6.2 Evaluate Powers With Integer Exponents

Student Text Pages
288–295

Suggested Timing
60–70 min

Materials and Technology Tools
- graphing calculators
- computers with Internet access

Related Resources
- BLM 6–4 Section 6.2 Evaluate Powers with Integer Exponents
- BLM 6–5 Section 6.2 Achievement Check Rubric

Teaching Suggestions

- This section lends itself particularly well to development by investigation.

Investigate

- Consider having students work in small groups. Each group could present part of the Investigate to the class.
- Emphasize the role of patterning in deriving the meaning of negative exponents. Students can use this patterning later if they have difficulty remembering the exponent rules.

Investigate Responses (pages 288-289)

1.

Power	Value	Power	Value	Power	Value
2^4	16	3^4	81	5^4	625
2^3	8	3^3	27	5^3	125
2^2	4	3^2	9	5^2	25
2^1	2	3^1	3	5^1	5
2^0	1	3^0	1	5^0	1

In each case as we continue to divide to produce the next term it appears that the value of a power with an exponent of 0 must be 1.

2. Any power that has an exponent of 0 must have a value of 1 regardless of the value of its base.

3.

Power	Value	Power	Value	Power	Value
2^4	16	3^4	81	5^4	625
2^3	8	3^3	27	5^3	125
2^2	4	3^2	9	5^2	25
2^1	2	3^1	3	5^1	5
2^0	1	3^0	1	5^0	1
2^{-1}	$\frac{1}{2}$	3^{-1}	$\frac{1}{3}$	5^{-1}	$\frac{1}{5}$
2^{-2}	$\frac{1}{4}$	3^{-2}	$\frac{1}{9}$	5^{-2}	$\frac{1}{25}$
2^{-3}	$\frac{1}{8}$	3^{-3}	$\frac{1}{27}$	5^{-3}	$\frac{1}{125}$
2^{-4}	$\frac{1}{16}$	3^{-4}	$\frac{1}{81}$	5^{-4}	$\frac{1}{625}$

4. a) For any two powers with the same base that have exponents that are opposite integers I find that the values of the powers will be reciprocals. The power with the positive exponent has a value greater than 1 and the power with the negative exponent has a value of 1 divided by the value of the first power.

b) From the table, $2^{-1} = \frac{1}{2}$, $2^{-2} = \frac{1}{2^2}$, $2^{-3} = \frac{1}{2^3}$, …. In general, $2^{-n} = \frac{1}{2^n}$

c) The pattern holds for all powers in the chart regardless of base. In general, $a^{-n} = \frac{1}{a^n}$.

5. According to the rule, $10^{-3} = \frac{1}{10^3}$. Using a calculator, both sides have a value of 0.001.
$10^3 = 1000$, so the rule works.

Examples

- For Method 1 in **Example 1**, be sure to include the second line for at least one part. Students need to understand what is happening here, rather than just memorizing a rule.
- For Method 2, students often confuse the negative key with the subtraction key on a graphing calculator. It might be wise to have students do an example or two, such as $-1 - (3)$. The first sign makes the integer negative, and is done with the negative key, while the second indicates an operation, and is done with the subtraction key.
- Many students have never used their scientific calculators to display fractions, as suggested in the Technology Tip. Review (or introduce) this operation with students. Consider grouping students who own the same calculator model together.
- **Example 3** offers several opportunities to reinforce the precise statement of the division rule.
- Students who have musical training could elaborate on **Example 4**. They could explain why there are 12 notes in a scale, and 8 notes in an octave. If you have access to a small keyboard, you could demonstrate the connection between mathematics and music. You might draw the sound waves to show the pattern in two waves where the frequency of one is 2^{-1} times the frequency of the other. Ask students how two notes that are an octave apart sound when played together. Can they draw a connection between the sound and the relationship of the wave frequencies?

Communicate Your Understanding

- Powers involving fractions are often stumbling blocks for many students. Check that students can follow **question C1** and show a step-by-step development.
$$\begin{aligned}\left(\frac{1}{2}\right)^{-3} &= \left(\frac{2}{1}\right)^{3}\\ &= 2^3\\ &= 8\end{aligned}$$
- Take some time to develop the answers to **question C2**. Many students may have difficulty determining the correct sign of the answer.
- The most straightforward approach to **question C3** is to attempt the pattern and then identify why it does not work.
- You may wish to use **BLM 6–4 Section 6.2 Evaluate Powers with Integer Exponents** for remediation or extra practice.

Communicate Your Understanding Responses (page 293)

C1 I can express $\frac{1}{2}$ as a power with a negative exponent.

$$\begin{aligned}\left(\frac{1}{2}\right)^{-3} &= (2^{-1})^{-3} && \text{Convert the fraction to a power then multiply exponents.}\\ &= 2^3\\ &= 8\end{aligned}$$

Or I can work from first principles.

$$\begin{aligned}\left(\frac{1}{2}\right)^{-3} &= \frac{1}{\left(\frac{1}{2}\right)^{3}}\\ &= \frac{1}{\left(\frac{1}{8}\right)}\\ &= 1 \times \frac{8}{1}\\ &= 8\end{aligned}$$

Note that a negative exponent applied to a base that is greater than 1 will result in a positive fractional value that is less than 1. However, a negative exponent applied to a fractional base will result in a value that is greater than 1.

Common Errors

- Students do not apply the division rules for powers correctly.

R_x Have students write out the subtraction, linking the numbers in the subtraction to the exponents in the question. Remind students of the rules for subtracting integers.

- When simplifying expressions involving powers, students have difficulty determining the correct sign for the answer.

R_x Review how to determine the sign of the answer when negative bases are involved.

- Some students do not apply the rules for integer subtraction properly for division that involves two negative exponents. For example:

$$\frac{2^{-2}}{2^{-5}} = 2^{-2-5} = 2^{-7}$$

R_x Have students show solutions for the first few questions in the exercise with only one operation per step. Students can also check their answers for reasonableness. $2^{-2} > 2^{-5}$ so the answer should be greater than 1.

- Some students make successive errors that cancel each other, such as:

$$3^{-1} = -3 = \frac{1}{3}$$

R_x Have students perform one operation in each step so they can check for errors more easily. Have students check their answers for reasonableness.

C2 **a)** Yes, the two expressions are equivalent.

$$(-3)^{-3} = \frac{1}{(-3)^3} = -\frac{1}{27} \qquad -3^{-2} = -\frac{1}{3^3} = -\frac{1}{27}$$

b) No, the expressions have opposite signs.

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9} \qquad -3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$$

c) Powers with negative exponents represent repeated division by the base not an actual negative value. 4^{-2} represents $\frac{1}{16}$ or a division by 4^2. In general a negative exponent creates a positive fraction, not a negative integer.

C3 No. The procedure in the Investigate required repeated divisions by the base of the power. In the case of a power of 0, I would be performing repeated divisions by 0, but the operation of division is not well defined for division by 0. There are two conflicting arguments. The first states that any power of 0 has a value of 0. The second is that any number divided by itself is 1, which is the key argument for demonstrating that $x^0 = 1$. In this case, the argument is no matter how many times you multiply 0 by itself you still get 0. This works well for positive integer exponents ($0^1 = 0^2 = 0^3 = \ldots = 0$) but does not work if the exponent is zero. The idea of cancelling matching factors to equal 1 does not hold either. Since 0^0 cannot equal 0 and 1, 0^0 is not defined.

Practise, Connect and Apply, Extend

- For **questions 1 to 3**, many students will memorize a pattern, and jump to the final answer. It is wise to insist on a step-by-step development for at least some parts of each question.
- For **question 5**, which involves decimals raised to negative exponents, remind students that conversion to a fraction is often the best first step.
- For **questions 7 and 8**, some students might combine too many steps in one line, leading to errors. Encourage multi-step solutions at this stage.
- For **question 9**, it is instructive to first write the decimal as a fraction.
- For **question 11**, recall that raising pitch by an octave implies a doubling, while lowering pitch by an octave implies halving.
- For **question 14**, it might be useful to review how a magic square works before extending the concept to a Magic Product Square.
- **Question 15** is an Achievement Check Question. Provide students with **BLM 6–5 Section 6.2 Achievement Check Rubric** to help them understand what is expected.

Achievement Check Sample Solution (page 295, question 14)

a) The product of each row, column, and diagonal is 8. Therefore, this is a Magic Product Square.

b)

2^4	2^{-3}	2^2
2^{-1}	2^1	2^3
2^0	2^5	2^{-2}

c) The sum of exponents in each row, column, and diagonal is 3.

d)

2^7	2^0	2^5
2^2	2^4	2^6
2^3	2^8	2^1

The sum of the exponents in each row, column, and diagonal is 12. The product of each row, column, and diagonal is 2^{12}. Therefore this is a Magic Product Square.

e) The cube of the middle square in the original question is 8. The cube of the middle square in part d) is 2^{12}. Both these answers were the products in their questions.

Ongoing Assessment

- **Question 14** is an Achievement Check question. Use **BLM 6–5 Section 6.2 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Perceptual–have students rewrite equations such as the one in **question 13** with values in place before they evaluate them

Motor–provide a calculator with large buttons

Memory–have students add the rules for evaluating integer exponents to the list they began in Section 6.1 and refer to the list as needed

Career Connections

- Students could research careers that make a connection between music and mathematics.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12–15
Reasoning and Proving	12
Reflecting	12, 13
Selecting Tools and Computational Strategies	1–15
Connecting	9, 11
Representing	6, 7, 9, 10
Communicating	12