Investigate the Sine Law

Teaching Suggestions

• Remind students of the labelling convention for triangles; uppercase letters are used to label the vertices and the corresponding lowercase letter is used to label the opposite side.
• After completing the Investigate, formalize both versions of the sine law. Ask students why there are two versions.
• If you are completing this section over two days, you may wish to concentrate on calculating lengths the first day and calculating angle measures the second day. Alternatively, assign only the Practise questions the first day and work on the Connect and Apply questions the second day.

Investigate

• If using *The Geometer's Sketchpad*®, have copies of **BLM T–2 *The Geometer's Sketchpad*® 3** or **BLM T–3 *The Geometer's Sketchpad*® 4** available for students. Students can use the **Tabulate** and **Add Table Data** features to build tables of data.
• If using paper and pencil, have students work in small groups and compare their results.
• Once students complete the Investigate, discuss the results as a class. Students coming from MPM2D will have seen the sine law before, but it will be new to students coming from MFM2P.

Investigate Responses (pages 202-204)

Method 1

1., 2. Answers may vary.

3. a) The three ratios are equal.
   b) In each quotient, the side length is divided by the sine ratio of the angle opposite that side.

4. Answers may vary.

a	b	c	∠A (°)	∠B (°)	∠C (°)	$\dfrac{a}{\sin A}$	$\dfrac{b}{\sin B}$	$\dfrac{c}{\sin C}$
10.68	9.57	7.92	74.58	59.76	45.66	11.08	11.08	11.08
12.74	9.57	10.17	80.37	47.77	51.86	12.92	12.92	12.92
12.92	9.57	10.50	79.95	46.85	53.21	13.12	13.12	13.12
14.47	9.57	11.31	87.35	41.34	51.31	14.49	14.49	14.49
16.39	9.57	12.37	95.77	35.53	48.70	16.47	16.47	16.47
16.39	9.57	12.37	95.77	35.53	48.70	16.47	16.47	16.47
15.91	9.57	10.37	105.75	35.38	38.87	16.53	16.53	16.53
15.82	9.57	9.62	111.06	34.37	34.58	16.95	16.95	16.95
15.82	9.57	9.62	111.06	34.37	34.58	16.95	16.95	16.95
11.79	9.18	9.62	77.61	49.54	52.85	12.07	12.07	12.07
6.05	6.67	6.80	53.37	62.26	64.37	7.54	7.54	7.54
6.05	6.67	6.80	53.37	62.26	64.37	7.54	7.54	7.54

5. As each new triangle is created, the measures of the angles and the side lengths change. The quotient of side length divided by the sine ratio of opposite angle changes from the previous triangle but is constant for all three quotients within any given triangle.

6. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Method 2

1., 2. Answers may vary.

3. a) The three ratios are equal.

    b) The angle is opposite the side in each ratio.

4. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Examples

- Ensure students understand they only need two ratios to solve problems with the sine law.
- Emphasize to students that the key to solving a problem such as the one in **Example 3** is to draw a clear, well-labelled diagram.

Communicate Your Understanding

- For **question C1**, part d), explain that the sine law can also be used with right triangles. Have students use their calculators to find that sin 90° = 1.
- For **question C3**, keystroke sequences depend on student's individual calculators.
- You may wish to use **BLM 4–7 Section 4.4 Investigate the Sine Law** for remediation or extra practice.

Communicate Your Understanding Responses (page 206)

C1 a) Yes, two angles and one side length are given. Subtract the measures of the two known angles from 180° to find the measure of the third angle. Then, substitute the values into the sine law and solve for x.

    b) No, no angles are given.

    c) Yes, a side opposite a known angle and an angle opposite a second known side are given. Substitute the values into the sine law and solve for x.

    d) Yes, two angles and one side length are given.

C2 Two angles and the length of the side opposite one of the known angles are given. From the sine law, $\dfrac{b}{\sin B} = \dfrac{d}{\sin D}$. Substitute the given values for B, D, and d and solve for b.

C3 Answers may vary. For example, 21 [×] 48 [sin] [÷] 25 [=] [2nd] [sin⁻¹] or [2nd] [sin⁻¹] [( ] [( ] 21 [×] [sin] 48 [ )] [÷] 25 [ )] [=]

Practise, Connect and Apply, Extend

- Encourage students to refer to the individual Examples for assistance.
- For the Practise questions, remind students to re-arrange their formulas to isolate the variable before evaluating, rather than evaluating the individual components before re-arranging.
- Remind students to sketch and label a diagram for each problem. Students should look for clues in the question to help them label the diagram.
- For **questions 8 to 11**, students must determine the measure of the third angle before using the sine law.
- For **question 13**, students should subtract 87° from 180°, let the unknown sides each be x, and use the sine law to calculate the length of x.

- **Question 14** provides a good opportunity to apply the sine law in a real-life setting. Kinaesthetic learners will benefit from completing this question. Have students work in pairs.
- For **question 19,** part b), the closest distance would be perpendicular to Jia Wei's direction of travel.
- **Question 20** requires students to visualize and work in three dimensions. Calculate the hypotenuse of each vertical triangle. Then use the 80° angle to find the distance between the two airplanes.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–20
Reasoning and Proving	5, 6, 10, 11, 13–16
Reflecting	16, 18–20
Selecting Tools and Computational Strategies	1–20
Connecting	7, 14–17
Representing	4, 9–11, 14, 16–19
Communicating	10, 11, 13–16