3.2 The Quadratic Formula

Student Text Pages
135–144

Suggested Timing
80 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM A–8 Application General Scoring Rubric
- BLM 3–5 Section 3.2 The Quadratic Formula

Teaching Suggestions

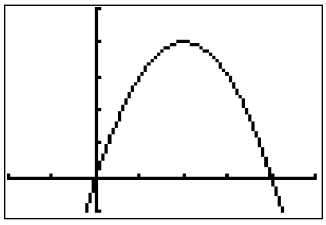
- Explain that the quadratic formula is an alternative way to solve a quadratic equation and that the quadratic formula is derived by completing the square on the general case, $y = ax^2 + bx + c$.
- Ensure students understand why there are between zero and two roots or solutions to a quadratic equation. Discuss the difference between approximate and exact roots and rounding errors.
- Some students may benefit from explicitly identifying the values of a, b, and c before they substitute into the quadratic formula.
- The use of appropriate technology can help illustrate the connection between the roots of an equation and the x-intercepts of the graph.

Investigate

- Distribute copies of **BLM G–1 Grid Paper**.
- Have students work though the Investigate in pairs. Students should fully explain each step of their solution.
- Have each student complete **step 8** individually and have their partner check their work.

Investigate Responses (page 135)

1. The x-intercepts are 0 and 4.



2. $-5x^2 + 20x = 0$
$-5x(x - 4) = 0$
$-5x = 0 \qquad x - 4 = 0$
$x = 0 \qquad x = 4$

3. The x-intercepts and the solutions to the equation are the same.

4. $y = -5x^2 + 20x$
$= -5(x^2 - 4x)$
$= -5[(x^2 - 4x + 4) - 4]$
$= -5(x - 2)^2 + (-5)(-4)$
$= -5(x - 2)^2 + 20$

5. $-5(x - 2)^2 + 20 = 0$
$-5(x - 2)^2 + 20 - 20 = 0 - 20$
$-5(x - 2)^2 = -20$
$\frac{-5(x - 2)^2}{-5} = \frac{-20}{-5}$
$(x - 2)^2 = 4$
$\sqrt{(x-2)^2} = \sqrt{4}$
$x - 2 = \pm 2$
$x - 2 = 2 \qquad x - 2 = -2$
$x = 4 \qquad x = 0$

6. The solutions in step 5 are the same as the x-intercepts in step 1 and the solutions in step 2.

7. This method should work for any values of a and c where c is divisible by a. The equation must first be expressed in vertex form before solving for x using the method in step 5.

8.

$$\begin{aligned} 5x^2 + 10x - 4 &= 0 \\ 5(x^2 + 2x) - 4 &= 0 \\ [5(x^2 + 2x + 1) - 1] - 4 &= 0 \\ 5(x + 1)^2 + 5(-1) - 4 &= 0 \\ 5(x + 1)^2 - 5 - 4 &= 0 \\ 5(x + 1)^2 - 9 &= 0 \\ 5(x + 1)^2 &= 9 \\ (x + 1)^2 &= \frac{9}{5} \\ x + 1 &= \pm\frac{3}{\sqrt{5}} \\ x &= \pm\frac{3}{\sqrt{5}} - 1 \end{aligned}$$

$x = \frac{3}{\sqrt{5}} - 1 \quad x = -\frac{3}{\sqrt{5}} - 1$

$$\begin{aligned} 3x^2 + 12x + 1 &= 0 \\ 3(x^2 + 4x) + 1 &= 0 \\ [3(x^2 + 4x + 4) - 4] + 1 &= 0 \\ 3(x + 2)^2 + 3(-4) + 1 &= 0 \\ 3(x + 2)^2 + (-12) + 1 &= 0 \\ 3(x + 2)^2 - 11 &= 0 \\ 3(x + 2)^2 &= 11 \\ (x + 2)^2 &= \frac{11}{3} \\ x + 2 &= \pm\sqrt{\frac{11}{3}} \\ x &= \pm\sqrt{\frac{11}{3}} - 2 \end{aligned}$$

$x = \sqrt{\frac{11}{3}} - 2 \quad x = -\sqrt{\frac{11}{3}} - 2$

Examples

- **Example 1 and 2** illustrate the mechanical workings of the formula.
- **Examples 3 and 4** show more practical applications of the formula. Students may need help determining what the question is asking them to do or what to solve for.

Communicate Your Understanding

- Have students work in pairs to answer each question. Discuss the solutions as a class.
- **Question C1** asks students to demonstrate the skills necessary to use the quadratic formula well by illustrating typical errors. Focus on the errors made and how to correct them.
- You may wish to use **BLM 3–5 Section 3.2 The Quadratic Formula** for remediation or extra practice.

Communicate Your Understanding Responses (page 142)

C1 The first error occurs in line 1. Since $b = 5$, the first term in the numerator should be -5.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4(-4)(1)}}{2(-4)} \\ &= \frac{-5 \pm \sqrt{25 + 16}}{-8} \\ &= \frac{-5 \pm \sqrt{41}}{-8} \end{aligned}$$

So, $x = \frac{-5 + \sqrt{41}}{-8}$ or $x = \frac{-5 - \sqrt{41}}{-8}$.

C2 Both equations can be solved using the quadratic formula, but a more efficient method is to factor each equation, set each factor equal to 0, then solve for x.

$$\begin{aligned} 2x^2 + 4x &= 0 \\ 2x(x + 2) &= 0 \\ 2x = 0 \quad x + 2 &= 0 \\ x = 0 \quad x &= -2 \end{aligned}$$

$$\begin{aligned} 2x^2 - 4 &= 0 \\ 2(x^2 - 2) &= 0 \\ x^2 - 2 &= 0 \\ x^2 &= 2 \\ x = \sqrt{2} \quad x &= -\sqrt{2} \end{aligned}$$

Common Errors

- Some students may neglect to consider the two operations, addition and subtraction, for the $\sqrt{b^2 - 4ac}$ component of the quadratic formula.

R$_x$ Have students write the equation as two separate roots (positive and negative) once the square root is evaluated.

Ongoing Assessment

- You may wish to use **BLM A–8 Application General Scoring Rubric** to assess students' responses to **question 5**.

Accommodations

Gifted and Enrichment–encourage students to derive the quadratic formula by completing the square for the general equation $y = ax^2 + bx + c$

Visual–encourage students to highlight *a*, *b*, and *c* in different colours

Memory–display a poster of the quadratic formula in the classroom

Practise, Connect and Apply, Extend

- **Questions 1 to 4** focus on mechanical skills. This is the main expectation. Proficiency with the arithmetic is critical for the remainder of the chapter. Remind students that the square root of a positive value has two possible solutions.
- The Connect and Apply questions follow the worked Examples closely and build on skills necessary for the remainder of the chapter. For **question 13**, students may wish to cut the corners off a sheet of paper and fold the paper into a box to better visualize the problem.
- The Extend questions are meant to further develop the importance of critical thinking and problem solving skills in mathematics. For **question 17**, you may need to remind students of the definition of reciprocal.

Literacy Connections

- Have students explain how the quadratic formula is related to the process of completing the square.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–10, 12–14
Reasoning and Proving	8, 9, 12, 13
Reflecting	14
Selecting Tools and Computational Strategies	4–7, 9–14
Connecting	5–9, 12, 13,
Representing	8–10, 13
Communicating	8, 14