5.2 Circles and the Sine Ratio

Student Text Pages
239–247

Suggested Timing
80 min

Materials and Technology Tools
- grid paper
- protractors
- scientific calculators
- computer with *The Geometer's Sketchpad®* (optional)
- Cabri Jr™ (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM A–9 Communication General Scoring Rubric
- BLM 5–5 Section 5.2 Communicate Your Understanding
- BLM 5–6 Section 5.2 Circles and the Sine Ratio

Teaching Suggestions

- Review the numbering of the quadrants of a coordinate grid.
- Search the Internet for videos illustrating the development of the sine ratio starting with a unit circle.
- Encourage students to draw diagrams to illustrate the contextual problems where possible.

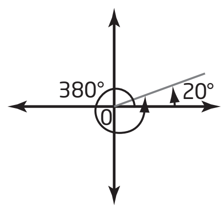
Investigate

- *The Geometer's Sketchpad®* or Cabri Jr™ can be used to illustrate and compare these angles.
- Once students have completed the Investigate, summarize the concepts of coterminal angles. These are important for students to be able to graph the sine function for angles greater than 360°.

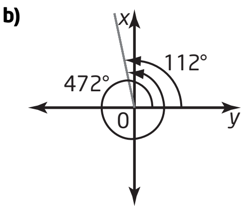
Investigate Response (page 239)

1. The numbers 360 and 720 refer to degrees of rotation. The numbers represent one and two full rotations.

2. a)



b)



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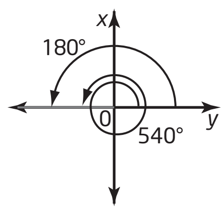

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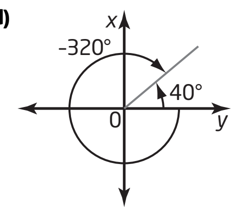


3. The angles in each pair in question 2 share the same terminal arm.

4. a) $\sin 20° = \sin 380° \doteq 0.342$ **b)** $\sin 112° = \sin 472° \doteq 0.927$

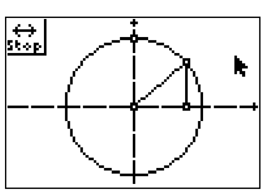
c) $\sin 180° = \sin 540° = 0$ **d)** $\sin 40° = \sin 320° \doteq 0.643$

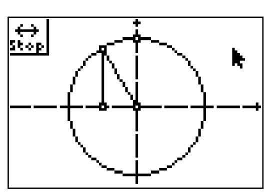
The sine ratios for each pair of angles are identical. This occurs because each pair of angles has a common coterminal arm. The y-coordinate for each terminal point must also be equal.

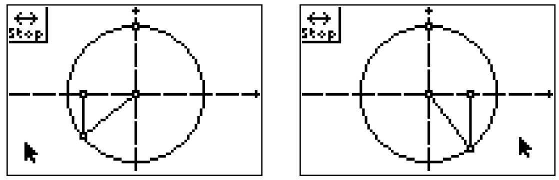
5. Answers may vary but must have a difference of 360° or a multiple of 360°.

Examples

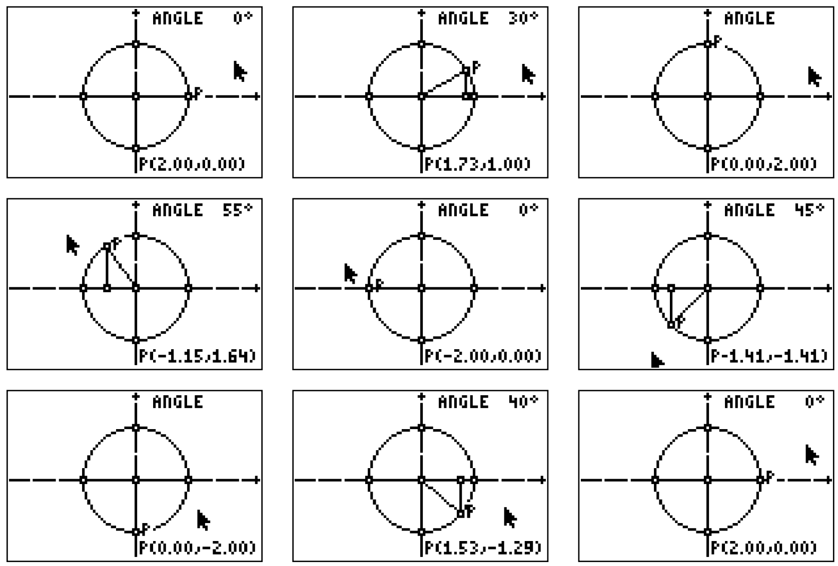
- **Example 1** is an excellent place for students to use Cabri Jr™ to illustrate the terminal arm rotating about the origin. You may wish to use the animation feature from Cabri Jr™ to illustrate the position of the terminal arm as it rotates counter-clockwise about the origin as a student activity or a teacher demonstration.







Students would create a table of values for (θ, y); then plot the points to produce the graph of $y = 2\sin\theta$. In the example below, one angle was chosen from each quadrant and from along each axis.
Note:
- Use a circle of radius 2 in order to easily drag the point, P.
- A limitation of Cabri Jr™ prevents the measuring of angles equal to or greater than 90°. For angles in quadrants 2, 3, and 4, tell students to record the angle in the quadrant that point P was in.



- Before discussing **Example 2**, have students use their calculators to evaluate cos 60° and cos 120°. Then have students try to sketch a right triangle for each. Ask students to explain why no triangle is possible for 120°. This sets the stage for using the coordinate system to help define trigonometric ratios for obtuse angles.
- For **Example 3**, have students verify the unit circle equation by substituting values for θ and evaluating. It is important that students understand the concept of a unit circle so they can understand the concepts in later sections.
- Review with students the quadrants in which sine and cosine are positive and negative. Have students visualize the location of the angle relative to the x- and y-axes.

Communicate Your Understanding

- Have students work in pairs to answer each question. Discuss the solutions as a class.
- You may wish to use **BLM 5–5 Section 5.2 Communicate Your Understanding** for students to answer **question C1**.
- You may wish to use **BLM 5–6 Section 5.2 Circles and the Sine Ratio** for remediation or extra practice.

Common Errors

- Some students may not go past one extra rotation of 360° when finding coterminal angles.

R_x Have students rotate the terminal arm more than once and ask what the coterminal angles would be for a given acute angle.

- Some students may have difficulty remembering how to determine the sign of the trigonometric ratios for each quadrant of the unit circle.

R_x Have students label the *x*-axis the cos θ-axis and the *y*-axis the sin θ-axis, then label the four quadrants based on the sign of cos θ and sin θ. They can also colour the quadrants to emphasize the signs.

Ongoing Assessment

- You may wish to use students' responses to the Communicate Your Understanding questions as a formative assessment tool. Use **BLM A–9 Communication General Scoring Rubric**.

Accommodations

Perceptual–encourage students to sketch diagrams when solving problems

Motor–give students extra time to complete the questions in this section

Memory–provide a chart with the vocabulary for this section and space to write the definitions

ESL–encourage students to use a translator to understand the meaning of the new words in this section

Communicate Your Understanding Responses (page 245)

C1

	First Quadrant	Second Quadrant	Third Quadrant	Fourth Quadrant
Sign of cos	positive	negative	negative	positive
Sign of sin	positive	positive	negative	negative
Sign of tan	positive	negative	positive	negative

C2 For any angle, if you move forward or backward by 360°, the new terminal arm will be in exactly the same position as the original. It follows that the terminal point on the unit circle must also be identical to the original. Because the sine ratio of the angle is simply the *y*-coordinate of this terminal point you will get identical *y*-values each time you shift by 360°. This creates a function with a period of 360°.

Practise, Connect and Apply, Extend

- Have copies of **BLM G–1 Grid Paper** available for students to use.
- Assign all of **questions 1 to 7**. They are good practice of the basic concepts and can be used as diagnostic tools. For **question 7**, remind students to communicate why the sign is positive or negative.
- **Question 8** is an excellent question to demonstrate students' understanding of coterminal angles and the relationship between quadrants and the sign of the trigonometric ratio.
- **Questions 11, 12, 16, and 17** are good real-world applications of the concepts.
- **Questions 13 to 15** should be assigned as mathematical applications of the concepts.
- **Questions 18 and 19** are interesting challenges that are accessible to students working at levels 3 and 4. For **question 18**, suggest that students begin by making a first quadrant right triangle, then transforming it to the third quadrant. For **question 19**, the area will be the fraction of the area of the circle, relative to the angle.

Literacy Connections

- Encourage students to make a glossary of terms associated with rotation angles.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	11, 12, 16–19
Reasoning and Proving	2, 6–8
Reflecting	n/a
Selecting Tools and Computational Strategies	3–5, 7, 9–19
Connecting	19
Representing	1, 3, 4, 8
Communicating	7, 8