3.4 Multiple Forms of Quadratic Functions

Student Text Pages
153–163

Suggested Timing
160 min

Materials and Technology Tools
- graphing calculators
- grid paper

Related Resources
- BLM G–1 Grid Paper
- BLM 3–7 Section 3.4 Multiple Forms of Quadratic Functions
- BLM 3–8 Section 3.4 Achievement Check Rubric

Teaching Suggestions

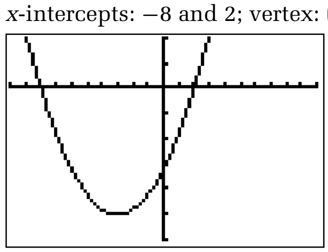
- You may need to spend two days on this section.
- On Day 1, use the Investigate to review completing the square and the quadratic formula. Lead the class in a discussion about step 7 of the Investigate. Work through Examples 1, 2, and 4 and then assign the Practice questions.
- On Day 2, review the main points from the previous day and continue to make connections with the intervals of increase/decrease and positive/negative. Then, work through Example 3. Ensure students fully understand and are able to explain the Key Concepts.

Investigate

- Work in small groups and discuss the relevant concepts and mathematics needed to solve each step.

Investigate Responses (page 153)

1. a) x-intercepts: -8 and 2; vertex: $(-3, -25)$; axis of symmetry: $x = -3$



b) By determining the midpoint or average of the x-intercepts, you can determine the x-coordinate of the vertex of a parabola.

2. a) $0 = x^2 + 6x - 16$

$0 = (x + 8)(x - 2)$

$x + 8 = 0 \qquad x - 2 = 0$

$x = -8 \qquad x = 2$

The x-intercepts are the same as those found in step 1 a).

b) The x-coordinate of the vertex is the midpoint between the x-intercepts which is $x = -3$. Substitute $x = -3$ into the equation to find the y-coordinate of the vertex.

$y = x^2 + 6x - 16$
$= (-3)^2 + 6(-3) - 16$
$= 9 - 18 - 16$
$= -25$

The coordinates of the vertex are $(-3, -25)$.

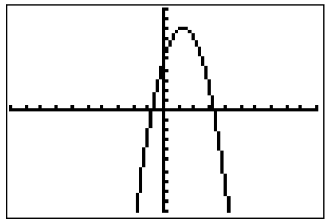
c) The axis of symmetry is a vertical line through the vertex. Therefore, $x = -3$ is the equation of the axis of symmetry.

3. a) $0 = x^2 + 6x - 16$
$0 = (x^2 + 6x + 9) - 9 - 16$
$0 = (x + 3)^2 - 25$
The coordinates of the vertex are $(-3, -25)$.

b) The coordinates from part a) and the coordinates from step 2 part b) are the same.

c) By completing the square of the quadratic function, the vertex form of a function is generated. It readily shows the coordinates of the vertex. It is a more direct method of finding the coordinates of the vertex.

4. a) x-intercepts: approximately -0.8 and 3.3; vertex: approximately $(1.2, 8)$; axis of symmetry: $x \doteq 1.2$



b) The expression is not factorable because the x-intercepts are non-terminating, non-repeating decimals. Also there is no pair of terminating decimals whose product is $a \times c$, or -10 and whose sum is b, or 5.

5. a) $-2x^2 + 5x + 5 = 0$; $a = -2$, $b = 5$, $c = 5$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-2)(5)}}{2(-2)}$$

$$= \frac{-5 \pm \sqrt{65}}{-4}$$

So, $x = \dfrac{5 + \sqrt{65}}{4}$ or $x = \dfrac{5 - \sqrt{65}}{4}$.

The x-intercepts are $x \doteq 3.3$ and $x \doteq -0.8$.

b) The x-coordinate of the vertex is the midpoint of the x-intercepts, $x \doteq 1.2$.

$$\begin{aligned} y &= -2x^2 + 5x + 5 \\ &\doteq -2(1.2)^2 + 5(1.2) + 5 \\ &\doteq -2.88 + 6 + 5 \\ &\doteq 7 \end{aligned}$$

vertex: approximately $(1.2, 8)$

6. a)

$$\begin{aligned} -2x^2 + 5x + 5 &= 0 \\ -2(x^2 - 2.5x) + 5 &= 0 \\ [-2(x^2 - 2.5x + 1.5625) - 1.5625] + 5 &= 0 \\ -2(x - 1.25)^2 + 3.125 + 5 &= 0 \\ -2(x - 1.25)^2 + 8.125 &= 0 \end{aligned}$$

vertex: $(1.25, 8.125)$

b) The coordinates of the vertex from step 5 part b) are approximately the same.

c) For the given quadratic function, completing the square to find the coordinates of the vertex is preferred because it is more accurate.

7. Explanations may vary. To find the coordinates of the vertex of a quadratic function in standard form, $y = ax^2 + bx + c$, given a quadratic expression of the form $ax^2 + bx + c$ that is factorable, I can graph the function, determine the x-intercepts from the graph, and determine the midpoint of the x-intercepts to find the x-coordinate of the vertex. By substituting the x-coordinate into the quadratic function, I can determine the y-coordinate of the vertex. I can also use the quadratic formula to determine the x-intercepts. Then I can find the midpoint of the x-intercepts to determine the x-coordinate of the vertex. By substituting the x-coordinate into function, the y-coordinate of the vertex can be determined. The quadratic formula method is easiest to use.

To find the coordinates of the vertex of a quadratic function in the standard form, $y = ax^2 + bx + c$, given that the quadratic expression $ax^2 + bx + c$ is not factorable, I can complete the square to obtain the vertex form of the function. The coordinates of the vertex can easily be found in the vertex form. The quadratic formula can also be used to determine the coordinates of the vertex as indicated above. Completing the square is the easier method.

Examples

- The problem in **Example 1** could also be solved using the quadratic formula. Encourage students who have difficulty factoring to use the quadratic formula to check their factoring.

Communicate Your Understanding

- The Communicate Your Understanding questions are subjective but some answers are more correct than others. That is the main point the students should be working towards.
- You may wish to use **BLM 3–7 Section 3.4 Multiple Forms of Quadratic Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 161)

C1 The intervals on which a function is positive or negative can be determined by analysing the behaviour of the function near the *x*-intercepts. The intervals for which a function is increasing or decreasing are determined by analysing the behaviour of the function near the vertex.

C2 Answers may vary. The vertex form of quadratic function is preferred because the vertex, direction of opening, axis of symmetry, and vertical intercept are readily given by the function.

Practise, Connect and Apply, Extend

- The Practise questions use the concept of completing the square to find the maxima/minima and/or the quadratic formula to find specific values at specific instances.
- Students are asked to sketch graphs for **questions 1 and 2**. Distribute copies of **BLM G–1 Grid Paper**.
- The Connect and Apply questions help consolidate the properties of quadratic functions. The questions add the notion of the respective intervals to the concept of completing the square.
- **Question 12** relates to the Chapter Problem. You may wish to have students flag their answers to this question for use in the Chapter Problem Wrap-Up.
- **Question 13** is the Achievement Check question. Provide students with **BLM 3–8 Section 3.4 Achievement Check Rubric** to help them understand what is expected.
- The Extend questions focus on finding equations given the properties. These may be difficult concept questions for some students.

Achievement Check Sample Solution (page 163, question 13)

a) The question could be solved graphically or algebraically.

$$\begin{aligned} h(t) &= -4.9t^2 + 529.2t - 50 \\ &= -4.9(t^2 - 108t + 2916 - 2916) - 50 \\ &= -4.9(t - 54)^2 + 14\,288.4 - 50 \\ &= -4.9(t - 54)^2 + 14\,238.4 \end{aligned}$$

The rocket releases the weather balloon when it reaches its maximum height of 14 238.4 m. This occurs 54 s after launch.

b) Find the earliest time at which the height of the rocket is 0 m.

$$h(t) = -4.t^2 + 529.2t - 50 = 0$$
$$-4.9(t - 54)^2 + 14\,238.4 = 0$$
$$(t - 54)^2 = 2905.795\,918$$
$$t - 54 = \pm 53.905\,434\,96$$
$$t \doteq 0.095 \text{ or } t \doteq 107.905$$

The second answer is when the rocket comes back down to Earth. The rocket is first above ground after 0.1 s. It is above ground (function is positive) on the time interval between 0.1 s and 107.9 s. It is below ground (function is negative) for the first tenth of a second.

c) The rocket falls back to Earth after 107.9 s.

d) The height is −50 m at $t = 0$, so the silo is 50 m below ground.

e) The rocket is climbing for the first 54 s ($0 < t < 54$) and then falling for the next 53.9 s ($54 < t < 107.9$).

Common Errors

- Some students may not be sure when a function has a maximum or minimum.

R_x Have students recognize that the *a*-value determines the direction of opening for a quadratic function. Encourage students to graph functions and relate the direction of opening of the parabola to the sign of *a*.

Ongoing Assessment

- **Question 13** is an Achievement Check. Use **BLM 3–8 Section 3.4 Achievement Check Rubric** to assess students' responses.

Accommodations

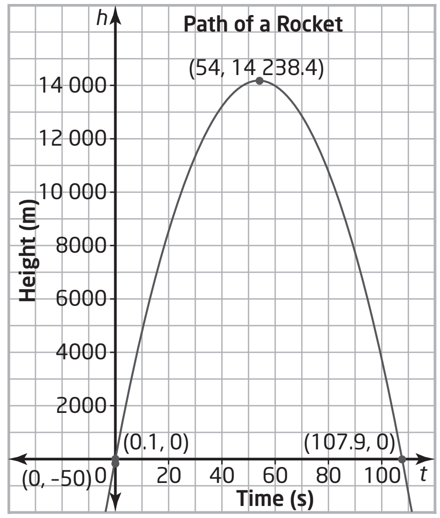
Gifted and Enrichment–provide students with several more complex quadratic functions and challenge them to write each equation in each of the different forms

Perceptual–encourage students to graph each function and relate the key features of the graph to the context of the problem

Language–allow students to give verbal responses to **question 10,** part e), **question 11,** part a), **and question 12,** part e)

f) The rocket takes 53.9 s to fall back to earth.

g)



Literacy Connections

- Have students write a journal entry describing the different forms of a quadratic function and the advantages and disadvantages of each form.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	13, 17
Reasoning and Proving	3, 9, 13–16
Reflecting	10, 14
Selecting Tools and Computational Strategies	1, 2, 4–8, 12, 13, 17
Connecting	13
Representing	n/a
Communicating	9–11, 13–16