# 6.6 Compare Linear, Quadratic, and Exponential Functions

**Student Text Pages**
319–325

**Suggested Timing**
60–70 min

**Materials and Technology Tools**
- rulers
- computers with *Fathom*™
- graphing calculators (optional)
- spreadsheet software (optional)

**Related Resources**
- BLM 6–9 Section 6.6 Compare Linear, Quadratic, and Exponential Functions
- BLM 6–10 Section 6.6 Using a Computer Algebra System

## Teaching Suggestions

- The appropriate selection of a modelling function is of great importance in statistical analysis of data derived from direct research. Finite differences or ratios are important tools in helping to determine an appropriate modelling function. These are especially useful in differentiating between quadratic and exponential growth.

### Investigate

- The Investigate reviews and consolidates skills that students have used before. Although you may be tempted to skip it to allow more time for the applications, consider using it as a solid summary of previous knowledge to set the stage for a proper analysis of more complicated problems.
- The Investigate also provides an opportunity for review of technology skills in working with lists of numbers. Graphing calculators, spreadsheets, and *Fathom*™ are all suitable technologies for this section.

**Investigate Responses (pages 319-320)**

**a), b)** **Relation A**

| $x$ | $y$ | First Differences | Second Differences | Ratio of $y$-values |
|---|---|---|---|---|
| −3 | −6 | 2 | 0 | $\frac{2}{3}$ |
| −2 | −4 | 2 | 0 | $\frac{1}{2}$ |
| −1 | −2 | 2 | 0 | 0 |
| 0 | 0 | 2 | 0 | undefined |
| 1 | 2 | 2 | 0 | 2 |
| 2 | 4 | 2 | | 1.5 |
| 3 | 6 | | | |

In Relation A, the first differences are equal. This makes the relation linear. The ratios of terms change continually and are not significant in classifying the relation. The graph has a slope of 2 and a $y$-intercept of 0. Its equation is $y = 2x$.

**Relation B**

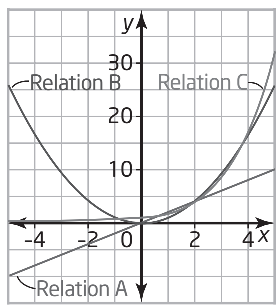
| $x$ | $y$ | First Differences | Second Differences | Ratio of $y$-values |
|---|---|---|---|---|
| −3 | 9 | −5 | 2 | $\frac{4}{9}$ |
| −2 | 4 | −3 | 2 | $\frac{1}{4}$ |
| −1 | 1 | −1 | 2 | 0 |
| 0 | 0 | 1 | 2 | undefined |
| 1 | 1 | 3 | 2 | 4 |
| 2 | 4 | 5 | | 2.25 |
| 3 | 9 | | | |

In Relation B, the second differences are equal. This makes the relation quadratic. The ratios of terms change continually and are not significant in classifying the relation. The graph has a vertex at (0, 0) and follows a pattern of perfect squares. Its equation is $y = x^2$.

**Relation C**

| $x$ | $y$ | First Differences | Second Differences | Ratio of $y$-values |
|---|---|---|---|---|
| −3 | 0.125 | | | |
| | | 0.125 | | 2 |
| −2 | 0.25 | | 0.125 | |
| | | 0.25 | | 2 |
| −1 | 0.5 | | 0.25 | |
| | | 0.5 | | 2 |
| 0 | 1 | | 0.5 | |
| | | 1 | | 2 |
| 1 | 2 | | 1 | |
| | | 2 | | 2 |
| 2 | 4 | | 2 | |
| | | 4 | | 2 |
| 3 | 8 | | | |

In Relation C, the first differences and second differences are identical to the $y$-values. This is a special relationship that is only observed for this particular relation. The key is that neither the first nor the second differences take on a constant value. The ratio of $y$-values is a constant of 2. This makes the relation exponential and the constant ratio will also provide the base of the exponential function. Its equation is $y = 2^x$.

**c)** 


The graphs confirm the classifications in part b).

**d)** Relation A has equation $y = 2x$. Relation B has equation $y = x^2$. Relation C has equation $y = 2^x$.

## Example

- Take time to demonstrate a tangent line to a curve, as well as its slope. There is value in using a straight edge to represent a dynamic physical model of the tangent line.
- The **Example** represents a function as an equation, a table of values, a graph, and in words in terms of the changing slope of a moving tangent line. Once the drawings and calculations are finished, take some time to make connections among the various representations. For example, compare the table of values and the graph in terms of intervals of increase and decrease.
- You can use dynamic geometry software to create a sketch that models the tangent line and its slope at a point on the curve. You can drag this point slowly from left to right, watching how the slope changes. As an extension, you can plot the slope of the tangent against $x$ such that it appears as a point. Then, turn on the Trace feature for the point to trace out the changes in the slope as the original point is dragged along the curve.

## Communicate Your Understanding

- Students could expand on **question C1** by researching world population growth.

- **Question C2** invites differing methods of solution, such as graphing, algebra, or a computer algebra system (CAS). If a CAS is available, this is a good opportunity to demonstrate its usefulness. After students have predicted all possible solutions, use the **solve(** function from the **F2** menu to find solutions. Reconcile any discrepancies. Take time to consider the negative solution, and its meaning. Supply students with copies of **BLM 6–10 Section 6.6 Using a Computer Algebra System**.
- You may wish to use **BLM 6–9 Section 6.6 Compare Linear, Quadratic, and Exponential Functions** for remediation or extra practice.

**Communicate Your Understanding Responses (page 322)**

**C1** World population growth is best modelled by an exponential relation. Any situation where the rate of growth depends on the size of the population tends to be exponential. Stating that a population grows by a certain percent each year specifies an exponential model and the growth factor needed for it.

The actual exponential model tends to be valid until there is a change in the conditions for growth. For example, major technological changes may introduce an increase in growth. Catastrophic disease can cause a decrease in growth. Overall an exponential model will provide the best fit for population growth.

**C2** The graphs of $y = 2x$, $y = x^2$, and $y = 2^x$ must all intersect when $x = 2$. This is because they all share a $y$-value of 4 for this value of $x$.

## Practise, Connect and Apply, Extend

- For **questions 1 to 3**, ensure that students can justify their answers.
- For **question 4**, take a moment to consider other models besides linear, quadratic, or exponential.
- For **question 6**, you can use an applet to demonstrate Newton's Law of Cooling for various initial temperatures. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links.
- For **question 7**, scuba tanks and regulators are inspected, and possibly overhauled, on a yearly basis. A diver whose equipment fails at depth is in serious trouble. This is the major reason for the "buddy system". Solo diving, although not illegal, is a dangerous activity.
- For **question 10**, natural gas can be liquefied, with difficulty, and presents hazards in transportation and storage. There is some controversy over using liquefied natural gas (LNG) as a motor fuel, transporting it in ships or railroad cars, or storing it in stationary tanks. This is a good opportunity to connect mathematics with social issues.
- For **question 11**, a real-world example of a structure like "hexaville" is a honeycomb in a beehive. Consider having students mark the count at each stage in the spread of the secret using a different colour to help avoid confusion or accidental double counts.

## Career Connections

- In previous sections, students have considered careers that involve the use of formulas. This section shows examples of analysing data by looking for patterns that resemble a type of function and making predictions based on that pattern. Have students gather information about careers that require examining patterns and making prediction based on them. What might be interesting about these careers?

## Common Errors

- Some students may be confused by Newton's Law of Cooling in **question 6**, especially the dual effects of the temperature of the liquid, and the difference between this temperature and the ambient temperature.

$R_x$ Use the applet mentioned in the question 6 note to demonstrate changes in the graph as a result of changing these temperatures, one at a time. Ask students to predict the changes before using the applet to demonstrate.

- Some students will count the same item more than once in a complex counting exercise, such as **question 12**.

$R_x$ Have students mark the items they count at each stage using a different colour to help avoid multiple counts.

## Accommodations

**Visual and Perceptual**–present the graphs in **question 2** on a separate set of axes to avoid confusion among different curves and colours

**Motor**–present the tables in **question 4** on a separate page. Include spaces for students to write the differences and ratios.

## Student Success

- Encourage students to examine functions in various forms, and to become familiar with what a particular function looks like as a graph, as a table of values, and as an equation. This will help them to distinguish quickly and accurately among quadratic, exponential, linear, and other functions.

## Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectations | Selected Questions |
|---|---|
| Problem Solving | 7–12 |
| Reasoning and Proving | 3, 5–13 |
| Reflecting | 6 |
| Selecting Tools and Computational Strategies | 4 |
| Connecting | 5–10 |
| Representing | 6 |
| Communicating | 1, 2, 4, 5, 7, 9–13 |