4.2 Use Trigonometry to Find Angles

Student Text Pages
192–196

Suggested Timing
80 min

Related Resources
- BLM A–8 Application General Scoring Rubric
- BLM 4–4 Section 4.2 Use Trigonometry to Find Angles
- BLM 4–5 Section 4.2 Achievement Check Rubric

Teaching Suggestions

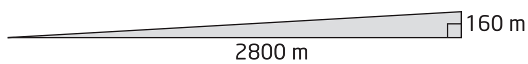
- Have students look at their individual calculators and describe the keystrokes needed to find the measure of an angle, given the value of a trigonometric ratio. For most calculators, it will be SHIFT $\tan^{-1}$ or INV $\tan^{-1}$.
- Indicate to the students that $\tan^{-1}A$ is called the inverse tan of angle A.

Investigate

- Remind students that the angle of elevation is measured from the horizontal (the ground) to the top of the tower.
- Allow students to compare the methods they used to calculate the angle of elevation.

Investigate Responses (page 192)

1.



2. Rachel should use the tangent ratio. She knows the lengths of the sides opposite and adjacent to the angle of elevation.

3. $\tan\theta = \frac{160}{2800}$

4. Isolate the angle by stating the inverse relationship. So, $\tan\theta = \frac{160}{2800}$ becomes $\theta = \tan^{-1}\left(\frac{160}{2800}\right)$. Most calculators require the use of the second function key in addition to the key for the trigonometric ratio to find the inverse of the ratio.

5. $\theta = \tan^{-1}\left(\frac{160}{2800}\right)$

$\theta \doteq 3.3$

The angle of elevation is approximately 3.3°.

6. First, find the distance from the observation point to the base of the building. Next, imagine a right triangle with its opposite side being the height of the building and the adjacent side being the distance to the base of the building and find the angle of elevation measured at the observation point.

$$\tan\theta = \frac{\text{height}}{\text{horizontal}}$$

$$\theta = \tan^{-1}\left(\frac{\text{height}}{\text{horizontal}}\right)$$

$$\theta = \text{angle of elevation}$$

Examples

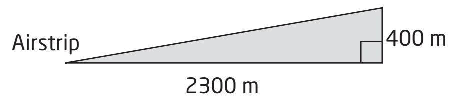
- **Examples 1 and 2** are similar in nature. In each case, students are asked to determine the measure of an angle given information about side lengths.
- For Example 2, ensure students know that to solve the triangle means to determine all the side lengths and angle measures.
- As a class, discuss the clues in a given problem that help in labelling a diagram. Discuss such terms as *surface, rise, run, horizontal,* and *vertical*.
- Students need to consider what is meant by direct distance, based on the other clues in a problem.

Communicate Your Understanding

- Suggest that students work in pairs to share their ideas.
- Some students may have difficulty with **question C3**. You may wish to suggest that they use the Pythagorean theorem to find the length of the longest side.
- You may wish to use **BLM 4–4 Section 4.2 Use Trigonometry to Find Angles** for remediation or extra practice.

Communicate Your Understanding Responses (page 194)

C1



C2 Answers may vary. For example, a ski run has a vertical drop of 10.16 m and a length of 23.69 m. Find the angle of elevation from the bottom to the top of the run. It is appropriate to use the sine ratio to solve the problem because the hypotenuse and opposite side in the right triangle are known.

C3 The trigonometric ratios only apply to right triangles. Substitute the side lengths into the Pythagorean theorem to check if △WXY is a right triangle.

$$\begin{aligned} LS &= 14^2 + 19^2 \\ &= 196 + 361 \\ &= 557 \end{aligned} \qquad \begin{aligned} RS &= 25^2 \\ &= 625 \end{aligned}$$

$$LS \neq RS$$

Since LS ≠ RS, △WXY is not a right triangle so the trigonometric ratios cannot be used to solve the triangle. Jennifer could use the Cosine Law and the Sine Law to solve △WXY.

Practise, Connect and Apply, Extend

- Ask students to refer to the relevant Examples prior to asking for assistance with these questions.
- Remind students to sketch and label a diagram for each question. Students should visualize the situation, sketch the diagram, label it with words, then with numbers.
- **Question 9** links to the Chapter Problem. You may wish to remind students to highlight this question for later referral in the Chapter Problem Wrap-Up.
- Students may approach the problem in **question 10** by solving for the angle of inclination with a 15 m base, or by solving for the length of a ramp with an angle of inclination of 6°.
- **Question 12** is an Achievement Check question. Provide students with **BLM 4–5 Section 4.2 Achievement Check Rubric** to help them understand what is expected.
- **Question 13** is a good problem-solving question that is accessible for students working at levels 3 or 4. It involves a conjecture and requires good communication skills.
- **Question 15** requires students to work backwards from the formula for the area of a triangle, in order to find the height and then the angle.

Common Errors

- Some students may substitute an angle measure or side length in the wrong place in a trigonometric ratio.

R_x This indicates that they do not understand the difference between an angle and a side length. Colour-code the placement of the components. Use coloured chalk or marker to show

$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}}$

Explain that the angle measure is in degrees and the side lengths are in units of length.

Ongoing Assessment

- **Question 12** is an Achievement Check question. Use **BLM 4–5 Section 4.2 Achievement Check Rubric** to assess students' responses.
- You may wish to use **BLM A–8 Application General Scoring Rubric** to assess students' responses to **question 7**.

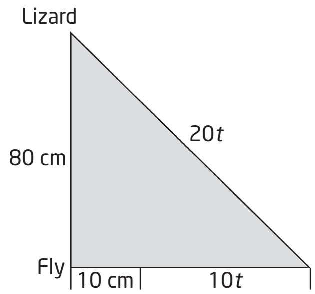
Accommodations

Gifted and Enrichment–have students make up and solve problems similar to those in **question 3**

Motor–give students extra time to complete the questions in this section and let them work with a partner when sketching diagrams

Language–allow students to explain their answers orally

Achievement Check Sample Solution (page 196, question 12)



Let t represent the time in seconds required for the lizard to intercept the fly. When the lizard begins to move, $t = 0$. The initial vertical separation is 80 cm. The horizontal distance travelled by the fly is expressed by $10 + 10t$. The distance travelled by the lizard along the hypotenuse is expressed by $20t$. Use the Pythagorean theorem to solve for t.

$$(20t)^2 = 80^2 + (10 + 10t)^2$$
$$400t^2 = 6400 + 100 + 20t + 100t^2$$
$$300t^2 - 20t - 6500 = 0$$
$$15t^2 - t - 325 = 0$$

I used a graphing calculator to solve for t; $t \doteq 4.688$ s. I ignored the negative root since time cannot be negative. When $t = 4.688$ the fly has moved 10 + 46.88 cm or approximately 57 cm.

Let θ represent the angle between the lizard's path and the original vertical line of separation.

$$\tan \theta = \frac{56.88}{80}$$
$$\theta = \tan^{-1}\left(\frac{56.88}{80}\right)$$
$$\theta \doteq 35.4°$$

The lizard moves at an angle of approximately 35.4° to intercept the fly.

Literacy Connections

- In their journals, have students describe two different ways to solve **question 10**.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–16
Reasoning and Proving	9, 10, 12, 13
Reflecting	12, 13
Selecting Tools and Computational Strategies	1–16
Connecting	8–10, 13–15
Representing	5–10, 12, 13
Communicating	5, 9, 10, 12, 13