3.3 Real Roots of Quadratic Equations

Student Text Pages
145–152

Suggested Timing
80 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM A–9 Communication General Scoring Rubric
- BLM 3–6 Section 3.3 Real Roots of Quadratic Equations

Teaching Suggestions

- You may wish to review question 3 in Prerequisite Skills to refresh students' skills with substitution and evaluation of expressions.
- Lead a class discussion on the relevance of the discriminant. Stress that the discriminant does not give the actual roots of the equation, just the number of roots.

Investigate

- Distribute copies of **–1** .
- Have students work through the Investigate in small groups.
- Ensure students make connections between the number of x-intercepts the relation has, the roots of the corresponding equation, and the value of the discriminant. Once students have completed the Investigate, have a class discussion about the difference between an equation, a relation, and the discriminant.

Investigate Responses (pages 145-146)

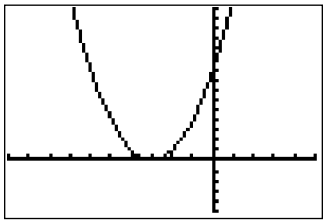
1. a) $y = x^2 - 1$; $a = 1$, $b = 0$, $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{0 \pm \sqrt{0^2 - 4(1)(-1)}}{2(-1)}$$
$$= \frac{0 \pm \sqrt{4}}{-2}$$

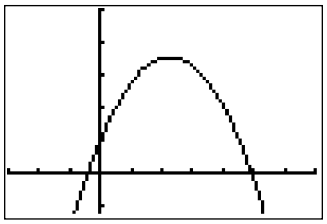
So, $x = 1$ or $x = -1$.

The roots of the quadratic equation and the x-intercepts of the graph are equal.

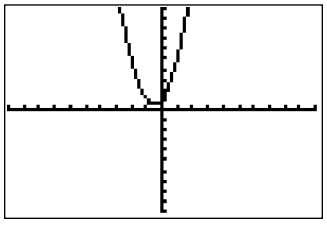
2. a) **i)** The x-intercept is -3. There is only one x-intercept.



ii) The x-intercepts are -0.5 and 4. There are two x-intercepts.



iii) There are no x-intercepts.



b) i) $0 = x^2 + 6x + 9$; $a = 1$, $b = 6$, $c = 9$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 - 36}}{2}$$
$$= \frac{-6}{-2}$$
$$= 3$$

So, $x = 3$.

ii) $0 = 2x^2 + 9x + 4$; $a = 2$, $b = 9$, $c = 4$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(4)}}{2(2)}$$
$$= \frac{-9 \pm \sqrt{49}}{4}$$
$$= \frac{-9 \pm 7}{4}$$

So, $x = -4$ or $x = -\frac{1}{2}$.

iii) $0 = 2x^2 + 2x + 1$; $a = 2$, $b = 2$, $c = 1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{-4}}{4}$$

The square root of a negative number is not defined in the real number system. So, there are no x-intercepts.

3. $D = b^2 - 4ac$

i) $0 = x^2 + 6x + 9$; $a = 1$, $b = 6$, $c = 9$

$D = 6^2 - 4(1)(9)$
$= 36 - 36$
$= 0$

ii) $0 = 2x^2 + 9x + 4$; $a = 2$, $b = 9$, $c = 4$

$D = 9^2 - 4(2)(4)$
$= 81 - 32$
$= 49$

iii) $0 = 2x^2 + 2x + 1$; $a = 2$, $b = 2$, $c = 1$

$D = 2^2 - 4(2)(1)$
$= 4 - 8$
$= -4$

4. When $D = 0$, there is only one x-intercept. When $D > 0$, there are two x-intercepts. When $D < 0$, there are no x-intercepts.

5. Sketches may vary.

Value of Discriminant	Number of Solutions	Number of x-intercepts	Graph
$b^2 - 4ac = 0$	1	1	See graph for 2. a) i)
$b^2 - 4ac > 0$	2	2	See graph for 2. a) ii)
$b^2 - 4ac < 0$	0	0	See graph for 2. a) iii)

Examples

- As you work through **1**, ask students how they would decide whether to solve a quadratic equation by graphing, by factoring, or using the quadratic formula.
- For **2**, ensure students understand that the x-intercepts of a parabola are the same as the roots of the related quadratic equation.

Common Errors

- Some students may make arithmetic errors, particularly with the potential for a double negative. For example, $b^2 - 4ac$ if a or c is negative.

R_x Have students work slowly through the Exercises and practise the multiplication of several positive and negative integers.

Ongoing Assessment

- You may wish to use **BLM A–9 Communication General Scoring Rubric** to assess students' responses to **question 10 or 11**.

Accommodations

Perceptual–encourage students to sketch an example of a quadratic function for each of the three cases for the discriminant: $D < 0$, $D = 0$, and $D > 0$.

Language–ask students to explain verbally how the discriminant is related to the quadratic formula

Memory–encourage students to use cue cards to remember the discriminant and the quadratic formula

Communicate Your Understanding

- **1** provides a quick review of the concepts.
- **2** is an excellent placemat activity outlining the advantages and disadvantages of each method of finding roots.
- You may wish to use **3** **6** **3.3** for remediation or extra practice.

Communicate Your Understanding Responses (page 150)

C1 $D = 6$, ; $D = -17$, ; $D = \frac{2}{3}$, ; $D = 0$,

C2

Method	Advantages	Disadvantages	Does It Always Work?
Graphing	displays roots	may not be accurate	yes
Factoring	quick and easy	equation not always factorable	no
Quadratic Formula	numerical accuracy	time-consuming	yes

Practise, Connect and Apply, Extend

- For **8**, students should make the connection between the value of the discriminant and the graph.
- **13** allows students to apply the concept of the discriminant to a real world context.
- **14** focuses on families of quadratic functions and ties in the concept of the discriminant.
- The Extend questions, in particular **15** **16**, allow students to delve deeper into the theoretical aspects of quadratic relations and the discriminant.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–10, 12–15, 17–19
Reasoning and Proving	3, 14, 16
Reflecting	12–14, 17–19
Selecting Tools and Computational Strategies	1, 2, 4–15, 17–19
Connecting	5, 6, 8–10, 12, 14, 15, 17–19
Representing	7, 8, 10, 13
Communicating	14, 16