4.5 Investigate the Cosine Law

Student Text Pages
210–215

Suggested Timing
80–160 min

Materials and Technology Tools
- computers with *The Geometer's Sketchpad*®

Related Resources
- BLM T–2 *The Geometer's Sketchpad*® 3
- BLM T–3 *The Geometer's Sketchpad*® 4
- BLM A–17 Learning Skills Checklist
- BLM 4–8 Section 4.5 Investigate the Cosine Law

Teaching Suggestions

- After completing the Investigate, summarize the cosine law. Do not have students memorize the three versions of the cosine law; rather, have them look at the pattern of how it is developed and develop their own memory device as a reminder.
- Rather than having students memorize the version of the cosine law that solves for the angle, have them rearrange the formula each time.
- If you are teaching this section over two days, you may want to concentrate on side measurements the first day and angles the second day. Alternatively, you could assign only the Practise questions the first day and the Connect and Apply and Extend questions the second day.

Investigate

- Have copies of **BLM T–2 *The Geometer's Sketchpad*® 3** or **BLM T–3 *The Geometer's Sketchpad*® 4** available for students.
- Remind students they can use the **Tabulate** and **Add Table Data** features to build tables of data.
- As students work through the Investigate, they should recognize that the Pythagorean theorem does not hold true for oblique triangles, but can be adapted using the $-ab \cos C$ term.
- For a ready-made activity, students can use *The Geometer's Sketchpad*® activity, 4.6 Investigate Cosine Law. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links.

Investigate Responses (pages 210-211)

1., 2. Answers may vary.

3. a) No. The values of c^2 does not equal the sum $a^2 + b^2$.

b) When $\angle C = 90°$, $c^2 = a^2 + b^2$. When $\angle C$ is acute, $c^2 < a^2 + b^2$ and when $\angle C$ is obtuse, $c^2 > a^2 + b^2$.

c) The Pythagorean theorem is true only for right triangles.

4. a) The values of $a^2 + b^2 - c^2$ and $2ab \cos C$ are equal.

b) The expression $a^2 + b^2 - 2ab \cos C$ is equal to c^2.

5. Answers may vary.

a	b	c	m∠ACB	$a^2 + b^2$	c^2	$a^2 + b^2 - 2ab \cos C$
9.55	3.36	10.08	89.10°	102.52	101.51	101.51
9.55	4.34	8.07	57.18°	110.00	65.12	65.12
9.55	6.30	5.51	33.36°	130.97	30.39	30.39
9.55	3.33	6.41	15.69°	102.29	41.13	41.13
9.55	3.87	11.60	112.53°	106.18	134.49	134.49
9.55	1.74	10.82	133.15°	94.26	117.01	117.01
9.55	6.03	5.71	34.46°	127.62	3260.00	3260.00
9.55	9.19	2.53	15.36°	175.75	6.40	6.40
9.55	5.54	6.08	36.65°	121.93	37.01	37.01
9.55	1.03	10.44	147.53°	9230.00	108.98	108.98
9.55	6.44	15.24	144.02°	132.71	232.28	232.28

6. From the table, the value for c^2 for any triangle is always equal to the value of the expression $a^2 + b^2 - 2ab \cos C$ for the same triangle.
7. The angle is located at the intersection of the sides a and b.
8. In general it appears that $c^2 = a^2 + b^2 - 2ab \cos C$ for any triangle. If $\angle C = 90°$, $\cos C = 0$, which returns this equation to the Pythagorean theorem.

Examples

- In **Example 1**, stress the location of the known sides relative to the known angle. The unknown side is opposite the known angle.
- In **Example 2**, the cosine law is used to find an angle when three sides are known.

Communicate Your Understanding

- Have students work in pairs to discuss the answers before taking them up as a class.
- **Question C1** is the key to understanding why the cosine law is needed.
- **Question C2** illustrates a typical error made by many students.
- You may wish to use **BLM 4–8 Section 4.5 Investigate the Cosine Law** for remediation or extra practice.

Communicate Your Understanding Responses (page 213)

C1 If I apply the sine law to the problem in Example 1, I obtain $\frac{15}{\sin P} = \frac{17}{\sin R} = \frac{q}{\sin 43°}$. If I choose any pair of ratios, I get an equation with two unknowns, which I cannot solve.

In Example 2, only the side lengths are given. I cannot use the sine law unless I know the measure of at least one angle in the triangle.

C2 Barbra's calculator follows the order of operations. In order to get the correct result, Barbra needs to include brackets to ensure that the entire numerator of the expression is divided by the entire denominator: $\cos^{-1}((5^2 + 6^2 - 8^2)/(2*5*6))$

C3 To find the length of side a: $a^2 = b^2 + c^2 - 2bc \cos A$

To find the measure of $\angle A$: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

To find the length of side b: $b^2 = a^2 + c^2 - 2ac \cos B$

To find the measure of $\angle B$: $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

Practise, Connect and Apply, Extend

- Have students refer to the appropriate Examples for assistance.
- For **question 4**, remind students of the labelling convention for triangles.
- For **question 5**, students should recognize that the angle of interest is at the hinge.
- Students should sketch and label diagrams for **questions 6** and **questions 11 to 13**.
- For **question 13**, students should recognize that they can use any side length they wish, as long as the triangle is equilateral.
- **Question 14** is a good problem-solving question accessible to students working at level 3 or 4. Students should divide the hexagon into six congruent triangles.

Common Errors

- Some students may substitute values incorrectly into the cosine law.

R_x Have students develop a memory device that helps them remember how to write the cosine law and how each side is substituted. The side being solved for must be opposite the angle in the formula. Draw an arrow from the indicated angle to the opposite side to help set up the cosine law.

Ongoing Assessment

- While students are working, circulate and see how well each works. This may be an opportunity to observe and record individual students' learning skills. Use **BLM A–17 Learning Skills Checklist**.

Accommodations

Gifted and Enrichment–challenge students to create three different problems that can be solved using the cosine law. Have students trade problems with a partner and solve their partners' problems.

Perceptual–encourage students to sketch and label diagrams when solving problems

Motor–encourage students to work in groups when using technology

Memory–allow students to use words instead of variables when developing the cosine law

ESL–encourage students to work together with their classmates when using *The Geometer's Sketchpad*® and to use their translators to understand the new words in this section

Literacy Connections

- Have students write a journal entry describing the cosine law in words and in symbols. Students should also describe how to use the cosine law to find the length of a side of a triangle and to find the measure of an angle.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–12, 14, 15
Reasoning and Proving	10, 11, 13, 14
Reflecting	12, 14, 15
Selecting Tools and Computational Strategies	1–15
Connecting	5–9, 11, 12, 14
Representing	4, 11, 12
Communicating	10, 11