6.5 Exponential Functions and Their Properties

Student Text Pages
312–318

Suggested Timing
60–70 min

Materials and Technology Tools
- graphing calculator
- grid paper (optional)

Related Resources
- BLM G–1 Grid Paper (optional)
- BLM 6–8 Section 6.5 Exponential Functions and Their Properties

Teaching Suggestions

- The Investigate introduces exponential relations directly from data collected by the students. This is a valuable approach, especially for the visual, hands-on learner.
- Technology such as a graphing calculator or graphing software allows the appropriate value of the base b to be found quickly, while presenting a changing visual representation of the relation as a consequence of the changes in the base.

Investigate

- You can simulate the telephone tree in the classroom. Select a student to play the part of the secretary who "calls" two other students. These two students stand up to show they have been called and then each select two other students. The process continues until everyone in the class is standing. Keep a tally on the blackboard.

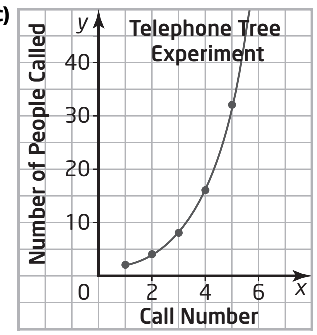
Investigate Responses (pages 312-313)

1. a) On the first call two people are called. On the second call, the two recipients phone two people each. Four people are called. On the third call, the four recipients phone two people each. Eight people are called.

b)

Call Number	1	2	3	4	5
People Called	2	4	8	16	32

c)



2. Need to restrict the values for both call number and number of people called to natural numbers. Decimal or negative values would not be meaningful.

3., 5. Let x represent the number of the call in the sequence. Let y represent the number of people called for the specified call number. I predict $y = 2^x$ will model the data. This relation is graphed on the data plot and provides a perfect match at the data points.

4. Have to try larger values of b if the curve is below the data or smaller values of b if the curve is above the data.

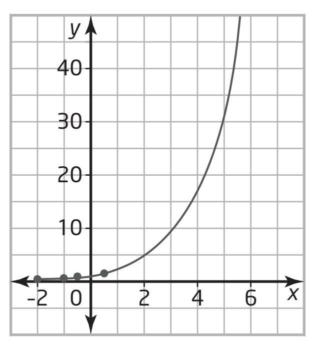
6. a) A base of 2 is reasonable because the number of calls made is always double the number made in the previous round. Each recipient makes two calls.

b) If each person called three people then the base of the exponential relation would be 3.

7. a)

x	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-2
$y = 2^x$	1.414	0.707	0.5	0.25

b) The four new points all lie on the curve defined by $y = 2^x$.



8. **a)** As I choose points farther to the left, the value of y gets smaller. The value will not reach zero because each step to the left results in a division by two of the previous positive y-value. Dividing a positive number by two gives a number that is smaller but will always remain positive.

b) Each step to the right doubles the y-value. Outside the context of the physical model there is no mathematical limit to how large the y-values may become.

Examples

- **Example 1** requires recall of concepts related to functions, and also introduces new ones. Review and apply the vertical line test for a function. Review the meanings of domain and range, and how they are expressed using set builder notation. Ensure that students understand why there are no x-intercepts. Introduce the concept of a horizontal asymptote. Use technology to show that the value of y will not reach zero, even for large negative values of x. Ensure that all of these concepts are well understood before moving on.
- **Example 2** demonstrates the influence of the base on the shape of the graph. Contrast bases greater than 1 with those less than 1. What happens if a base equal to 1 is used?
- Students will have worked with mapping diagrams (**Example 3**) and function machines (**Example 4**) in earlier grades.

Communicate Your Understanding

- Ask students to predict their answers to **question C1** before sketching the graphs. Then they should graph the functions on the same set of axes.
- **Question C2** is not a trivial question. Students tend to think of exponents as having only whole number or integral values. Students will need to think of the exponent as taking on real number values. It might be helpful for students to begin with a few possible positive values of the base a, then select a negative value and look for values of x that will and will not yield an answer.
- You may wish to use **BLM 6–8 Section 6.5 Exponential Functions and Their Properties** for remediation or extra practice.

Communicate Your Understanding Responses (page 316)

C1 Assign values $a = 2$ and $b = 3$ and prepare a small table to identify the intervals requested. The conclusions drawn will be valid for any choice of a and b as defined.

x	-2	-1	0	1	2
$y = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$y = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

The functions will have equal value when $x = 0$. For positive values of x, the function with the larger base will have the greater value: $f(x) < g(x)$ for $x > 0$. For negative values of x, the function with the smaller base will have the greater value: $f(x) > g(x)$ for $x < 0$.

C2 When I define an exponential function, $f(x) = a^x$, I expect a continuous function that moves smoothly from point to point along a steadily increasing or decreasing path. As long as I assign a positive value to the base of the function a, this is what I get.

If I define a as a negative value two key problems arise. The first has to do with the even/odd property of powers. If I consider only natural numbers for the values of x I will find that the magnitude of the y-values increases

Common Errors

- Some students may mistake quadratic growth for exponential growth.

R_x Have students take measurements and calculate ratios to confirm the type of relation.

- Some students will conclude that a relation is a function because the mapping diagram given shows no multiple values of *y* for a given value of *x*.

R_x Emphasize that a mapping diagram is more useful in showing that a relation is not a function. Since it contains only a finite number of ordered pairs, it may appear to be a function by missing values that show it is not a function.

- Some students may determine a possible relation for a function machine from a subset of the ordered pairs given. It works for these subsets, but not for the remaining points. For example, any two points can be used to generate a linear relation. However, if the real relation is quadratic, it will not correctly predict a third ordered pair.

R_x Ensure that students check the relation for all given ordered pairs.

Accommodations

Spatial–give students who have trouble visualizing a telephone tree small items such as paper clips that they can use to build one. Each paper clip can represent one person called.

ESL–the problems in this section contain some words that may not yet be in students' vocabularies: *potential*, *wavelength*, *silt*, *asteroid*. Have students identify any words they are not familiar with and find their meanings by asking another student or by using a dictionary before they attempt the problems.

exponentially but the signs of the *y*-values keep switching. For example, a base of −2 would produce the pattern: −2, 4, −8, 16, ... I lose the sense of a continuously increasing or decreasing relation.

The second problem has to do with even and odd roots. Consider the exponent 1.5. This may be interpreted as $\frac{3}{2}$ which is interpreted as the square root of the base cubed. If the base is negative I am attempting to find the square root of a negative value, which is not defined. As a result there will be numerous holes in the graph where the value of the power cannot be evaluated.

As a result, I cannot use negative values for the base of an exponential function, even though individual calculations are possible for some expressions of this type.

Practise, Connect and Apply, Extend

- **Question 1** can be done as a team competition. Divide the class in half, and have each student write down the matches. Have teams exchange papers, and quickly determine the correct answers. The team with the most correct answers wins.
- For **question 2**, students should predict the answers first, then use calculators or graphs to check their predictions.
- For **questions 4 and 5**, take time to have selected students explain their reasoning to the class.
- For **questions 7 and 8**, the effects of depth on light intensity and colour when scuba diving are apparent in photographs and videos taken by scuba divers.
- The media error suggested in **question 9**, part b), is very common. You can find other examples in newspapers, magazines, or other media reports. These articles are excellent sources for real-world, contextual questions for tests and summative assessments.

Literacy Connections

- Use **question 9**, part b), as an example of why you have to read texts critically. Show students other examples of misrepresented or misleading statistics or have students gather examples.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	7– 11
Reasoning and Proving	2, 6, 9–11
Reflecting	1, 10
Selecting Tools and Computational Strategies	2, 7, 9, 10
Connecting	6–11
Representing	4, 5, 7, 8, 10
Communicating	3, 6, 9, 11