6.4 Model Data With Exponential Functions

Student Text Pages
305–311

Suggested Timing
60–70 min

Materials and Technology Tools
- 24 number cubes in a plastic container
- grid paper
- computers with Internet and E-STAT access
- graphing calculators
- slide rule (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 6–7 Section 6.4 Model Data with Exponential Functions

Teaching Suggestions

- Probability experiments like the one in the Investigate provide a hands-on demonstration of a process that can be modelled using an exponential function.

Investigate

- Number cubes can be purchased cheaply in quantity at most dollar stores. A faster version of this investigation flips a large number of coins, removing any that turn up heads.
- Although the experiment can be simulated on software such as *Fathom*™ or with the random number generator on a graphing calculator, using physical number cubes helps emphasize to students the real-life and spontaneous nature of the process.

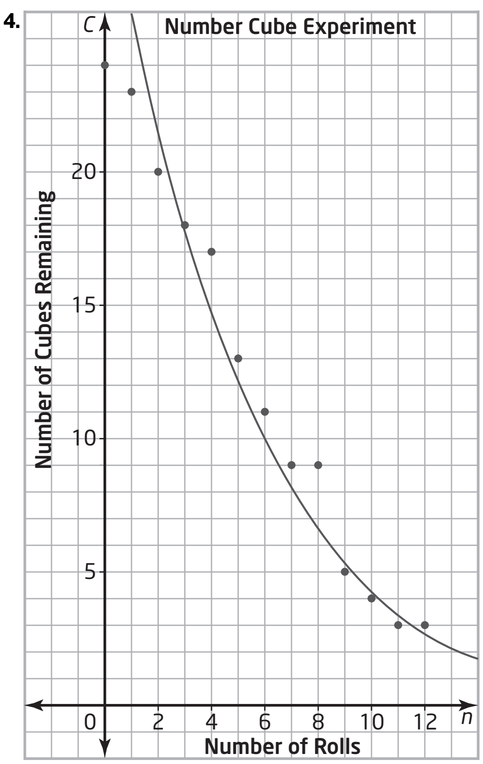
Investigate Responses (pages 305–306)

Results may vary. For example:

1. to 3.

Number of Rolls	Number of Cubes Remaining
0	24
1	23
2	20
3	18
4	17
5	13
6	11
7	9
8	9
9	5
10	4
11	3
12	3

4.



The slope of the graph is negative at all times. Moving from left to right, the graph becomes flatter. In terms of integers, this means the slope is increasing but in terms of magnitude the size of the slope values becomes smaller.

5. For this data set it appears that the half-life is about four or five rolls. A long-term average should approach the theoretical probability for the experiment resulting in an average half-life of slightly less than 4 rolls.

Examples

- **Example 1** extends the notion of finite differences to ratios, allowing students to identify an exponential relationship from a table of numbers. This is similar to the way they identified linear and quadratic relationships in earlier courses using finite differences.
- **Example 2** shows some of the data that may be downloaded from Statistics Canada through the E-STAT portal. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links. The solutions show three methods that use technology and a pencil and paper method. You are not expected to use all of them. Select one or two as convenient.
- You may also review the ready-made *Fathom*™ file 6.4 Example 2 CPI on the Web site.

Communicate Your Understanding

- The verification of Moore's Law in **question C1** makes another good student project, combining Internet research with mathematical modelling using exponential functions.
- You may wish to use **BLM 6–7 Section 6.4 Model Data with Exponential Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 309)

C1 This is a good example of exponential growth. I have a regular time interval for which I can use the same multiplier to find the number at the end of the interval based on the number at the beginning of the interval. If I examine the ratios for any set of values which are based on evenly spaced times I will find a common ratio. These are the necessary characteristics for an exponential relation.

C2 Given a graph of a relation:

- If the graph is a straight line having a constant slope for any two points chosen on the line, that the relation must be linear.
- If the graph is a parabola, it must be a quadratic relation. If I am not certain whether the shape is a parabola I can prepare a table of values to test.
- If the graph increases continuously from a very small positive slope to a progressively larger positive slope, it might be exponential growth. I can prepare a table of values to test. Or, it might be exponential decay if the graph decreases continuously with a very large negative slope that becomes progressively flatter without changing sign.

Given an equation of a relation:

- If the variable has exponent 1, then the relation is linear. This may include equations written in the form, $y = mx + b$ or $Ax + By + C = 0$ or variants of these.
- If the expression is a polynomial of degree 2 then the relation is quadratic. The relation could be expressed in vertex form, standard form, or factored form.
- If an expression has a positive base with an exponent containing a linear variable then it is an exponential relation.

Given a table of values for a relation:

- If the first differences are constant, it is a linear relation.
- If the second differences are constant, it is a quadratic relation.
- If the ratios of consecutive terms are constant, it is an exponential relation.

Practise, Connect and Apply, Extend

- In **question 1**, a table of values is a useful tool for analysis. Students can check for first and second differences, and ratios, and classify the situation according to the appropriate function model.
- For **question 2**, a quadratic model can appear to be exponential over short distances. Encourage students to take measurements from the graph to construct a table of values and then analyse the table using differences and ratios.
- For **question 3**, it might be necessary to discuss why the cooling curve for a liquid should be exponential. When the liquid is hot, there is a large difference in temperature between the liquid and its surroundings. Hence, it cools at a fast rate. As its temperature drops, the temperature difference also drops, resulting in a slower rate of cooling.
- For **question 4**, a demonstration using a slide rule will add interest to this question.
- For **question 5**, go to *www.mcgrawhill.ca/functionsapplications11* and follow the links for an applet that demonstrates chain reactions.
- Before assigning **question 7**, ensure that you have demonstrated E-STAT in class so that students are familiar with the interface. They will need to navigate through several screens, making choices along the way, before obtaining their data.
- **Question 9** is intended to be done without developing an exponential model for the cooling water but rather by reasoning. To have the water cool to 24° after an additional 8 h would require a linear relationship not an exponential relationship.
- **Question 10** is a variation on Zeno's paradox. Another interesting variation is known as Thompson's Lamp. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links for more information.

Literacy Connections

- Students could write creative or humorous problems based on the mathematics in **questions 9 and 10**. These problems could be posted for others to enjoy.

Career Connections

- Have students research the high school and university courses required for a bachelor of science in pharmacology.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5, 6, 8–11
Reasoning and Proving	1–6, 8, 9, 11
Reflecting	n/a
Selecting Tools and Computational Strategies	4, 6, 10
Connecting	6–9
Representing	3– 7
Communicating	1– 5, 7–11

Common Errors

- Some students may mistake a graph of a quadratic or other growth for exponential growth.

R_x Have students take measurements from the graph and then use a table of values and ratios to confirm exponential growth.

- Some students have difficulty determining the type of relation because they plot too few data points.

R_x Ensure students plot a large number of data points that cover a wide domain so they can clearly see what type of relation the graph represents.

Accommodations

Visual and Perceptual–the graphs in **question 2** could be presented on separate axes to avoid confusion among intersecting curves and colours

Language–consider providing starter sentences to guide students as they write explanations in several of the questions in this section