6.7 Exponential Growth and Decay

Student Text Pages
326–333

Suggested Timing
60–70 min

Materials and Technology Tools
- graphing and scientific calculators
- grid paper (optional)
- computers with *The Geometer's Sketchpad®* (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 6–11 Section 6.7 Exponential Growth and Decay
- BLM 6–12 Section 6.7 Achievement Check Rubric

Teaching Suggestions

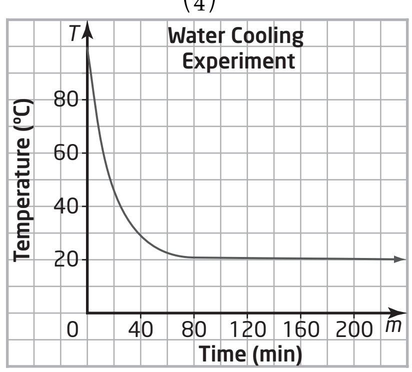
- Most real-world situations that can be modelled using exponential relations will require some translation and stretching of the simple exponential function. The equations of these relations become increasingly complicated to represent. However, the same features occur time after time, such as a vertical shift. Point out these features, and continually revisit them as you work through the section.

Investigate

- The intent of the Investigate is to demystify the rather complicated relation presented to model the temperature of a cup of coffee as it cools over time. This structure will be seen again as the section progresses.
- Technology makes it easy to see the effect of each parameter in the relation, as in **steps 3** to **5**.
- The concept in **step 5** is important, since it allows students to build models for time steps other than 1 unit. Ensure that this useful concept is fully understood. The Chapter Problem Wrap-Up may also make use of this concept.
- For **step 6**, a ready-made graph using *The Geometer's Sketchpad®* is available. Go to *www.mcgrawhill.ca/functionsapplications11* and follow the links.
- Have students express the domain and range using set builder notation. Students are required to restrict the domain and range to construct a model that accurately represents the data.

Investigate Responses (pages 326-327)

1. In addition to a value for the base and an expression for the exponent I have also added a constant, 20, at the end of the function and multiplied the usual power by a coefficient, 80 at the beginning.
2. When $m = 0$, $T(0) = 100$. This corresponds to an initial temperature of 100°C when $m = 0$ min.
3. $T(60) = 80\left(\frac{3}{4}\right)^{\frac{60}{5}} + 20 \doteq 22.5$ $\quad T(300) = 80\left(\frac{3}{4}\right)^{\frac{300}{5}} + 20 \doteq 20$ $\quad T(1200) = 80\left(\frac{3}{4}\right)^{\frac{1200}{5}} + 20 \doteq 20$

   At a time of 1 h, the water is still slightly warmer than room temperature. Larger time values produce a constant value of 20° from the function. This corresponds to the fact that the water will eventually reach and remain at room temperature.
4. The parameter 80 corresponds to the initial difference in temperature between the water and the room. The parameter 20 corresponds to the room temperature.
5. If $m = 5$ then $80\left(\frac{3}{4}\right)^{\frac{5}{5}} = 60$. If $m = 10$ then $80\left(\frac{3}{4}\right)^{\frac{10}{5}} = 45$.
6. 

Common Errors

- Some students may confuse a growth model with a decay model.

R_x Have students inspect the base and exponent of the modelling function. If the base is greater than 1, and the exponent is positive, the equation represents a growth model. Summarize the four possibilities in a table:

	Exponent Positive	Exponent Negative
Base greater than 1.	growth model	decay model
Base greater than 0, but less than 1.	decay model	growth model

- As models become more complex, some students may forget to include a set of brackets around the exponential expression when evaluating the function on a calculator.

R_x Reinforce the extra set of brackets each time students are asked to make such a calculation.

Ongoing Assessment

- While students are working, circulate to observe how well each works. This is an opportunity to observe and record individual student's learning skills.
- **Question 9** is an Achievement Check question. Use **BLM 6–12 Section 6.7 Achievement Check Rubric** as a summative assessment tool.

7. The domain of this function in the real world would be restricted to the time interval between when the cup of water reached its maximum heat and the time at which the temperature of the room changed or the time the water was dumped. The range will have a firm upper limit of 100°C and a variable lower limit, which depends on the temperature of the room.

Examples

- Students often confuse the growth rate with the base of the exponential function. Use additional examples, if necessary, to ensure that this concept is well understood. The second example shown below models a stock that is decreasing in value.
  - The population of a town is modelled using the function $P = 20\ 000(1.025)^t$. The base of the function is 1.025. The growth rate is 0.025.
  - The value of a stock is modelled using the function $V = 1000(0.95)^t$. The base of the function is 0.95. The growth rate is −0.05.
- For **Example 1**, a real experiment in bacterial growth is limited by the food supply available. Although the growth at first is exponential, it then decreases as the food supply is used up. The familiar logistic curve results. Students may bring up this point.
- For **Example 2**, students are asked to use the exponential model in reverse. Technology simplifies the "guess and check" approach. Students can refine answers correct to several significant digits in a few minutes. Alternatively, the **intersect(** function on a graphing calculator will yield the answer in one step.

Communicate Your Understanding

- **Question C1** is an example of how reading critically can help students understand what is really being said in a text.
- The answer to **question C2** should include an example of growth, and one of decay, showing the sequence of numbers generated by each of the bases.
- After discussing **question C3**, you may want to introduce the concept of the logistic model in a general way.
- You may wish to use **BLM 6–11 Section 6.7 Exponential Growth and Decay** for remediation or extra practice.

Communicate Your Understanding Responses (page 330)

C1 Compare the ratios of values: $\frac{212}{200} = 1.06$, $\frac{228}{212} \doteq 1.075$, $\frac{248}{228} \doteq 1.088$
By a strict definition this is not an exponential relation because the ratios are not identical. On the other hand the ratios are very similar so I plotted the points. I could draw a single exponential curve of best fit that was a close approximation.

C2 In an exponential model, the base is the factor by which each y-value is multiplied to produce the next y-value. Multiplying by a value greater than 1 gives a product that is larger than the value I multiplied. As a result, a base value greater than 1 guarantees an increasing function or exponential growth. Multiplying by a value less than 1 gives a product that is smaller than the value multiplied. As a result, a base value less than 1 guarantees a decreasing function or exponential decay.

C3 The growth of bacteria requires nutrients. Also cells die and produce toxins. If the nutrient medium is not replenished, the bacteria will run out of food. Increased levels of toxins also affect the bacteria's survival rate and ability to divide. The typical population curve for a Petri dish culture actually has an exponential growth phase, a stable phase (flat line) at the top followed by an exponential decay phase as the bacteria die off in the absence of nutrients.

Accommodations

Gifted and Enrichment–students could write crime scenarios and solutions similar to **question 5** using other exponential functions they have encountered in this chapter. They can share their problems with the class to solve.

Memory–supply students with the table in Common Errors to refer to as they solve growth and decay problems

Student Success

- Help students to develop facility manipulating exponential equations to achieve desired graphs. At first, it will be difficult for them to predict how to manipulate a function so that it matches a given graph but with practice they will become adept at knowing what to add or multiply to achieve the result.

Practise, Connect and Apply, Extend

- **Question 4** makes a connection between mathematics and medicine. Determining the magnitude of an overdose is important to the selection of the appropriate medical treatment.
- **Question 5** makes a connection between mathematics and detection of crime, similar to popular television programs such as CSI.
- **Question 6** can be extended to consider decisions made by business executives. Storing the seafood at higher temperatures decreases refrigeration costs, but increases spoilage rates. A proper analysis will determine the optimum temperature to maximize revenues.
- For **question 7**, americium-241 is a man-made, silvery metal that sells for about $1500 US per gram. It emits primary alpha radiation, which does not penetrate human skin. This is why smoke detectors do not pose a health risk.
- **Question 9** is an Achievement Check question. Provide students with **BLM 6–12 Section 6.7 Achievement Check Rubric** to help them understand what is expected.

Achievement Check Sample Solution (page 332, question 9)

a) The carpet has an area of 2 m^2 to start. After one wish the length is now 1 m and the width is $\frac{2}{3}$ m so the area is $\frac{2}{3}$ m^2. After two wishes the length is now $\frac{1}{2}$ m and the width is $\frac{4}{9}$ m so the area is $\frac{4}{9}$ m^2. As a formula, the area, A, is:

$$A = 2\left(\frac{1}{2}\right)^n\left(\frac{2}{3}\right)^n$$
$$= 2(3^{-n})$$

where n is the number of wishes granted.

b) 200 cm^2 = 0.02 m^2

Solve the equation.

$$2(3^{-n}) = 0.002$$
$$3^{-n} = 0.01$$
$$3^{-n} = \frac{1}{100}$$
$$3^n = 100$$

$3^4 = 81$ and $3^5 = 243$, so Sinbad will receive 4 wishes.

Literacy Connections

- Students can use point form notes to summarize the important facts of the crime in **question 5** before they attempt to solve the problem.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	1–13
Reasoning and Proving	3, 5, 8, 10, 12
Reflecting	8
Selecting Tools and Computational Strategies	1–13
Connecting	1–8, 10–13
Representing	4–9, 11–13
Communicating	3, 5, 10, 12

Student Text Page
334–335

Suggested Timing
20 min

Materials and Technology Tools
- graphing calculator

Accommodations

Visual–provide access to a calculator with a large (or projected) display

Perceptual–if some students have trouble entering the values from a table, consider having them work with a classmate for this activity

Exponential Regression on a Graphing Calculator

Specific Expectations

2.2

Teaching Suggestions

- Students should be familiar with linear and quadratic regressions from previous courses. If this is not the case, consider using examples for linear and quadratic regressions from earlier grades before attempting an exponential regression.
- **Step 1**, part b), places the regression equation in variable Y1. You can choose a different location, if desired. In part d), you can also use the **value** function from the **CALC** menu (press 2nd, then **TRACE**) to obtain a precise selection for *x*.
- In **step 2**, part b), the **ZoomStat** feature will automatically select window settings appropriate for the data being graphed. In part c), note that the exponential regression always assumes an exponent of *x*. It is not possible to use intervals of more than 1, so it does not match the model given in the question.