2.5 Solve Quadratic Equations by Factoring

Student Text Pages
108–113

Suggested Timing
75 min

Materials and Technology Tools
- graph paper
- algebra tiles
- calculators or computers with Computer Algebra System (CAS)

Related Resources
- BLM 2–8 Section 2.5 Solve Quadratic Equations by Factoring
- BLM 2–9 Section 2.5 Achievement Check Rubric

Teaching Suggestions

- The purpose of this section is for students to apply the factoring strategies they have acquired throughout the chapter to solve quadratic equations that model real-life contextual situations. The focus is on application of skills. Students should have sufficient mastery of the algebraic methods introduced earlier in the chapter, leaving the rest of the course time for higher order thinking required to solve the exercise problems.
- Students should by now realize that not all quadratic expressions are factorable. In the next chapter, they will learn how to solve quadratic equations that are not factorable.

Investigate

- In this Investigate, projectile motion is modelled by a factorable quadratic equation. Students discover the meaning of the mathematical roots (t-intercepts) as they relate to the physical nature of the problem.
- It should also be reinforced that sometimes it is necessary to place limits on a mathematical model when parts of its domain do not apply to the situation it is describing. For example, negative t-intercepts have mathematical meaning but no physical meaning.

Investigate Responses (pages 108-109)

1. $h(t) = -5t(t - 2)$

2. a) The zeros of the function are 0 and 2. These are the times when the ball is at a height of 0 m. Start measuring time at $t = 0$ when the ball is thrown. The second zero at $t = 2$ identifies the time when the ball lands.

b) The t-coordinate of the vertex is located halfway between the zeros.

$$t = \frac{0 + 2}{2}$$
$$= 1$$

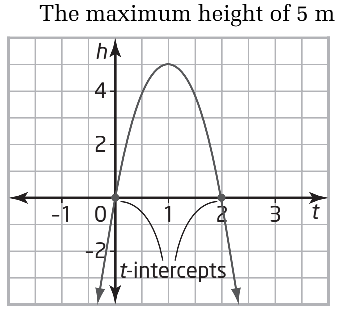
Substitute this time value into either form of the function to find the height at this time.

$$h(1) = -5(1)^2 + 10(1)$$
$$= 5$$

The vertex of the parabola is at (1, 5). It is a maximum because the value of a in the quadratic function is negative and the parabola opens downward.

The maximum height of 5 m is reached at a time of 1 s.

3.



4. a) When $t = 0$,

$$h(0) = -5(0)^2 + 10(0)$$
$$= 0$$

$t = 0$ is a zero of the function.

b) When $t = 2$,
$h(2) = -5(2)^2 + 10(2)$
$= -20 + 20 = 0$
$t = 2$ is a zero of the function.

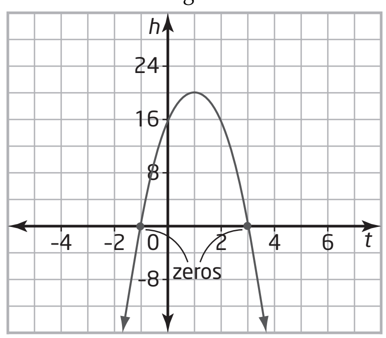
5. **a)** $h(t) = -5(t + 1)(t - 3)$

b) The zeros of the function are the times that make the values of the factors equal to 0. The zeros are −1 and 3.

c) The variable t represents the number of seconds that have passed since the ball is thrown. The value $t = 3$ represents the time when the ball lands.

d) The of $t = -1$ does not make sense for this situation because the starting time of the motion is at time $t = 0$. The function of the height is defined for positive values of time when the ball is actually in the air.

e) The t-coordinate of the vertex is located halfway between the zeros.
$t = \frac{-1 + 3}{2}$
$= 1$
We substitute this time value into either form of the function to find the height at this time.
$h(1) = -5(1)^2 + 10(1) + 15$
$= 20$
The vertex of this function is (1, 20). It is a maximum. The ball reaches its maximum height of 20 m at a time of 1 s after it is thrown.

6.


Examples

- The focus of **Example 1** is on the processes of determining algebraically the roots of a quadratic equation in factored form. The method is based on the zero property: if $a \times b = 0$, then at least one of a or b must equal zero. This typically gives rise to/up to two solutions, which are directly related to the horizontal intercepts of the corresponding quadratic function.
- **Example 2** illustrates that when quadratic equations are presented in expanded form, they must be factored first in order to solve for the roots. This is a good time to review and model the process of selecting factoring strategies.
- **Example 3** provides the opportunity to apply many of the key skills of the chapter in the context of a real-life problem. Students should be encouraged to restrict the domain and range of such a model, as befitting the nature of the situation being described, and to reject inadmissible solutions.

Communicate Your Understanding

- Consider having students discuss these questions in pairs or small groups. Then, have them write individual responses in their journal or notebook.
- You may wish to use **BLM 2–8 Section 2.5 Solve Quadratic Equations by Factoring** for remediation or extra practice.

Communicate Your Understanding Responses (page 111)

C1 **a)** If a product has a value of 0, then at least one of its factors must be equal to zero. If this were not true, then both of the factors would be non-zero. The product of two non-zero numbers would also be non-zero, which contradicts our assumption that the product is zero.

b) Answers may vary. For example, $4b = 0$. Since 4 is non-zero, divide both sides of the equation by 4 to obtain the result $b = 0$. For $(-2)a = 0$, -2 is non-zero. Divide both sides of the equation by -2 to obtain the result $a = 0$.

C2 If the quadratic function is factorable, factor the quadratic expression using an appropriate strategy, then set each factor equal to zero, and solve for the x-intercepts of the function. If the quadratic function is not factorable, graph the function and approximate the values of the x-intercepts of the graph, which are zeros of the function.

C3 When you model a real-life situation using a quadratic function, there is usually a restricted interval of values for which the function applies to the situation. If you solve the corresponding quadratic equation and find a solution that is outside the interval of meaningful values for the variable, the solution is inadmissible.

A typical situation would be the height of a stone in **Example 3**. The function that models the height is based on the time that begins at $t = 0$. Any solution for time that is a negative value would be inadmissible.

Practise, Connect and Apply, Extend

- Consider having students work in pairs or small groups so that they can discuss the problems before transcribing individual solutions. Some students may also benefit from exploring these problems using graphing technology.
- For **question 1**, initially, students may benefit from explicitly writing and solving two distinct linear equations in order to solve for the roots. Eventually, they should be able to do this mentally.
- In **question 3**, students will need to recall their factoring strategies. This is an additional opportunity to consolidate factoring skills.
- For **questions 4 and 5**, suggest that working backward as a viable strategy.
- Students will discover in **questions 6 and 7** that at times there could be a more efficient approach. It is important to understand that if they move the -49 to the other side and take the square root of both sides, then they must consider both positive and negative square roots in order to identify the two roots of the equation.
- In **questions 9 and 10**, there is a strong emphasis on connecting the mathematics to the situations being modelled. Students are challenged to demonstrate deep understanding of how the algebra connects to the key features of a graph, what these imply about the real situation, and also how the mathematical model should be restricted, as needed. **Question 10** is an Achievement Check question. Provide students with **BLM 2–9 Section 2.5 Achievement Check Rubric** to help them understand what is expected.
- **Question 11** is a good introduction to the material of Chapter 3.

Common Errors

- Some students may get confused with the concepts of roots and factors.

R_x Have students refer to the tile models to reinforce that a polynomial is an expression that can be written as a product of two factors. When such an expression is set equal to zero, then factoring allows a process for identifying solutions to the resulting equation.

- Some students may fail to reject inadmissible solutions.

R_x Have students reflect on the answers they obtain and consider whether each answer has real meaning within the context of the problem.

Ongoing Assessment

- **Question 10** is an Achievement Check question. Use **BLM 2-9 Section 2.5 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Motor–have students use technology for graphing and use CAS to explore patterns

Memory–have students use their flowcharts created in Section 2.4

Achievement Check Sample Solution (page 113, question 10)

a) The tree would be 12 m tall if Biff is at the top. This is found by substituting $t = 0$ into the quadratic function $h(t) = -2t^2 + 2t + 12$.

$h(0) = -2(0)^2 + 2(0) + 12$
$= 12$

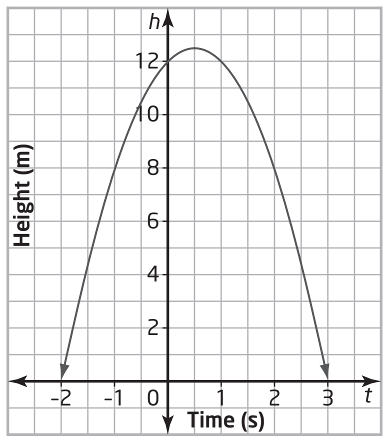
b) Find the t-intercepts of the function or the roots of the equation $0 = -2t^2 + 2t + 12$.

$0 = -2t^2 + 2t + 12$
$0 = -2(t^2 - t - 6)$
$0 = -2(t - 3)(t + 2)$
$t = 3$ or $t = -2$

When $h(t) = 0$ (ground level), t is -2 or 3. The value of -2 for time t does not make sense in this scenario. So, it takes a branch 3 seconds to reach Rocco.

c) The vertex is at (0.5, 12.5). The vertex tells you that the maximum height of the branches is 12.5 m and this occurs at a time equal to 0.5 s.

d)



The zeros are at $t = -2$ and $t = 3$. The first zero is inadmissible as it gives a negative value of time.

e) Biff is tossing the branches up into the air. At the beginning, he is sitting at a height of 12 m up the tree and the maximum height reached is slightly higher than 12 m. The vertex of the graph (the maximum height) occurs after the starting time.

f) There may be air resistance acting on the branches against the force of gravity. This would slow their descent somewhat like the effect of a parachute.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	8–11
Reasoning and Proving	1, 2, 4, 5, 7–11
Reflecting	3, 5, 9
Selecting Tools and Computational Strategies	1–3, 5–7, 9, 10
Connecting	3, 8, 9
Representing	3, 4, 6, 10, 11
Communicating	3–5, 7–11