5.4 Investigate Transformations of Sine Curves

Student Text Pages
254–263

Suggested Timing
160–240 min

Materials and Technology Tools
- CBR™ (calculator-based ranger)
- chalk
- graphing calculators
- grid paper
- link cable

Related Resources
- BLM G–1 Grid Paper
- BLM T–6 Using the CBR™ (Calculator-Based Ranger)
- BLM 5–9 Section 5.4 Investigate Transformations of Sine Curves
- BLM 5–10 Section 5.4 Achievement Check Rubric

Teaching Suggestions

- Each Investigate is very time-consuming. You may wish to carry out Investigate A over two days, one for translations and the other for stretches and reflections.
- It is important that students learn transformations by investigation using technology. The visual approach allows students to see the transformation and to communicate what they see. You can then assess their understanding based on their responses.
- In this course, it is important to remember that each transformation (vertical stretch, horizontal translation, and vertical translation) is applied individually, not in combination, when graphing by hand. Horizontal stretches (i.e. changes in period) are not included in the expectations. With the use of technology, students may interpret applications of sine curves containing multiple transformations.

Investigate

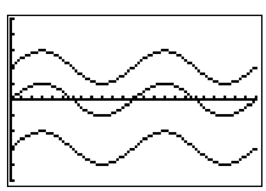
- Distribute copies of **BLM G–1 Grid Paper**.
- For **Investigate A**, have students summarize each transformation in a chart after completing each part of the Investigate. Then provide plenty of examples of each type of transformation.
- You may wish to have students change the calculator to **simul** mode (see notes in Section 5.3, Investigate). Then have them stop and start the graph by pressing ENTER. Have students note the relative positions of the maximum values for the two graphs. Students should see that there is a 90° phase shift to the right.
- **Investigate B** relates a distance-time graph to the sine function, which is a new concept for students. This Investigate will help kinesthetic and visual learners understand the relationship in a clearer manner. Have students work in pairs. If necessary, allow students to try the activity more than once to get a good curve. Rather than using chalk on the floor, you may wish to use a hoola-hoop or masking tape.

Investigate Responses (pages 254-357)

Investigate A

A: Vertical Translations

5. a)



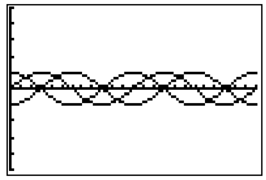
b) The three graphs are identical with respect to amplitude, period, and domain. They differ in terms of their vertical location.
$y = \sin x$ is horizontally bisected by $y = 0$ and has range $\{y \in \mathbf{R} \mid -1 \le y \le 1\}$.
$y = \sin x + 2$ is horizontally bisected by $y = 2$ and has range $\{y \in \mathbf{R} \mid 1 \le y \le 3\}$. $y = \sin x - 3$ is horizontally bisected by $y = -3$ and has range $\{y \in \mathbf{R} \mid -4 \le y \le -2\}$.

6. The graph of $y = \sin x + 4$ will be horizontally bisected by $y = 4$ and will have range $\{y \in \mathbf{R} \mid 3 \le y \le 5\}$. The graph of $y = \sin x - 1$ will be horizontally bisected by $y = -1$ and will have range $\{y \in \mathbf{R} \mid -2 \le y \le 0\}$.

7. The graph of $y = \sin x + c$ is a vertical translation by c units of the graph of $y = \sin x$.

B: Horizontal Translations

2. a)



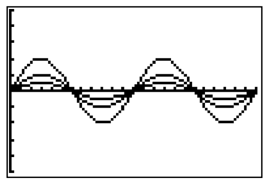
b) The graphs are identical in terms of period, amplitude, domain, range, and the horizontal line that bisects them. They differ in the location of the x-intercepts and the maximum and minimum points. The basic sine graph is shifted left or right.

3. The graph of $y = \sin (x - 30°)$ is a horizontal shift right by 30° of the graph of $y = \sin x$. The graph of $y = \sin (x + 30°)$ is a horizontal shift left by 30° of the graph of $y = \sin x$. This is confirmed when checked with a graphing calculator.

4. The graph of $y = \sin (x - d)$ is a horizontal translation by d units of the graph of $y = \sin x$. Note that a positive value of d results in a right shift but will be seen in the equation preceded by a minus sign.

C: Vertical Stretches or Compressions

2. a)



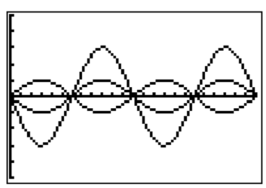
b) The graphs are identical in terms of period, domain, and the horizontal line that bisects them. They differ in terms of amplitude and range.
The graph of $y = 2 \sin x$ has amplitude 2 and range $\{y \in \mathbf{R} \mid -2 \le y \le 2\}$.
The graph of $y = \frac{1}{2}\sin x$ has amplitude $\frac{1}{2}$ and range $\{y \in \mathbf{R} \mid -\frac{1}{2} \le y \le \frac{1}{2}\}$.

3. The graph of $y = 3 \sin x$ will be a vertical stretch of the graph of $y = \sin x$ by a factor of 3. The amplitude will be 3 and the range will be $\{y \in \mathbf{R} \mid -3 \le y \le 3\}$. The graph of $y = \frac{1}{3}\sin x$ will be a vertical compression of the graph of $y = \sin x$ to $\frac{1}{3}$ of its original height. The amplitude will be $\frac{1}{3}$ and the range will be $\{y \in \mathbf{R} \mid -\frac{1}{3} \le y \le \frac{1}{3}\}$. This is confirmed when checked with a graphing calculator.

4. For positive values of a, the graph of $y = a \sin x$ is a vertical stretch by factor a when a is greater than 1 and is a vertical compression of the amplitude by factor a when a is between 0 and 1.

D: Vertical Reflections and Stretches or Compressions

2. a)



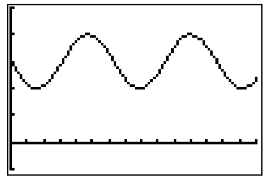
b) The graphs are identical in terms of period, domain, and the horizontal line that bisects them. They differ in terms of amplitude, range, and initial direction.
The graph of $y = -\sin x$ has identical range and amplitude to $y = \sin x$. However, it is reflected in the x-axis.
The graph of $y = -3 \sin x$ has identical range and amplitude to $y = 3 \sin x$. It is a reflection in the x-axis of $y = 3 \sin x$.

3. The graph of $y = -2 \sin x$ will be a reflection in the x-axis of the graph of $y = 2 \sin x$ as described in Part C above. All of its characteristics except for initial direction will be shared with $y = 2 \sin x$. The graph of $y = -\frac{1}{2} \sin x$ will be a reflection in the x-axis of the graph of $y = \frac{1}{2}\sin x$ as described in Part C above. All of its characteristics except for initial direction will be shared with $y = \frac{1}{2} \sin x$. This is confirmed when checked with the graphing calculator.

4. For negative values of a, the graph of $y = a \sin x$ is a vertical stretch of factor a and a reflection in the x-axis when a is less than -1 and a reflection in the x-axis and a vertical compression of the amplitude to the absolute value of a when a is between 0 and -1.

Investigate B

5. Graphs may vary depending on the speed at which the student walks. What the graphs should have in common will be distance readings which start at 3 m, shrink to a minimum of 2 m, increase to a maximum of 4 m and return to 3 m to repeat the pattern as many times as the speed chosen will allow. A possible graph is shown.



6., 7. The distance from the wall to the centre of the circle is constant. As you walk on the circular path your distance from the horizontal axis is determined by a sine function. The measured distance is the difference between the fixed distance and the sine value at any time.

8. Based on the angle of rotation, x, the function for the distance to the wall is $y = 3 - \sin x$ or $y = -\sin x + 3$.

9. **a)** If the walk started at the point closest to the wall the graph would shift left by 90°. A possible equation would be $y = 3 - \sin(x + 90)$.

b) In this case if the student walks at the same speed the graph from the CBR would change in two ways. The shape and horizontal axis would remain the same. The amplitude would reduce from 1 m to 0.5 m. The period with respect to time will be cut in half because the circumference of the circle has been cut in half. However, if the graph is based on rotation, the speed factor is not significant. The new equation would be $y = 3 - 0.5 \sin x$, assuming a start from 3:00 o'clock.

c) The centre of the circle is now only 2.5 m from the wall but all other variables are unchanged. The graph will be identical to the first except the horizontal axis will now be at 2.5 m instead of 3 m. The new equation will be $y = 2.5 - \sin x$.

Examples

- You may wish to work through the Examples before doing Investigate B, in order to consolidate students' learning from Investigate A.
- Stress the key features of the transformed sine curve: period, amplitude, phase shift, domain, and range.
- Have students draw the graph of $y = \sin x$ as a reference curve for graphing the transformed curve. Using coloured pencils for the graphs also helps visual learners.

Communicate Your Understanding

- Have students work on **question C1** alone. Encourage students to discuss their ideas for **question C2** with a partner.
- Discuss the answers as a class before assigning homework questions.
- You may wish to use **BLM 5–9 Section 5.4 Investigate Transformations of Sine Curves** for remediation or extra practice.

Communicate Your Understanding Responses (page 260)

C1 **a)** $y = -\sin x$ is a reflection in the x-axis of $y = \sin x$.

b) $y = \sin x + 7$ is a vertical translation up 7 units of $y = \sin x$.

c) $y = \sin(x - 40°)$ is a horizontal translation by 40° to the right of $y = \sin x$.

C2 When I graphed transformations of $y = x^2$ I plotted the basic parabola. I then applied vertical stretches based on the value of a, horizontal shifts based on the value of h, and vertical shifts based on the value of k. The same principles can be applied to graph transformations of $y = \sin x$. Treat the starting point of (0, 0) in the same way as the vertex (0, 0) of $y = x^2$ and transform one period using the values given for a, h, and k. Once one period has been graphed, repeat the pattern as often as required.

Common Errors

- Some students may apply the horizontal phase shift in the incorrect direction.

R_x Have students refer to transformations of parabolas. Because the transformation is applied to the *x*-coordinate, before the function is applied, it is the inverse of the way it is read.

Ongoing Assessment

- **Question 16** is an Achievement Check. Use **BLM 5–10 Section 5.4 Achievement Check Rubric** to assess students' responses.

Accommodations

Gifted and Enrichment–give students opportunities to create and design their own problems involving the transformation of sine functions. Challenge students to apply multiple transformations to the graph of $y = \sin x$ and write the equation of the result.

Student Success

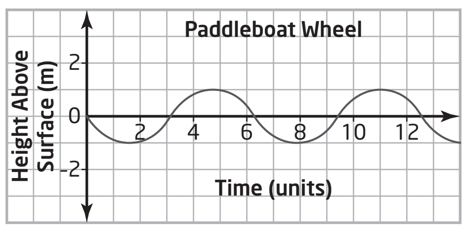
- Have students write a journal entry to explain how transformations of quadratic functions and transformations of sine functions are related.

Practise, Connect and Apply, Extend

- The Practise questions consolidate learning of the key concepts. All questions should be assigned. Use the students' answers as a diagnostic tool to determine their level of understanding of transformations.
- Many Connect and Apply questions focus on a single key feature. **Questions 7 to 11, and 13** cover the basic concepts.
- **Question 12** develops students' understanding of the periodic nature of the sine function.
- **Question 14** refers to the Chapter Problem. You may wish to have students flag their solution so they can refer to it in the Chapter Problem Wrap-Up.
- For **question 15**, remind students to refer to 0 as the starting point of the clock hand.
- **Question 16** is an Achievement Check question. Provide students with **BLM 5–10 Section 5.4 Achievement Check Rubric** to help them understand what is expected.
- For **question 17**, suggest that students graph the function first.
- For **questions 18 and 19**, suggest to students that they visualize all scenarios of how the situation could occur.

Achievement Check Sample Solution (page 263, question 16)

a)



b) The equation is $y = -\sin x$.

c) If the wheel rotates counterclockwise, the graph would be reflected in the *x*-axis and the equation would be $y = \sin x$.

d) If the radius were 2 m, the amplitude would be doubled. The graph would be vertically stretched by a factor of 2 and the new equation would be $y = -2 \sin x$.

e) Other examples of machinery with motion that can be modelled by a periodic sine function are car pistons, oil derricks, clock pendulums, gears, and so on. The equations would all have a sine function shape with various amplitudes and periods.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	10, 14–19
Reasoning and Proving	4, 10, 12, 14, 16
Reflecting	16
Selecting Tools and Computational Strategies	1–3, 7–9, 11, 13, 19
Connecting	14–16
Representing	1, 5–9, 13, 15–18
Communicating	2–4, 12, 14, 16