7.2 The Compound Interest Formula

Student Text Pages
355–361

Suggested Timing
75–110 min

Materials and Technology Tools
- graphing calculators
- calendars (optional)

Related Resources
- BLM 7–5 Section 7.2 The Compound Interest Formula
- BLM 7–6 Section 7.2 Achievement Check Rubric

Teaching Suggestions

- If technology is available, have students work through the section using both technology and pencil and paper methods. Ultimately, technology tools are needed to solve financial problems involving compound interest. Pencil and paper methods may be beneficial for students to understand the concepts being presented.

Investigate

- **Method 1** should be done as a teacher demonstration to illustrate how the compounding of interest works and how the compound interest formula is derived.
- Have students work through **Method 2** to reinforce the concepts illustrated in Method 1.

Investigate Responses (pages 355–357)

Method 1

1.

Year	Amount at Start of Year ($)	Amount at End of Year ($)
1	5000	$P(1 + rt)$ $= 5000(1.07)$
2	$5000(1.07)$	$P(1 + rt)$ $= 5000(1.07)(1.07)$ $= 5000(1.07)^2$
3	$5000(1.07)^2$	$P(1 + rt)$ $= 5000(1.07)^2(1.07)$ $= 5000(1.07)^3$
4	$5000(1.07)^3$	$P(1 + rt)$ $= 5000(1.07)^3(1.07)$ $= 5000(1.07)^4$
5	$5000(1.07)^4$	$P(1 + rt)$ $= 5000(1.07)^4(1.07)$ $= 5000(1.07)^5$
6	$5000(1.07)^5$	$P(1 + rt)$ $= 5000(1.07)^5(1.07)$ $= 5000(1.07)^6$

2. Using the pattern in the table, the amount at the end of Year 10 is represented by the expression $5000(1.07)^{10}$. Since $5000(1.07)^{10} \doteq 9835.76$, the amount at the end of Year 10 is $9835.76.

3. **a)** The expression $5000(1.07)^n$ models the amount of the investment at the end of Year n.

b) When $n = 10$, the expression becomes $5000(1.07)^{10}$, which is the same as the expression used in **question 2** to find the amount at the end of Year 10.

4. The rate of growth of an investment earning compound interest must be exponential because the amount is calculated by repeated multiplication by the factor $(1 + rt)$.

5. For an investment with an initial value of P earning interest at a rate of r per year, compounded annually, for n years, the amount A can be modelled by the formula $A = P(1 + rt)^n$.

Method 2

3. The scatter plot is in the form of an exponential curve.
5. The rate of growth of an investment involving compound interest is exponential.
6. The exponent in the expression represents the number of years of investment.
8. a) The investment earns $350 in the 1st year.
   b) The investment earns $490.89 in the 6th year.
   c) The investment earns $643.46 in the 10th year.
   d) The investment earns $4898.19 in the 40th year.

Examples

- You may wish to have students work through both examples. **Example 1**, which involves an annual compounding situation, is a straight forward presentation of the use of the compound interest formula. **Example 2** involves compounding where the compounding period is a fraction of a year and, thereby, requires a deeper understanding on the student's part.
- Explain to students that time lines help visualize the compounding periods.
- Ask students to write down the values of i, P, and n before substituting into the formula so that they can look back at what they are substituting, and why.

Communicate Your Understanding

- Explain to students that the answer to **question C1** can be yes and no, depending on the interest rate. If the interest rates are the same, the compound interest investment will generate more interest. If the interest rate of the simple interest investment is higher, the interest generated may outperform the compound interest investment.
- **Question C3** is important for determining if the student has made the connection between the previous work on exponential functions and compound interest, a common example of exponential growth.
- You may wish to use **BLM 7–5 Section 7.2 The Compound Interest Formula** for remediation or extra practice.

Communicate Your Understanding Responses (page 359)

C1 If two investments pay the same interest rate per year over the same term, the investment earning compound interest will always generate more interest than the investment earning simple interest.

If two investments pay different interest rates per year, it is possible for simple interest to outperform compound interest over the same term when the simple interest rate is high enough to outperform the effect of compounding.

For two investments, one involving a higher simple interest rate and the other involving a lower compound interest rate, the compound interest earned will eventually outperform the simple interest earned given sufficient time. This is because compounded interest grows exponentially at an increasing rate while simple interest grows at a fixed rate.

C2 The amount of compound interest earned depends on four variables: principal, annual interest rate, compounding period, and term of investment.

The greater the principal (the original invested amount), the greater the interest earned.

The higher the annual interest rate, the greater the interest earned.

The shorter the time interval for a compounding period, the greater the interest earned.

The longer the term of the investment, the greater the interest earned.

C3 Exponential growth involves the repeated multiplication of a starting value by a growth factor or common ratio. The amount of a compound interest investment grows by repeated multiplication of the principal by the factor $(1 + i)$, or by raising $(1 + i)$ to the exponent n, where n is the number of compounding periods. So, compound interest is an example of exponential growth.

Practise, Connect and Apply, Extend

- Depending on time, not all parts of all questions need be assigned.
- **Questions 1 and 2** are the keys to determining student understanding of the concepts. Students who are unable to complete these questions will find problem-solving in later questions extremely difficult. Reinforcement of the concepts may be necessary for some students.
- **Question 3** may help students who struggle with questions 1 and 2. A more hands-on approach of drawing time lines may help with student understanding. An alternative would be to provide students with calendars. Students can go through the months or years and determine maturity dates and final values that way.
- You may stress that at this point, as in many areas of mathematics, calculating "the answer" to an application question simply means ending the task. What all of these calculations lead to is financial decision making, something that students will be doing for the rest of their lives.
- **Question 16** is an Achievement Check question. Provide students with **BLM 7–6 Section 7.2 Achievement Check Rubric** to help them understand what is expected.
- **Questions 17** consolidates many of the concepts from this section. Allow students to solve this problem using methods of their choice, with or without technology.

Achievement Check Sample Solution (page 361, question 16)

a) **A:** $P = 6000$, $i = 0.07$, $n = 1$

$$\begin{aligned} A &= P(1 + i)^n \\ &= 6000(1 + 0.07)^1 \\ &= 6000(1.07) \\ &= 6420 \end{aligned}$$

B: $P = 6000$, $i = \frac{0.065}{2} = 0.0325$, $n = 2$

$$\begin{aligned} A &= P(1 + i)^n \\ &= 6000(1 + 0.0325)^2 \\ &= 6000(1.0325)^2 \\ &= 6396.34 \end{aligned}$$

b) Answers may vary. Loan B is better for Ryanne. It is difficult to predict whether compounding semi-annually at a lower interest rate would make up for the difference in interest rate. Since the amount at the end of the first year shows that loan B pays less interest, compounding for a longer term will only intensify this difference.

c) **A:** $A = 6000(1.07)^5$ **B:** $A = 6000(1.0325)^{10}$

d) **A:** $A = \$8415.31$ **B:** $A = \$8261.37$

e) Loan B is better for Ryanne as she would have to pay back a smaller amount.

f) Assume that Ryanne chooses loan B. After 3 years, she has to pay back

$$\begin{aligned} \$6000(1.0325)^6 + \$700 &= \$7269.28 + \$700 \\ &= \$7929.28 \end{aligned}$$

$7969.28 is less than $8261.37. Ryanne should pay off her loan early to save $292.09.

Common Errors

- Some students may struggle with finding the number of compounding periods in **questions 1 and 2**.

R$_x$ Have students draw time lines or go through months and years in a calendar to find the number of compounding periods.

- Students may forget to calculate the interest rate for a compounding period other than a year.

R$_x$ Remind students to read each problem carefully and write down key words that will remind them to calculate a correct interest rate to use.

Ongoing Assessment

- You may wish to collect students' responses to the Communicate Your Understanding questions to use as a formative assessment tool.
- **Question 16** is an Achievement Check question. Use **BLM 7–6 Section 7.2 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Spatial–give students a handout of the tables in **questions 1 and 2**

Motor–provide a calculator with large buttons

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	8, 10, 14, 16
Reasoning and Proving	8–13, 16
Reflecting	6, 8, 10, 12, 14–17
Selecting Tools and Computational Strategies	1, 2, 4, 5, 7–17
Connecting	6–10, 12–14, 16, 17
Representing	3, 14
Communicating	6, 9–12, 16