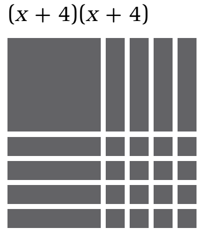
c) The length and width are equal.

d) **i)** The polynomial $x^2 + 4x + 4$ represents the area of the model. The length and width are equal and are represented by the expression $x + 2$.
ii) The polynomial $x^2 + 6x + 9$ represents the area of the model. The length and width are equal and are represented by the expression $x + 3$.

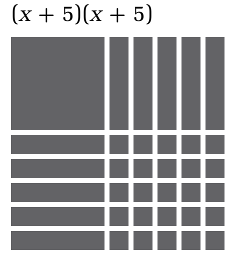
2. a), b)

Polynomial	Factor (length)	Factor (width)
$x^2 + 2x + 1$	$x + 1$	$x + 1$
$x^2 + 4x + 4$	$x + 2$	$x + 2$
$x^2 + 6x + 9$	$x + 3$	$x + 3$
$x^2 + 8x + 16$	$x + 4$	$x + 4$
$x^2 + 10x + 25$	$x + 5$	$x + 5$

c) $(x + 4)(x + 4)$



$(x + 5)(x + 5)$



The number of tiles needed to construct each model with the predicted length and width matches the corresponding expression in the table.

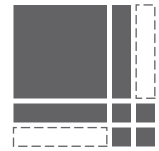
d) The trinomials are called perfect square trinomials because each is formed by expanding the square of a linear binomial. The rectangular arrangement of the tiles forms a perfect square in each case.

e) Notice first that the values of the coefficients for the x-term are consecutive even numbers. The values of the constant terms are consecutive square numbers. The value of the coefficient for the x-term is twice the square root of the value of the constant term.

3. a) Polynomials i), iii), and iv) are not perfect square trinomials because in each polynomial, the value of the coefficient for the x-term is not twice the square root of the value of the constant term. Polynomial ii) is a perfect square trinomial because the value of the coefficient (12) for the x-term is twice the square root (6) of the value of the constant term (36).

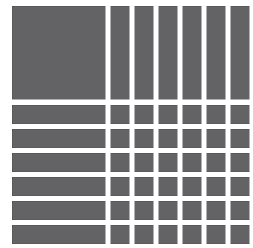
b) i) $x^2 + 2x + 4$

You cannot build a rectangular model with the same length and width using 1 x^2-tile, 2 x-tiles, and 4 unit tiles. Two more x-tiles are required.



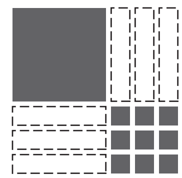
ii) $x^2 + 12x + 36$

You can build a rectangular model with the same length and width using 1 x^2-tile, 12 x-tiles, and 36 unit tiles.



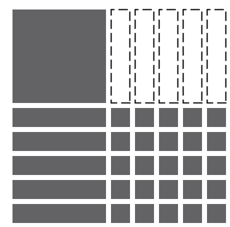
iii) $x^2 + 9$

You cannot build a rectangular model with the same length and width using 1 x^2-tile and 9 unit tiles. Six more x-tiles are required.



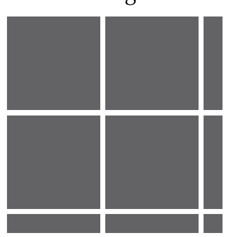
iv) $x^2 + 5x + 25$

You cannot build a rectangular model with the same length and width using 1 x^2-tile, 5 x-tiles, and 25 unit tiles. Five more x-tiles are required.



4. a) $x^2 + 20x + 100$ is a perfect square trinomial because the value of the coefficient (20) for the x-term is twice the square root (10) of the value of the constant term (100). The expression can be factored as $(x + 10)^2$.

b) $4x^2 + 4x + 1$ is also a perfect square trinomial. By factoring or by constructing a model, it can be shown that $4x^2 + 4x + 1 = (2x + 1)^2$.



5. a) i) An algebra tile model that represents a perfect square trinomial must have its length and width represented by the same expression. For example, see the model in question 4.

ii) An algebraic expression is a perfect square trinomial if it is written in the form $ax^2 + bx + c$ such that: a (coefficient for the x^2-term) and c (constant term) are positive perfect square numbers; b (coefficient for the x-term) is twice the product of the square roots of a and c; the sign before bx can stay as positive or negative in the linear factor. For example,
$16x^2 + 56x + 49 = (4x + 7)^2$ is perfect square trinomial.
$16 = 4^2$ and $49 = 7^2$ are perfect square numbers.
$56 = \sqrt{16 \times 49}$
$16x^2 - 56x + 49 = (4x - 7)^2$ is also a perfect trinomial.

b) To factor a perfect square trinomial written in the form $ax^2 + bx + c$:
Take the positive square roots of a and c. So, $a = p^2$ and $c = q^2$.
b is twice the product of the square roots of a and c. So, $b = 2pq$.
The operation sign before bx can stay negative in the linear factor.
$$ax^2 + bx + c = p^2x^2 + 2pqx + q^2$$
$$= (px + q)^2$$
The same strategy applies to $ax^2 - bx + c$. For example, to factor the perfect square trinomial $16x^2 - 56x + 49$: Take the positive square root of 16, $\sqrt{16} = 4$. Take the positive square root of 49, $\sqrt{49} = 7$. Twice the product of the square roots 4 and 7 is $2 \times 4 \times 7 = 56$. The sign before $56x$ can stay as negative in the linear factor.
$$16x^2 - 56x + 49 = 4^2x^2 - 2(4 \times 7)x + 7^2$$
$$= (4x - 7)^2.$$
Check by expanding and simplifying.

Examples

- In **Example 1**, an initial inspection of the coefficients helps to decide whether or not the trinomial is perfect square polynomial. Eventually, students are expected to be able to perform this analysis mentally. Students should be encouraged to (perhaps mentally) expand their factored result to check that they have correctly factored the trinomial.
- Students may benefit from a review of a difference of squares in **Example 2**. Sometimes, a difference of squares can be “hidden” by a common factor that must be factored out first, as in part c) or by a term of degree higher than 2, as in part d). Sometimes, factoring a difference of squares can be applied more than once, as in part d). Students may wonder why a sum of squares cannot be factored. They will have the opportunity to explore this idea in greater depth in questions 14 and 15.
- **Example 3** models the process of selecting an appropriate strategy when an expression is given in a random fashion. This is a more realistic mathematical context. Remind students to always look for common factors first.
- Multi-step factoring is the focus of **Example 4**. Often, factoring a polynomial results in factors that can be factored further. Once an expression is factored, students should be encouraged to inspect the factors to see if any of the factors is factorable. If so, students should select and apply an appropriate strategy to factor the given expression fully.
- In **Example 5**, algebraic reasoning (factoring) is applied to identify a key feature of the graph of a quadratic function. Students should see the connection between the algebraic properties of the equation of a quadratic function and the geometric properties of its graph.

Communicate Your Understanding

- Have students discuss **questions C1 and C2** in pairs or in small groups. Then, have them write individual responses in their journal or notebook. Have algebra tiles available.
- Consider having students answer **question C3** using a think-pair-share strategy.
- **Question C4** could form the basis of a performance task. You may wish to have students create the flowcharts on chart paper and post them in the classroom. Student flowcharts could be assessed according to accuracy, clarity, and ease of use.
- You may wish to use **BLM 2–7 Section 2.4 Select and Apply Factoring Strategies** for additional skill consolidation.

Communicate Your Understanding Responses (page 105)

C1 **a)** An algebraic expression is a perfect square trinomial if it is written in the form $ax^2 + bx + c$ such that: a (coefficient for the x^2-term) and c (constant term) are positive perfect square numbers; b (coefficient for the x-term) is twice the product of the square roots of a and c; the sign before bx can stay as positive or negative in the linear factor.

b) Answers may vary. For example, $9x^2 - 30x + 25$ is a perfect square trinomial. $9x^2 - 30x + 25 = (3x - 5)^2$

c) Answers may vary. For example, $x^2 + 5x + 9$ is not a perfect square trinomial. $\sqrt{1} = 1$ and $\sqrt{9} = 3$. The coefficient 5 for the x-term is not equal to twice the product of the square roots of 1 (1) and 9 (3), i.e., $2 \times (1) \times (3) \neq 5$.

C2 **a)** A binomial in the variable x is a difference of squares if it can be written as $p^2x^2 - q^2$, where p and q are integers, and its factored form is: $p^2x^2 - q^2 = (px + q)(px - q)$. Or, it can be written as $p^2 - q^2x^2$, where p and q are integers, and its factored form is: $p^2 - q^2x^2 = (p + qx)(p - qx)$.

b) Answers may vary. For example, $9x^2 - 16$ and $16 - 9x^2$ are binomials that are differences of squares.
$9x^2 - 16 = (3x + 4)(3x - 4)$ and $9 - 16x^2 = (3 + 4x)(3 - 4x)$

c) Answers may vary. The following binomials are not differences of squares. $x^2 + 16$: The operation in between the two perfect square numbers must be minus. $9x - 4$: The variable must also be a perfect square such as x^2. $x^3 - 25$: The variable must also be a perfect square such as x^2, x^4, x^6, etc.

C3 **a)** Since the coefficient of the x^2-term in $x^2 - 2x - 35$ is 1, apply the techniques of decomposition. Find two integers that have a sum of -2 and a product of -35. The integers are -7 and 5. $x^2 - 2x - 35 = (x - 7)(x + 5)$.

b) You can recognize $y^2 - 25$ as a difference of squares. $\sqrt{y^2} = y$ and $\sqrt{25} = 5$. Write the expression as the sum and difference of the square roots of each term. $y^2 - 25 = (y + 5)(y - 5)$.

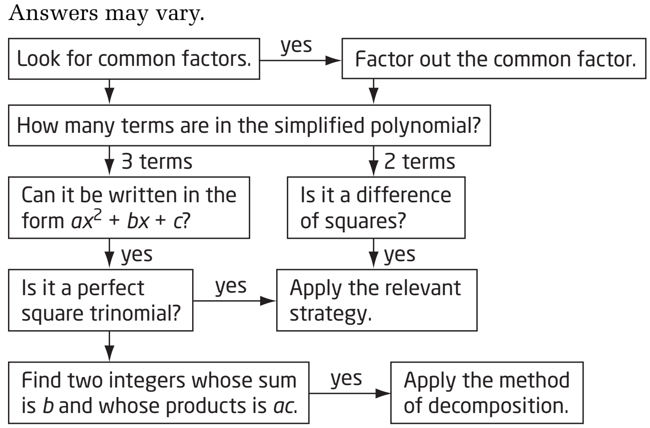
c) The binomial $5u^2 + 10u$ is not a difference of squares. Look for a common factor for the terms. The common factor is $5u$. Factor out the common factor. $5u^2 + 10u = 5u(u + 2)$.

d) To factor $k^2 + 8k + 16$, use one of two strategies. Find two integers that have a sum of 8 and a product of 16. The two integers are 4 and 4. $k^2 + 8k + 16 = (k + 4)(k + 4)$, or $(k + 4)^2$. Recognize the expression as a perfect square trinomial and apply the strategy for factoring such expressions. $\sqrt{16} = 4$. The value of the coefficient (8) for the k-term is twice the square root (4) of the value of the constant term (16). $k^2 + 8k + 16 = (x + 4)^2$

e) Apply the techniques of decomposition when $a \neq 1$ to factor $2w^2 + w - 6$. Find two integers with a sum of 1 and a product of -12. The integers are 4 and -3.

$$\begin{aligned} 2w^2 + w - 6 &= 2w^2 + 4w - 3w - 6 \\ &= 2w(w + 2) - 3(w + 2) \\ &= (w + 2)(2w - 3) \end{aligned}$$

C4 **a)** Answers may vary.



b) For $2x^2 + 4x + 10$, look for common factors and factor out the common factor 2 so that $2x^2 + 4x + 10 = 2(x^2 + 2x + 5)$. For $m^2 - 49$, check if $m^2 - 49$ is a difference of squares. Write $m^2 - 49 = m^2 - 7^2$. For $36d^2 - 12d + 1$, check if it is a perfect square trinomial by checking "Is $b = 2\sqrt{a} \times \sqrt{c}$?" or "Is $12 = 2 \times 6 \times 1$?"

Common Errors

- Some students may have trouble deciding what factoring strategy to apply in a given situation.

R_x Have students review their factoring flowcharts with you and provide them suggestions for improvement, as needed. Encourage students to use the flowchart to help them select a strategy.

- Some students may not be able to factor expressions fully.

R_x Have students recall the golden rule for factoring: Always look for common factors first! Once they have factored the expression, check if any of the factors can be factored further.

Accommodations

Visual–have students use algebra tiles to illustrate the factors

Motor–have students use CAS to explore patterns

Memory–have students use index cards for the two patterns described for a perfect square trinomial and a difference of squares

Student Success

- Encourage students to keep a copy of the flowcharts they have created to help them identify which strategies to try when factoring polynomials. These charts will be very useful when they work on the next section.

Practise, Connect and Apply, Extend

- For **questions 1 to 3**, some students may have difficulty remembering the patterns for perfect square trinomials and difference of squares. Have them write out these patterns on index cards and use these as a guide when factoring.
- For **questions 4 and 9**, students should be encouraged to use their factoring flowchart created for **question C4** in Communicate Your Understanding.
- **Question 5** illustrates why it is wise to look for common factors first, as the factored polynomial has smaller coefficients to deal with.
- **Question 6** illustrates the power of applying the pattern for a perfect square trinomial where the coefficients are relatively large.
- In **questions 7 and 8**, students are challenged to make connections between algebraic and features of a quadratic function and the geometric features of its graph. Some students may benefit from sketching the graphs first.
- CAS is a powerful tool to explore patterns such as those in **question 10**.
- In **questions 11, 12, and 19**, students are challenged to make connections between the algebraic features of a quadratic function and the geometric features of its graph. Some students may benefit from using technology to explore possible solutions.
- Students should see from their work on **questions 14 and 15** why it is not possible to factor a sum of squares.
- For **question 17**, the successful student will likely have a strong grasp of the inverse relationship between factoring and expanding.
- **Question 18** requires some sophisticated algebraic reasoning. By factoring the given equation, a particular case, students will see that the solutions are 2 and $\frac{1}{2}$. To explore the general case, students should let the two solutions be k and $\frac{1}{k}$, and work backward to see how a, b, and c are related to k.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12–14, 16–19
Reasoning and Proving	5–8, 10–14, 18, 19
Reflecting	5, 6, 10, 11, 16–19
Selecting Tools and Computational Strategies	1–6, 9
Connecting	n/a
Representing	7, 8, 13–17
Communicating	5, 6, 10, 11, 13, 18, 19