4.6 Make Connections With the Sine Law and the Cosine Law

Student Text Pages
216–221

Suggested Timing
160 min

Related Resources
- BLM A–4 Presentation Checklist
- BLM 4–9 Section 4.6 Make Connections With the Sine Law and the Cosine Law
- BLM 4–10 Section 4.6 Achievement Check Rubric

Teaching Suggestions

- One of the key skills in solving problems involving trigonometry is choosing the correct tool. In this section, students must first determine which tool, the trigonometric ratios, the sine law, or the cosine law, to use to solve the problem.
- On the first day use Example 1 to consolidate learning. On the second day, review Example 2 and apply problem-solving skills to contextual problems.

Investigate

- Have students work in pairs to complete the Investigate. Have them compare their answers to **question 2** with another pair of students.
- As a class, discuss the students' findings before moving on to the Examples.

Investigate Responses (page 216)

1. **Situation A:** From the sine law, $\frac{c}{\sin 52°} = \frac{12}{\sin 69°}$. This can be solved for c.
From the cosine law, $c^2 = a^2 + 12^2 - 2(a)(12) \cos 52°$. This equation contains two unknowns and cannot be solved for c.
Situation B: From the sine law, $\frac{p}{\sin 32°} = \frac{19}{\sin \text{Q}} = \frac{17}{\sin \text{R}}$. All pairs of ratios give an equation with two unknowns. The sine law cannot be used to solve for p.
From the cosine law, $p^2 = 19^2 + 17^2 - 2(17)(19) \cos 32°$. This can be solved for p.
Situation C: From the sine law, $\frac{7.3}{\sin \text{J}} = \frac{9.0}{\sin \text{K}} = \frac{8.5}{\sin \text{L}}$. This equation contains two unknowns and cannot be solved for ∠L.
From the cosine law, $\cos \text{L} = \frac{7.3^2 + 9.0^2 - 8.5^2}{2(7.3)(9.0)}$. This can be solved for ∠L.
Situation D: From the sine law, $\frac{\sin \text{W}}{61} = \frac{\sin 74°}{73}$. This can be solved for ∠W.
From the cosine law, $\cos \text{W} = \frac{73^2 + x^2 - 61^2}{2(73)x}$. The equation contains two unknowns and cannot be solved for ∠W.

2. Diagrams may vary. To solve a triangle, you need to have a minimum of three pieces of information, one of which must be the length of a side. Use the number of angles given to decide which tool to use: the sine law or the cosine law.
- **No angle measures are known.** Given the lengths of all three sides, use the cosine law to find the measure of an angle.
- **One angle measure is known.** Given the measure of one angle and the lengths of two sides, use the sine law or the cosine law depending on the location of the known angle relative to the two known sides.
  - **The angle is contained between the two given sides.** Use the cosine law to determine the length of the third side. If the measure of an angle is requested, use the cosine law to determine the length of the third side then use the sine law to determine the angle.
  - **The angle is not contained between the given sides.** Given the angle and the opposite side length, use the sine law to determine the measure of the angle opposite the other known side. Then find the measure of the third angle and the length of the third side.
- **Two angle measures are known.** Given the measures of two angles and the length of one side, determine the measure of the third angle, if necessary, and use the sine law to determine the lengths of the other two sides.

Examples

- In **Example 1**, once the cosine law has been used to determine *p*, students can use the sine law or the cosine law to determine the measure of either ∠R or ∠Q. Then, the measure of the third angle can be determined using the angle sum theorem, the sine law, or the cosine law. Students should choose the method they find easiest.
- In **Example 2**, it is important for students to sketch and label a diagram before deciding how to solve the problem.

Communicate Your Understanding

- Allow students to think about their responses to these questions individually before discussing them as a class.
- In **question C2**, students must analyse the situation and devise a plan to solve a problem.
- You may wish to use **BLM 4–9 Section 4.6 Make Connections With the Sine Law and the Cosine Law** for remediation or extra practice.

Communicate Your Understanding Responses (page 219)

C1 Use the sine law or the cosine law to solve problems involving side lengths and angle measures of triangles that are not right triangles.

C2 First, use the sine law to find the measure of ∠B. Then, use the measures of ∠B and ∠C to determine the measure of ∠A. Then, with the measure of ∠A, use the sine law to find the length of side *a*.

Practise, Connect and Apply, Extend

- Encourage students to refer to the Examples for assistance with the Practise questions.
- For **question 3**, ensure students label their diagrams properly. Suggest that they plan their steps to help decide whether to use the trigonometric ratios, the sine law, or the cosine law.
- **Question 8** links to the Chapter Problem. You may wish to suggest that students highlight their solution to refer to in the Chapter Problem Wrap-Up.
- For **question 9**, students should divide the loonie into 11 congruent triangles. Students' approach to solving **question 10** should be similar.
- For **question 12**, students should use the distance formula to calculate the length of each side.
- **Question 16** is an Achievement check question. Provide students with **BLM 4–10 Section 4.6 Achievement Check Rubric** to help them understand what is expected.
- For **question 17**, students may find it helps to draw the diagram on a grid showing the compass points.
- For **question 18**, students must determine if ∠ADE and ∠ADC, and ∠ACD and ∠ACB are pairs of supplementary angles.

Common Errors

- Some students may select an inappropriate trigonometric tool.

R_x Allow students to follow through with their mistakes and see that their method will not work. Suggest they try another trigonometric ratio or law instead. You may wish to have them review the Investigate.

Ongoing Assessment

- **Question 16** is an Achievement Check. Use **BLM 4–10 Section 4.6 Achievement Check Rubric** to assess students' responses.

Accommodations

Gifted and Enrichment–encourage students to use different strategies to solve mathematical problems and to extend their knowledge of patterns in mathematics

Perceptual–let students work with an educational assistant, if possible, or with a student who can help to explain the concepts

Spatial–have students work with a partner who can assist with sequencing steps

Language–allow students to give verbal responses to some of the questions in this section

Memory–encourage students to review how to use the trigonometric functions on their calculators

Achievement Check Sample Solution (page 221, question 16)

a) In △ABE:

By the cosine law:

$$\cos BAE = \frac{8^2 + 8.5^2 - 7^2}{2(8)(8.5)}$$

$$BAE = \cos^{-1}\left(\frac{8^2 + 8.5^2 - 7^2}{2(8)(8.5)}\right)$$

$$\angle BAE \doteq 50.09°$$

$$\cos AEB = \frac{8^2 + 7^2 - 8.5^2}{2(8)(7)}$$

$$AEB = \cos^{-1}\left(\frac{8^2 + 7^2 - 8.5^2}{2(8)(7)}\right)$$

$$\angle AEB \doteq 68.66°$$

By angle sum property:

$$\angle ABE = 180° - 50.09° - 68.66° = 61.25°$$

By angle sum property:

$$\angle BEC \doteq 180° - 2(83.85) \doteq 12.3°$$

In △BCE:

By the cosine law:

$$\cos CBE = \frac{0.75}{7}$$

$$\angle CBE = \cos^{-1}\left(\frac{0.75}{7}\right)$$

$$\angle CBE \doteq 83.85°$$

By properties of isosceles triangles: ∠BCE = ∠CBE

In △CDE:

By the cosine law:

$$\cos CED = \frac{0.5}{7}$$

$$\angle CED = \cos^{-1}\left(\frac{0.5}{7}\right)$$

$$\angle CED \doteq 85.90°$$

By properties of isosceles triangles: ∠CED = ∠CDE

By angle sum property:

$$\angle BEC \doteq 180° - 2(85.90) \doteq 8.2°$$

b) Answers may vary.

c) Minimum measures would be at least one side length and two more measures, either angles or side lengths or one of each, for each triangle.

d) Answers may vary.

Literacy Connections

- Have students use examples to explain when it is appropriate to use each of the trigonometric ratios, the sine law, and the cosine law.

Career Connections

- Have students discuss what they know about a career as an avionics technician. As an extension, have students research this career and other similar careers, and present their findings to the class. You may wish to use **BLM A–4 Presentation Checklist** to assess students' presentations. Using their research, have each student discuss:
- You may wish to have students include their research in their portfolios. For more career resources for your students, see the McGraw-Hill Ryerson Web site at *www.mcgrawhill.ca/functionsapplications11*.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	4, 6–18
Reasoning and Proving	5, 6, 8, 15, 16, 18
Reflecting	15, 16, 18
Selecting Tools and Computational Strategies	1–18
Connecting	4–6, 8, 9, 11–18
Representing	3, 4, 6, 7, 9–17
Communicating	5, 6, 8, 9, 12, 15, 16