5.1 Periodic Functions

Student Text Pages
232–238

Suggested Timing
80 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM A–4 Presentation Checklist
- BLM 5–3 Section 5.1 Periodic Functions
- BLM 5–4 Section 5.1 Achievement Check Rubric

Teaching Suggestions

- Periodic functions will be a new topic for most students. Define the word *periodic*, so students have an idea of what a graph may look like and relate the term to something familiar.
- You may wish to graph examples of periodic functions in nature, such as a series of photographs of a tide at various times of the day, a rotating wheel with a spot highlighted and tracked, the ebb and flow of blood as charted, or the cyclic nature of animal populations.
- Look for examples on the Internet to display in class.

Investigate

- Engage students in a discussion about periodic functions. Discuss the cyclical nature of tides.
- Students may work individually or in pairs.
- Encourage students to communicate in their own words, without emphasizing proper mathematical language at this point.

Investigate Response (page 232)

1. The graph shows water depths, which are rising and falling at regular intervals. The maximum and minimum depths for each interval are very similar.
2. A periodic function repeats a pattern of y-values on a regular interval of the domain. In this case there is a repeated pattern of water depth versus time.
3. **a)** Estimates may vary. **i)** approximately 12 m **ii)** approximately 2 m **iii)** There are two possible interpretations for the time between high tides. If all the peaks are considered to be approximately equal, the time between high tides is about 12 h. Or, noting that every other high tide is a bit higher than the high tide previous, then approximately every 24 h. **iv)** Same times as answer for part iii) only for low tides.
   b) the water depths from one high tide to the next high tide
4. Answers may vary but must include situations for which a pattern of measurements is repeated exactly over intervals of the domain, which have equal length.

Examples

- In **Example 1**, stress that for a function to be periodic each cycle must match precisely.
- An alternative definition of amplitude is *displacement from rest*.
- For **Example 2,** parts a) and b), ensure students understand that there are many values of x that give the same value of f.
- **Example 2** is an introduction to trigonometric functions. It is important that students understand the terminology and the periodic nature of the curve.

Communicate Your Understanding

- Have students answer the questions individually.
- You may wish to have students use their responses as journal entries.
- Discuss the answers as a class before moving on to the exercises.
- You may wish to use **BLM 5–3 Section 5.1 Periodic Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 235)

C1 **a)** A periodic function repeats its y-values at regular intervals. For the function shown, the same pattern of y-values repeats every 4 s.

b) The most reliable way to find the period of the function is to find the difference in x-values between two successive minimum values or two successive maximum values.

c) The amplitude of the function is half the distance from the maximum to minimum y-values. In this case:

$$\text{amplitude} = \frac{4 - 0.6}{2}$$
$$= 1.7$$

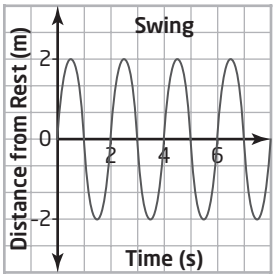
C2 The height values would increase and decrease with successive maximum and minimum values spaced at regular time intervals. However, over time the amplitude would get smaller and the weight would come to rest. The graph would be similar to the one in Example 1, part a).

Practise, Connect and Apply, Extend

- **Questions 1 to 3** are very important as diagnostic tools to see how well students understand the terminology and if they can demonstrate their knowledge of periodic functions. For **question 1**, remind students to justify their reasoning by providing evidence for their answer.
- There is a good mixture of applications in Connect and Apply. Try to assign a variety of questions that involve provided graphs and data.
- **Question 6** is a good diagnostic question that tests students' understanding of amplitude.
- **Question 7** should be assigned, as it is a relatively easy data-related question that supports understanding of the concepts of periodicity and amplitude.
- **Question 8** refers to the Chapter Problem. You may wish to have students flag their answers to this question for use in the Chapter Problem Wrap-Up.
- The scatter plots for **questions 7 and 11** may be produced by hand or with a graphing calculator. Alternatively, you may wish to have students draw the scatter plots by hand, then use a graphing calculator to check. You may wish to have copies of **BLM G–1 Grid Paper** available.
- **Question 12** is an Achievement Check question. Provide students with **BLM 5–4 Section 5.1 Achievement Check Rubric** to help them understand what is expected.
- **Question 13** might be easier to answer if students picture a strip of paper being pulled to the right as the rider goes up and down on the ride.

Achievement Check Sample Solution (page 238, question 12)

12. a)



b) The swing moves back and forth in a regular motion. Assume that the distance is not changing so that the width of each swing is always 4 m.

c) The swing might follow a dampened periodic path when the child is first starting out or when the child slows down to stop swinging. If an outside force were placed on the swing, there would be another interval in time

when the path is increasing or decreasing irregularly.

d) A teeter-totter goes up and down in a periodic fashion if you measure the height of the end off the ground. A merry-go-round goes around in a periodic fashion if you measure the position of a point on the merry-go-round compared to a fixed point. A child using the monkey bars might swing from bar to bar in a periodic fashion if you measure the height from the ground.

Literacy Connections

- Have students write a journal entry describing different examples of real-world situations that can be modelled using a periodic function.

Career Connections

- Have students discuss what they know about a career as a physical oceanographer. As an extension, have students research this career and other similar careers, and present their findings to the class. You may wish to use **BLM A–4 Presentation Checklist** to assess students' presentations. Using their research, have each student discuss:
  - The duties of a physical oceanographer.
  - The type of education and training needed for this career.
  - Similar careers.
  - The differences in the training and education required for a similar career.
- You may wish to have students include their research in their portfolios. For more career resources for your students, see the McGraw-Hill Ryerson Web-site at *www.mcgrawhill.ca/functionsapplications11*.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	8–13
Reasoning and Proving	1, 6–12
Reflecting	7
Selecting Tools and Computational Strategies	2–4, 7, 11
Connecting	4–12
Representing	7, 11–13
Communicating	5–12

Common Errors

- Some students may have difficulty determining the difference between the period of a periodic function and its cycle.

R_x Explain to students that the period is the horizontal distance from the start of one cycle to the start of the next cycle. A cycle is the part of a periodic function that repeats. It is one complete pattern, including both the horizontal and vertical components, of a periodic function.

Ongoing Assessment

- **Question 12** is an Achievement Check. Use **BLM 5–4 Section 5.1 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Gifted and Enrichment–have students describe a situation that could be modelled by each graph in **questions 1 and 2**

ESL–allow students extra time in class to understand the meaning of the words in this section. Encourage students to use a translator to ensure that they understand the meaning of the words being used and the context.

Student Success

- Link this topic to science. Have students identify periodic functions or relations they have seen in science class that exhibit periodic behaviour.