6.1 The Exponent Rules

Student Text Pages
280–287

Suggested Timing
60–70 min

Materials and Technology Tools
- calculators

Related Resources
- BLM 6–3 Section 6.1 The Exponent Rules

Teaching Suggestions

- Calculators, especially scientific calculators, may vary greatly in how calculations with exponents are entered. Before beginning the investigation, check that all students are familiar with using their calculators to multiply and divide expressions that involve exponents.

Investigate

- Consider having students work in pairs or small groups to complete the investigation.
- Emphasize the role that patterning plays in deriving the exponent rules. Students can use these patterns if they become confused by applying the roles, especially when under pressure, such as on a test.
- Summarize the three exponent rules as a class after students finish the Investigates.
- Students often state the division rule as "subtract the exponents". This leads to errors later when they run into questions where the exponent in the denominator is greater than the exponent in the numerator, and they are tempted to always subtract the lesser from the greater. A more precise statement is "subtract the exponent in the denominator from the exponent in the numerator". Encourage students to use this more precise formulation.

Investigate Responses (pages 280-282)

Investigate A

1. a) $3^5 \times 3^4 = 19\ 683$. This value is equivalent to 3^n when $n = 9$.

b) The value of $n = 9$ for the single power was the sum of the exponents 5 and 4.

c) When two powers with a common base are multiplied they will form a single power whose exponent is the sum of the exponents from the two original powers.

d) Examine the product. $7^2 \times 7^4 = 117\ 649$; $7^6 = 117\ 649$. Compare exponents: $2 + 4 = 6$

e) Yes.

f) Examples may vary.

2.

Product	Expanded Form	Number of Factors	Single Power
$3^5 \times 3^4$	$(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$	9	3^9
$2^3 \times 2^2$	$(2 \times 2 \times 2) \times (2 \times 2)$	5	2^5
$5^4 \times 5^3$	$(5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5)$	7	5^7
$7^2 \times 7^4$	$(7 \times 7) \times (7 \times 7 \times 7 \times 7)$	6	7^6

3. In each case the sum of the exponents in the product is equal to the exponent in the single power. I can express the product of two powers with the same base as a single power with the same base and an exponent equal to the sum of the exponents from the powers in the product. Algebraically: $a^m \times a^n = a^{m+n}$.

Investigate B

1. a) $\frac{4^5}{4^3} = 16$. This value is equivalent to 4^n when $n = 2$.

b) The value of $n = 2$ for the single power was the difference of the exponents 5 and 3.

c) When two powers with a common base are divided they will form a single power whose exponent is the difference of the exponent from the power in the numerator minus the exponent from the power in the denominator.

d) Examine the quotient: $\frac{2^8}{2^5} = 8$; $2^3 = 8$. Compare exponents: $8 - 5 = 3$

e) Yes.

f) Examples may vary.

2.

Quotient	Expanded Form	Number of Factors	Single Power
$\frac{4^5}{4^3}$	$\frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4}$	2	4^2
$\frac{3^7}{3^2}$	$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$	5	3^5
$\frac{5^4}{5^2}$	$\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$	2	5^2
$\frac{2^8}{2^5}$	$\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$	3	2^3

3. In each case the difference of the exponents in the quotient is equal to the exponent in the single power. I canexpress the quotient of two powers with the same base as a single power with the same base and an exponent equal to the difference of the exponent from the power in the numerator minus the exponent from the power in the denominator.

Algebraically: $a^m \div a^n = a^{m+n}$ or $\frac{a^m}{a^n} = a^{m-n}$

Investigate C

1.

Power of a Power	Expanded Form	Number of Factors	Single Power
$(2^3)^4$	$(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$	12	2^{12}
$(4^2)^3$	$(4 \times 4) \times (4 \times 4) \times (4 \times 4)$	6	4^6
$(3^3)^2$	$(3 \times 3 \times 3) \times (3 \times 3 \times 3)$	6	3^6
$(5^3)^3$	$(5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5)$	9	5^9

2. In each case the exponent in the single power is the product of the two exponents in the power of a power. I can rewrite a power of a power as a single power by keeping the same base and replacing the exponent with the product of the two original exponents.

3. a) Applying my rule, I predict $(-2)^2(-2)^3 = (-2)^5$. A calculator gives a value of -32 for both expressions.

b) Applying my rule, I predict $\frac{(-3)^4}{(-3)^2} = (-3)^2$. A calculator gives a value of 9 for both expressions.

c) Applying my rule, I predict $[(-5)^3]^2 = (-5)^6$. A calculator gives a value of 15 625 for both expressions.

Examples

- As students work through **Example 1**, ask them to justify the selection of rules at each step.
- Remind students to use the proper order of operations. Post the rules for BEDMAS on a bulletin board for easy reference.
- For **Example 2**, students may need to be reminded of the meaning and calculation of probability. For parts b) and c), you could have the class flip coins and compare the experimental probability to the theoretical probability.

Common Errors

- Some students will subtract the lesser exponent from the greater exponent when dividing powers.

R$_x$ Reinforce the precise statement of the division rule and have students subtract the exponent in the denominator from the exponent in the numerator.

- When simplifying expressions involving powers, students have difficulty determining the correct sign for the answer.

R$_x$ Review how to determine the sign of the answer when negative bases are involved.

- In questions involving fractions, such as **question 10**, students will be tempted to drop the brackets around the fraction, especially in the last line.

R$_x$ You can demonstrate that this is not mathematically correct using **question 10**, part b) as an example to show that $\left(\frac{4}{5}\right)^2$ is not the same as $\frac{4^2}{5}$.

- Students make errors when cancelling exponents, such as

$$\frac{3^4}{3^2} = \frac{3^{\cancel{4}\,2}}{3^{\cancel{2}\,1}} = \frac{9}{3} = 3$$

R$_x$ Have students evaluate the original expression and the one with cancelled exponents to see that they are not equal.

- Students might cancel the bases instead of the exponents. For example:

$$\frac{3^4}{3^2} = \frac{\cancel{3}^4}{\cancel{3}^2} = \frac{4}{2} = 2$$

R$_x$ Show students this example and ask them to explain why this is not a valid procedure.

Communicate Your Understanding

- The truth of the conjecture in **question C1** can be tested by simply evaluating each side using a calculator. However, it is worthwhile to expand each side using repeated multiplication and show why it works.

$$(3^2)^4 = (3^4)^2$$
$$(3 \times 3)^4 = (3 \times 3 \times 3 \times 3)^2$$
$$(3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3) = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$$
$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

- **Question C2** illustrates one of the most common errors that students make when dividing powers. It is worth taking the time to show why subtracting the lesser exponent from the greater exponent does not work all of the time.
- For **question C3**, take some time to discuss the prediction of the answer's sign. This becomes more important when students work with negative exponents. Some will have difficulty understanding why an expression like $(2)^{-3}$ does not result in a negative answer.
- You may wish to use **BLM 6–3 Section 6.1 The Exponent Rules** for remediation or extra practice.

Communicate Your Understanding Responses (page 285)

C1 $(3^2)^4$ must have the same value as $(3^4)^2$. I can support my answer in two ways. I can evaluate both sides, getting a value of 6561 for each. Or I can state both expressions as single powers, each of which must be 3^8.

C2 Sandor found the difference between the two exponents but subtracted the exponent for the numerator from the exponent for the denominator. For any quotient the order in which the division is performed is important. Sandor should have subtracted the exponent for the denominator from the exponent for the numerator. The correct answer is 12^{-4} or $\frac{1}{12^4}$.

C3 The value of $(-2)^3$ must be a negative integer. In this case I am taking the product of an odd number of negative factors, which must have a negative result. I know that the product of an even number of negative factors must be positive, while the product of an odd number of negative factors must remain negative. The value of a positive exponent indicates the number of factors in the expansion of the power. It follows that any power formed by a negative base raised to an even positive exponent must have a positive value while any such power raised to an odd positive exponent must have a negative value.

Practise, Connect and Apply, Extend

- Reinforce the prediction of the answer's sign in problems that involve negative bases, such as **question 1**, part c), and part d) of **questions 2 and 3**.
- For **questions 4 to 6**, caution students against combining too many operations in one step.
- In **question 7**, some students may have difficulty in making the jump from numerical bases to algebraic bases. It may be necessary to do an example or two as a class.
- In **question 8**, ensure that students put brackets around both the numerator and denominator, although the former is not really required. Demonstrate what happens if the expression is entered into a calculator without the brackets.
- In **question 11**, the use of an exponent as a scale is often confusing to students. It may be useful to expand the discussion on the Richter scale, giving examples.

Accommodations

Perceptual–provide students with a copy of the probability chart in **question 12** on which they can highlight or circle the favourable outcomes in order to count them

Memory–have students make a list of the exponent rules from their investigations and refer to it as needed

ESL–have students work with a classmate to isolate the parts of **question 11** that are required to find a solution

Student Success

- Encourage students to perform only one step on each line until they are very comfortable with the exponent rules and how they affect an expression.

- In **question 14**, binary numbers are usually written as zeros and ones. For example, 1101 represents $1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$ or $8 + 4 + 0 + 1 = 13$. This notation will be used in the next section. There is no need to introduce it here.

Career Connections

- The Richter scale is used by seismologists. Have students watch for other exponential functions throughout this chapter that are used by people in different fields. They could research the qualifications necessary for these careers.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12, 13, 16
Reasoning and Proving	8, 11, 13, 16
Reflecting	15, 16
Selecting Tools and Computational Strategies	1–7, 8, 10, 12, 14–16
Connecting	11
Representing	9, 12, 14
Communicating	8, 11, 13, 16