Task: Working with Realistic Exponential Functions

Student Text Pages
340–341

Suggested Timing
45–75 min

Materials and Technology Tools
- grid paper and rulers
- graphing calculators

Related Resources
- BLM 6–18 Chapter 6 Task Rubric

Accommodations

Motor–encourage students to use technology for graphing

Language–have students work in pairs, especially to write a word problem in **question 7**, part a).

Ongoing Assessment

- Use **BLM 6–18 Chapter 6 Task Rubric** to assess student achievement.

Specific Expectations

1.3, 1.4, 1.6, 2.2, 2.3

Teaching Suggestions

- Discuss the simulation in **Part I** with the class, making sure everyone understands what takes place.
- After students have completed **question 2**, have them describe their graphs. Using their graphs and the table of values, have them describe the type of function that is represented. As **question 3** points out, the first few values appear exponential. Ask students to explain why the later values may not follow this exponential pattern. (The number of people in the room is finite, so people start to shake hands with other infected people because there are fewer and fewer uninfected people in the room.)
- Consider having students work in pairs to develop ideas of other relationships that can be represented by exponential functions in **Part II**. They can then work individually (or with a classmate if they need language help) to illustrate and describe the function and to write and solve the word problem.
- Have students share the problems they create so that others can solve them.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- Do the explanations make sense?
- Are descriptions thorough?
- Are the word problems creative? Do they contain enough information to solve the problem?
- Are the calculations correct?
- Do the graphs present the data correctly?

Level 3 Notes

- Student presents a reasonable exponential relationship.
- Student describes exponential functions accurately, using appropriate vocabulary.
- Student includes accurate, fully-labelled graphs.
- Word problem is relevant and includes all necessary data.
- Student thoroughly explains and justifies the solution.

Level 3 Sample Response

Part I

1.

Signal Number, s	Number of People Infected, I	Ratio
0	1	--
1	2	2
2	4	2
3	8	2
4	16	2
5	32	2
6	62	1.938
7	118	1.903
8	226	1.915
9	402	1.779
10	658	1.637
11	896	1.362
12	1006	1.123
13	1022	1.016
14	1024	1.002

2. See graph for question 4, parts b) and c).

3. a) The ratio between the successive numbers of people infected is 2 for the first five values. Since the ratios are constant, the relation appears to be exponential.

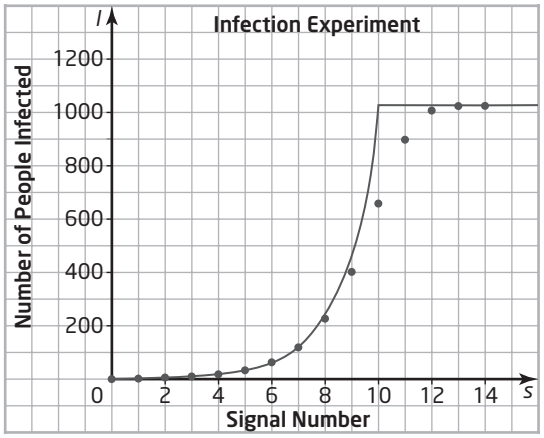
b) $I = 2^s$

c) i) The function is doubling with each successive signal.
ii) The slope is small for $s < 0$.
iii) The slope is great for $s > 0$.

4. a)

s	$I = 2^s$
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

b), c)



5. a) No. After the 10th signal, the growth rate of the number of people infected starts to decrease, so it does not grow exponentially.

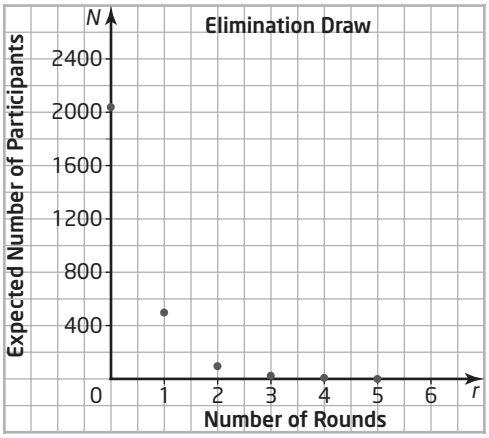
b) The model representing the spread of infection approximately fits an exponential function over the domain $\{s \in \mathbf{N} \mid 0 \leq s \leq 8\}$ and the range $\{s \in \mathbf{N} \mid 1 \leq I \leq 226\}$.

Part II

6. Answers may vary. My exponential function models a charity elimination draw.

a) $N = 2048\left(\frac{1}{4}\right)^r$, where N is the expected number of participants left after r rounds.

b)



c) ratio: 0.25, domain where slope is small: $\{r \in \mathbf{N} \mid r \geq 3\}$, domain where slope is great (in the negative direction): $\{r \in \mathbf{N} \mid r \leq 3\}$

7. a) A charity elimination draw is held with 2048 people participating. Each round, each person tosses a coin twice. If the coin does not come up heads twice, the participant is eliminated. How many rounds are expected before the number of participants is reduced to 2?

b) Algebraic solution: I substituted 2 for N in the equation for the expected number of participants left and simplified.

$$N = 2048\left(\frac{1}{4}\right)^r$$

$$\mathbf{2} = 2048\left(\frac{1}{4}\right)^r$$

$$\frac{1}{1024} = \left(\frac{1}{4}\right)^r$$

I know that $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ so $r = 5$.

Graphical solution: I looked for the point on the graph where $N = 2$ and found it was when $r = 5$.

What Distinguishes Level 2

- Student presents an exponential relationship, but it does not fit the data well.
- Student partially describes exponential functions, or uses non-standard vocabulary.
- Graphs show the general shape of the function only.
- Word problem may be missing important information.
- A solution is provided, but is not fully explained or justified.

What Distinguishes Level 4

- Student presents one or more exponential relationships that have been tailored to fit the data.
- Student describes exponential functions accurately and thoroughly, using appropriate vocabulary.
- Student includes detailed graphs.
- Word problem is creative and relevant, including all necessary data.
- Solution shows insight into the complexities of the situation.