3.1 Complete the Square

Student Text Pages
124-134

Suggested Timing
80-160 min

Materials and Technology Tools
- algebra tiles
- graphing calculators
- grid paper

Related Resources
- BLM G–1 Grid Paper
- BLM 3–3 Section 3.1 Complete the Square
- BLM 3–4 Section 3.1 Achievement Check Rubric

Teaching Suggestions

- Connect the process of completing the square to finding the maxima/minima of a quadratic relation and its applications to everyday situations (i.e. the path of a ball, arches in architecture, etc.).
- You may wish to complete this section over two days. On the first day, focus on the mastery of the process of completing the square with and without fractions. On the second day, apply the process to solve problems related to business or science.

Investigate

- Have students work through the Investigate in small groups. Distribute algebra tiles to each group. Have students model and explain the solutions to **steps 3 and 4** to each other.
- Ask students to explain how adding or subtracting unit squares creates a perfect square. Then ask students how they can use algebra to complete the square.
- Students may take turns creating their own questions and having another group member solve it. Alternatively, each student in the group could complete one step in the solution.

Investigate Responses (pages 124-125)

1. a) Arrange one x^2-tile, two x-tiles and one unit tile into a rectangle with side lengths $x + 1$.

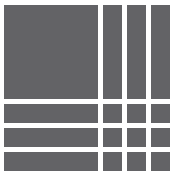
$x^2 + 2x + 1 = (x + 1)^2$

b) Arrange one x^2-tile, four x-tiles and four unit tiles into a rectangle with side lengths $x + 2$.

$x^2 + 4x + 4 = (x + 2)^2$

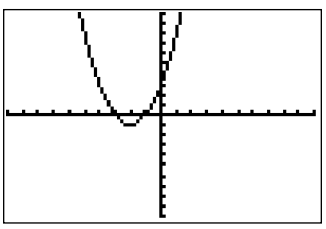
c) Arrange one x^2-tile, six x-tiles and nine unit tiles into a rectangle with side lengths $x + 3$.

$x^2 + 6x + 9 = (x + 3)^2$



2. a) Arrange one x^2-tile, four x-tiles and three unit tiles into a rectangular arrangement. One unit tile is needed to complete the rectangle.

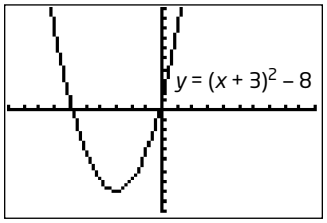
c) The function $y = x^2 + 4x + 3$ is equivalent to $y = (x + 2)^2 - 1$ when the second function is expanded and simplified. Both functions yield the same graph.



3. a) Arrange one x^2-tile, six x-tiles and one unit tile into a rectangular arrangement. Add eight more unit tiles to the arrangement to complete the rectangle. Then add eight negative unit tiles to the right side to keep the model of the equation balanced and equivalent to the original.

$y = x^2 + 6x + 1$
$= (x + 3)^2 - 8$

b) The function $y = x^2 + 6x + 1$ is equivalent to the function $y = (x + 3)^2 - 8$. This can be verified graphically.



4. Explanations and diagrams may vary. See the answer to step 3, part a) for an illustrative example. To write a function in standard form to a function in vertex form, express the first two terms as a trinomial that is a square binomial. Subtract the last term of the squared binomial from the original expression and simplify.

For example,

$y = x^2 + 6x + 1$
$= (x^2 + 6x + 9) - 9 + 1$
$= (x + 3)^2 - 8$

Examples

- If you complete this section over two days, you may wish to work through **Example 4** on the second day.

Communicate Your Understanding

- Have students answer these questions on their own. You may wish to have students use their responses as journal entries. Discuss the answers as a class before moving on to the exercises.
- Ensure students understand that the sign on a determines if the function has a maximum or a minimum.
- You may wish to use **BLM 3–3 Section 3.1 Complete the Square** for remediation or extra practice.

Communicate Your Understanding Responses (page 131)

C1 First I would remove a common factor of 2 from the first two terms.

$y = 2x^2 + 12x + 7$
$y = 2(x^2 + 6x) + 7$

Next, I would complete the square for the factored expression. I would do this by adding the square of half of 6, $\left(\frac{6}{2}\right)^2 = 9$. I would also subtract 9 times the common factor 2, or 18 from the expression.

$y = 2(x^2 + 6x + 9) - 18 + 7$

Finally, I would simplify the expression.

$y = 2(x + 3)^2 - 11$

C2 If $a > 0$, then the function has a minimum. If $a < 0$, then the function has a maximum. Since $a = 2$, the function has a maximum.

C3 Completing the square would be useful when trying to determine a function to model the surface area of a square garden or parking lot.

Common Errors

- Some students may forget to multiply the last term inside the brackets by the term outside the brackets.

R_x Have students draw arrows to represent the distributive property and work slowly through the Examples and exercises.

Ongoing Assessment

- **Question 19** is an Achievement Check question. Use **BLM 3–4 Section 3.1 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Visual–encourage students to use algebra tiles as they work through the algebraic process of completing the square

Language–encourage students to verbally describe the steps they follow as they complete the square

Motor/Memory–give a quadratic function in standard form. Then, provide each step in the process of completing the square on randomly ordered slips of paper. Encourage students to arrange the slips of paper in the correct order.

Student Success

- Link this topic to geometry. Have students compare the algebraic process of completing the square to the process they follow to complete the square using algebra tiles.

Practise, Connect and Apply, Extend

- The Practise questions provide a good opportunity for skill mastery and algebraic manipulation with and without fractions.
- You may wish to have copies of **BLM G–1 Grid Paper** available for **questions 11, 13, and 14**.
- The Connect and Apply questions put into context the usefulness of the procedure of completing the square. Encourage students to draw a sketch or use appropriate technology to visualize the situation.
- Students who have difficulty with **questions 16 and 17** should work through Example 4 and ensure they fully understand the solution.
- **Question 18** relates to the Chapter Problem. You may wish to have students flag their answers to this question for use in the Chapter Problem Wrap-Up.
- **Question 19** is an Achievement Check question. Provide students with **BLM 3–4 Section 3.1 Achievement Check Rubric** to help them understand what is expected.
- To complete **question 21**, students must have a good understanding of the process of completing the square.

Achievement Check Sample Solution (page 134, question 19)

a) Let x be the chosen number: $x^2 - 8x + 35$

Complete the square to find the minimum result and the value of x at the minimum.

$x^2 - 8x + 16 - 16 + 35 = (x - 4)^2 + 19$

The minimum result of 19 occurs when 4 is the chosen number.

b) Let x be the chosen number: $375 - 10x - x^2$

Rewrite the expression so the terms appear in descending powers of x. Then complete the square.

$$\begin{aligned} -x^2 - 10x + 375 &= -(x^2 + 10x) + 375 \\ &= -(x^2 + 10x + 25) + 25 + 375 \\ &= -(x + 5)^2 + 400 \end{aligned}$$

The maximum result of 400 when -5 is the chosen number.

Literacy Connections

- Have students describe how to complete the square to write a quadratic equation of the form $y = x^2 + bx + c$ in vertex form.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12–20
Reasoning and Proving	2, 5, 8, 10, 18
Reflecting	15–21
Selecting Tools and Computational Strategies	3, 4, 6, 7, 9, 12–20
Connecting	12–19
Representing	11, 13, 14, 16, 18, 19, 21
Communicating	18