4.1 Use Trigonometry to Find Lengths

Student Text Pages
186–191

Suggested Timing
80 min

Materials and Technology Tools
- drinking straws
- metre sticks or tape measures
- protractors
- string
- tape
- weights (e.g., paper clips)

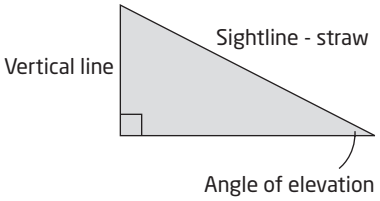
Related Resources
- BLM A–6 Knowledge and Understanding General Scoring Rubric
- BLM A–11 Group Work Assessment Recording Sheet
- BLM A–12 Group Work Assessment General Scoring Rubric
- BLM 4–3 Section 4.1 Use Trigonometry to Find Lengths

Teaching Suggestions

- Begin with a brief review of the primary trigonometric ratios (sine, cosine, and tangent),
- As a class, discuss ways to measure the angle of elevation of an inaccessible location.
- Ensure students understand the terms *angle of elevation*, *angle of inclination*, *angle of depression*, and *angle of declination*.

Investigate

- Have students work in pairs to complete the Investigate.
- Guide students through the process of building the clinometers. Ensure students tape the string to the centre of the base of the protractor.
- As a class, discuss how students might approach the task of measuring the angle of elevation, and how they plan to overcome the problem of holding the clinometer at eye level.
- Note that the angle being measured is the angle between the line of sight and the vertical. This angle must be between 0° and 90°. The angle of elevation is the angle between the line of sight and the horizontal. This is the complement of the measured angle.



- After students have measured their angles and returned to class, have them finish their calculations and the remaining questions in the Investigate.
- Once students have completed the Investigate, engage them in a class discussion of the results. Discuss the accuracy of their measures and ways to improve their findings.
- For an alternative Investigate, students can use *The Geometer's Sketchpad*® activity, 4.1 Investigate Trig ratios. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links.

Investigate Responses (pages 186-187)

2. To find the height of the object, I must measure the distance along the ground from my location to the bottom of the object being measured. I also need the measure of one acute angle, either the angle from vertical measured on the clinometer or the angle of elevation measured between the line of sight and the horizontal.

3. A clinometer is a measurement tool used to find the angle between a sight line and a plumb line, or vertical line. A homemade clinometer is not very precise. It is difficult to fix the string exactly at the centre of the protractor and the drinking straw may not be perfectly straight or mounted perfectly on the straight edge of the protractor. This might lead to measurement error even if the calculations are done correctly.

4. Answers may vary.

Common Errors

- Some students may use the incorrect trigonometric ratio when solving a problem.

R_x Remind students about SOH CAH TOA and how it can be used. Relative to the given angle, what do they know and what are they looking to find?

- Some students may have difficulties solving equations where the unknown is in the denominator.

R_x Draw arrows crossing over the equal sign to show the paths of the components when re-arranging the equation. Explain that this is why it is sometimes called cross-multiplying.

Ongoing Assessment

- Circulate as students work through the Investigate to see how well they understand the concepts. Use **BLM A–11 Group Work Assessment Recording Sheet** or **BLM A–12 Group Work Assessment General Scoring Rubric**.
- You may wish to use **BLM A–6 Knowledge and Understanding General Scoring Rubric** to assess students' responses to **question 3** of the Investigate.

5. Answers may vary. For example, Faisal and Baha measured an angle of 74° on their clinometer combined with a distance of 20 m from their sighting point to the bottom of their object. They used the tangent ratio to calculate the object's height.

$\frac{h}{20} = \tan 16°$

$h = 20(\tan 16°)$

$h \doteq 5.7$

The object's height is about 5.7 m.

6. Answers may vary.

7. Answers may vary.

Examples

- Emphasize to students the differences between **Examples 1 and 3** and **Example 2**. In Examples 1 and 3, the variable is in the numerator. In Example 2, the variable is in the denominator. Discuss why the solutions have different methods.
- Compare the use of the different trigonometric ratios in the examples. Remind students that the memory device, SOH CAH TOA, may be helpful in deciding which ratio is appropriate for a given problem (See Communicate Your Understanding, question C3).

Communicate Your Understanding

- Have students discuss their answers with their partners from the Investigate. Then have a class discussion to consolidate students' understanding.
- For **question C2**, students may find it helps to draw and label a diagram to represent the situation. Some students may label the diagram incorrectly because they do not understand the description of the situation.
- You may wish to use **BLM 4–3 Section 4.1 Use Trigonometry to Find Lengths** for remediation or extra practice.

Communicate Your Understanding Responses (page 189)

C1 Yes. The angle of depression is measured down from a horizontal and the angle of elevation is measured up from a horizontal. The horizontals are parallel, so by the parallel line theorem (alternate angles), the angle of elevation is equal to the angle of depression.

C2 From the perspective of the angle of inclination I know the length of the opposite side. To find the length of the hypotenuse of the triangle, I can use the sine ratio. This will give the length of the surface of the ramp.

$\sin 12° = \frac{1.8}{\text{surface}}$

$\text{surface} = \frac{1.8}{\sin 12°}$

$\text{surface} \doteq 8.7$

To find the length of the side of the triangle adjacent to the angle of inclination, I can use the tangent ratio. This will give the length of the base of the ramp.

$\tan 12° = \frac{1.8}{\text{base}}$

$\text{base} = \frac{1.8}{\tan 12°}$

$\text{base} \doteq 8.5$

The ramp is about 8.7 m long. It covers a horizontal distance of about 8.5 m.

C3 SOH-CAH-TOA are the first letters for the three primary trigonometric ratios:

$\text{Sin}\ \theta = \frac{\text{Opposite}}{\text{Hypotinuse}}$ $\text{Cos}\ \theta = \frac{\text{Adjacent}}{\text{Hypotinuse}}$ $\text{Tan}\ \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Accommodations

Gifted and Enrichment–challenge students to show more than one solution for solving the problems in this section

Perceptual–encourage students to draw diagrams when solving problems

Language–allow students to work in pairs to provide support in reading the questions

Memory–encourage students to review how to use the functions on their scientific and graphing calculators

Student Success

- Have students compile an ongoing journal of examples of problems involving right triangles. Students should include a minimum of six examples: at least one example for each of the trigonometric ratios to solve for a side length and at least one example for each of the trigonometric ratios to solve for an angle measure.

Practise, Connect and Apply, Extend

- Encourage students to refer to the Examples to remind them of the appropriate method and trigonometric ratio to use.
- Stress the importance of sketching a diagram for each question and labelling it carefully.
- In **question 10**, some students may use two different trigonometric ratios rather than the Pythagorean theorem. Students who do this should not round until the final step to improve the accuracy of their calculations.
- **Question 11** links to the Chapter Problem. You may wish to remind students to highlight this question for later referral in the Chapter Problem Wrap-Up.
- **Questions 15 and 16** are good problem solving questions that are accessible to all students' abilities.
- In **question 17**, students first need to use $\triangle ABC$ to determine the length of BC, since it is the common side of the two triangles.
- In **question 18**, suggest to students that they make two congruent right triangles.

Literacy Connections

- Explain to students that a mnemonic is a memory device. SOH-CAH-TOA is a mnemonic that is frequently used to remember the primary trigonometric ratios. Have students work individually to develop their own mnemonic for the trigonometric ratios, then present their mnemonic to the class.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	4, 5, 7–18
Reasoning and Proving	7, 9, 11, 16
Reflecting	13, 15
Selecting Tools and Computational Strategies	1–18
Connecting	5, 8–14, 17
Representing	3–5, 7, 9–14, 16
Communicating	7, 9, 11, 15, 16