# 5.3 Investigate the Sine Function

**Student Text Pages**
248–253

**Suggested Timing**
80 min

**Materials and Technology Tools**
- clear tape
- compasses
- elastic bands
- graphing calculators
- grid paper
- large sheets of grid paper
- markers
- paper plates
- pieces of paper 14.0 cm by 21.5 cm
- protractors
- rulers
- tennis balls

**Related Resources**
- BLM G–1 Grid Paper
- BLM A–17 Learning Skills Checklist
- BLM 5–7 Section 5.3 Investigate B
- BLM 5–8 Section 5.3 Investigate the Sine Function

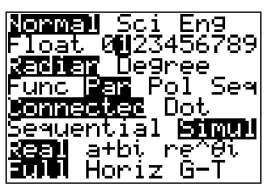
## Teaching Suggestions

- Discuss the opening paragraph and the picture. Ask if satellites travel in a wave and if not, how they do travel. Investigate A will help explain why the path resembles a wave. This Investigate is helpful for kinesthetic learners.
- Investigate B is key to understanding the key features of a sine curve—period and amplitude—in addition to seeing that many values of a sine function repeat or have opposite signs. You may wish to have students work in pairs. This Investigate is helpful for visual learners.

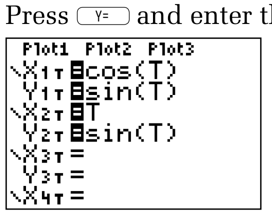
### Investigate

- **Investigate A** requires a tennis ball for each student. The piece of paper is a half-sheet of letter-sized paper. Alternatively, golf balls can be used along with one-sixth of a sheet of paper.
- The lengthwise line represents the equator of the ball. Make sure it is centred.
- The measure of the angle of the elastic band is not important. Its purpose is to compare the results when steeper angles are used.
- **Investigate B,** Method 1, has students generate data to plot a sine function. You may wish to copy **BLM 5–7 Section 5.3 Investigate B** for students to complete. Have copies of **BLM G–1 Grid Paper** available.
- You may wish to have students complete both methods, with and without a graphing calculator. Have students simulate the *y*-position on the circle graph to produce a sine graph as it rotates through 360°.
- You may wish to use an alternative approach to Investigate B, Method 2. Follow these instructions to view the unit circle and the sine curve simultaneously on a graphing calculator.

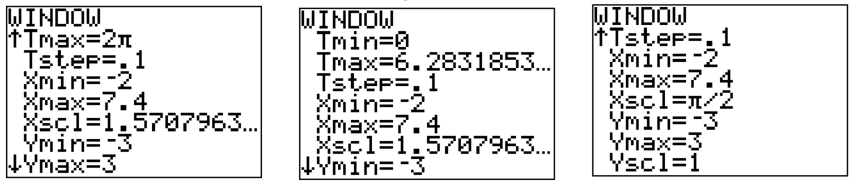
**1.** Press MODE and enter these settings.




**2.** Press Y= and enter these settings.

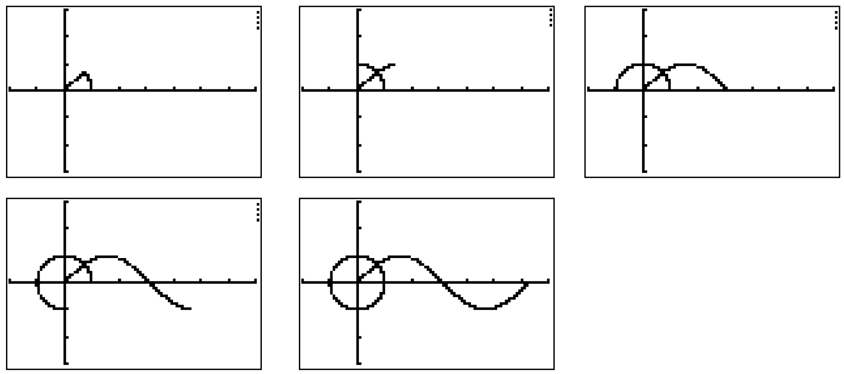


3. Press WINDOW and enter these settings.




Note:
- The circle has radius = 1 unit, so its circumference is $2\pi$ units.
- The T-values increase in increments of 0.1 units to a maximum of $2\pi$ units, or one complete revolution.
- The graphs are defined by *x* and *y* in terms of the period T.

4. Press GRAPH.



5. To stop the graph, press ENTER. To continue, press ENTER.

## Investigate Response (pages 248–249)

**Investigate A**

**5., 6.** The traced curve should cross the equator at 0°, 180°, and 360°. It will reach a maximum displacement at 90° and a minimum displacement at 270°. Depending on where the elastic was placed, these values may be shifted but the relative positions will remain the same.

**7.** The function is periodic. The line represents the displacement from the equator for one orbit. As the satellite follows the same path repeatedly, the same pattern will be followed.

**8.** The satellite is following a circular path around a spherical object. Its motion can be divided into 360° per rotation. During the rotation, the path moves progressively farther away from and closer to the equator to both the north and south. This corresponds to the pattern followed by a typical sine curve. What is not certain is whether it is a perfect fit or an approximation. There are inherent flaws in the method of measurement.

**9.** Steeper angles will result in similar curves with greater amplitude.

**Investigate B**

**Method 1**

3.

| Angle, $x$ | Height | Height ÷ Radius, $y$ |
|---|---|---|
| 0° | Measurements may vary. | 0 |
| 15° | | 0.26 |
| 30° | | 0.5 |
| 45° | | 0.71 |
| 60° | | 0.87 |
| 75° | | 0.97 |
| 90° | | 1 |
| 105° | | 0.97 |
| 120° | | 0.87 |
| 135° | | 0.71 |
| 150° | | 0.5 |
| 165° | | 0.26 |
| 180° | | 0 |
| 195° | | −0.26 |
| 210° | | −0.5 |
| 225° | | −0.71 |
| 240° | | −0.87 |
| 255° | | −0.97 |
| 270° | | −1 |
| 285° | | −0.97 |
| 300° | | −0.87 |
| 315° | | −0.71 |
| 330° | | −0.5 |
| 345° | | −0.26 |
| 360° | | 0 |

4. The result should be two periods of a sine graph.
5. I measured the vertical displacement from the horizontal axis of the terminal points for angles intersecting a circle and divided these values by the radius. This is an exact model of the definition of the sine ratio.
6. **a)** The amplitude of a sine function is 1.
   **b)** The period of a sine function is 360°.
   **c)** The $x$-intercepts of a sine function are 0°, 180°, and 360°.
   **d)** The $y$-intercept of a sine function is 0.
   **e)** The domain of the function is all real numbers. Imagine angles of any size either positive or negative as repeated rotations around a circle.
   **f)** The range of the function is $\{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$.
   **g)** The function is increasing for $0° < x < 90°$ and $270° < x < 360°$. The function is decreasing for $90° < x < 270°$.
7. A good strategy would be to add 360° to each angle in the original table. These are coterminal angles so all of the ratios will be identical. These angles will give the ordered pairs for a second period up to 720°.

**Method 2**

See the corresponding questions in method 1 for solutions.

## Communicate Your Understanding

- Encourage students to discuss answers with a partner. Take up their responses as a class before assigning the homework.
- You may wish to use **BLM 5–8 Section 5.3 Investigate the Sine Function** for remediation or extra practice.

**Ongoing Assessment**

- While students are working through the Investigates, circulate and see how well each works. This may be an opportunity to observe and record individual students' learning skills. Use **BLM A–17 Learning Skills Checklist**.

**Accommodations**

**Motor**–encourage students to work in pairs or small groups

**Language**–simplify instructions and provide additional scaffolding for the Investigates

**Communicate Your Understanding Responses (page 252)**

**C1** **a)** Measure time for the horizontal axis and the directed displacement of the pendulum from the midline for the vertical axis.

**b)** It is not immediately obvious that a sine function will be a good model because the function is not generated by the motion of a point moving around a circle and the angle is not being measured from 0° to 360°. However, if I do not worry too much about the input values not matching up to angles I see that the output values are periodic as the pendulum moves back and forth and range between −1 and 1 just like a sine function. If I match the time units up to the appropriate angles then the sine function should work well to approximate the displacement of the pendulum.

**C2** This method is based on the symmetry of a sine function. Once I see the pattern of marking the angles, which are 30° before and after the multiples of 180° at a height of ± 0.5, I can quickly mark the ordered pairs without using a calculator to figure out each sine ratio needed. Once I know the pattern it should make the task easier.

## Practise, Connect and Apply, Extend

- The Practice questions consolidate understanding of the key features of a sine curve. All questions should be assigned. **Question 4** is a simple application of the sine curve and provides a real reference to the concepts. If time permits, take up the Practise questions in class before students leave for the day.
- **Question 6** refers to the Chapter Problem. You may wish to suggest to students that they flag their answer so they can refer to it in the Chapter Problem Wrap-Up.
- **Question 7** is another simple application of the sine wave. It requires students to know their special angles.
- For **question 9**, students should make a table or use a graphing calculator as in Investigate B. Ask a follow-up question: "If the sine curve relates to the vertical displacement from a horizontal surface, how can you relate the cosine curve to real life?" (Horizontal displacement from a vertical surface.)

## Literacy Connections

- Have students write a journal entry describing the key features of a sine curve. Ensure they include a diagram.

## Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectations | Selected Questions |
|---|---|
| Problem Solving | 6–9 |
| Reasoning and Proving | 4–6 |
| Reflecting | 7 |
| Selecting Tools and Computational Strategies | 1, 6, 7, 9 |
| Connecting | 8 |
| Representing | 1–6, 8, 9 |
| Communicating | 1, 4–6, 9 |