8.1 Future Value of an Ordinary Simple Annuity

Student Text Pages
382–389

Suggested Timing
75–150 min

Materials and Technology Tools
- computer with Internet access
- TVM solver
- spreadsheet software

Related Resources
- BLM 8–3 Section 8.1 Future Value of an Ordinary Simple Annuity
- BLM 8–4 Section 8.1 Achievement Check Rubric

Teaching Suggestions

- Emphasize that this chapter builds upon the skills and the concepts learned in the previous chapter. Competence with the skills learned in the previous chapter will enhance the likelihood of student success in this chapter.
- Ensure that students understand the difference between an annuity calculation and a one-time, compound interest calculation.
- Graph the spreadsheet data in Investigate B to highlight the growth of the final value of an annuity versus that of a one-time investment. As an alternative, students could graph the value at the end of the year versus time by hand. Provide grid paper, if necessary. Having students plot the increases without the use of technology may enhance their understanding of the rapid growth for multiple payments.
- As this is the first section of the chapter, assign and take up the Practise questions before moving on to more challenging, context-based problems. For some students, more than the suggested time may be required to ensure that the concept of an annuity has been understood.

Investigate

- Students may work individually or in pairs
- Ensure that instructions are read clearly and followed in detail.
- Discuss the answers as a whole group to ensure that the larger concepts have been grasped by students. Stress the connection to the key concepts in Chapter 7.

Investigate Responses (pages 382-384)

Investigate A

Method 1

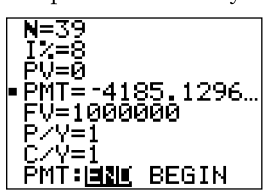
Answers and conclusions will be identical to those obtained by using Method 2.

Method 2

Solutions are based on an assumed age of 16 years for the student.

1. 15-year old: enter $n = 40$. 16-year old: enter $n = 39$. 17-year old: enter $n = 38$.

7. A 16-year old who wishes to have $1 000 000 by the age of 55 would need to invest about $4185.13 per year, assuming an interest rate of 8% per year, compounded annually.

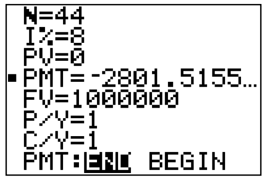


A 15-year old who wishes to have $1 000 000 by the age of 55 would need to invest about $3860.16 per year, assuming an interest rate of 8% per year, compounded annually.

A 17-year-old who wishes to have $1 000 000 by the age of 55 would need to invest about $4538.94 per year, assuming an interest rate of 8% per year, compounded annually.

8. To reach the goal by age 60: 15-year-old: enter $n = 45$; 16-year-old: enter $n = 44$; 17-year-old: enter $n = 43$.

A 16-year-old who wishes to have $1 000 000 by the age of 60 would need to invest about $2801.52 per year, assuming an interest rate of 8% per year, compounded annually.

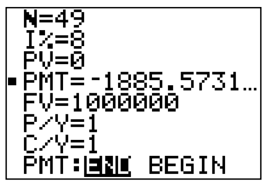


A 15-year-old who wishes to have $1 000 000 by the age of 60 would need to invest about $2587.28 per year, assuming an interest rate of 8% per year, compounded annually.

A 17-year-old who wishes to have $1 000 000 by the age of 60 would need to invest about $3034.14 per year, assuming an interest rate of 8% per year, compounded annually.

To reach the goal by age 65: 15-year-old: enter $n = 50$; 16-year-old: enter $n = 49$; 17-year-old: enter $n = 48$.

A 16-year-old who wishes to have $1 000 000 by the age of 65 would need to invest about $1885.57 per year, assuming an interest rate of 8% per year, compounded annually.



A 15-year-old who wishes to have $1 000 000 by the age of 60 would need to invest about $1742.86 per year, assuming an interest rate of 8% per year, compounded annually.

A 17-year-old who wishes to have $1 000 000 by the age of 60 would need to invest about $2040.27 per year, assuming an interest rate of 8% per year, compounded annually.

9.

Years Until Retirement	Annual Payment Amount ($)
38	4538.94
39	4185.13
40	3860.16
43	3034.14
44	2801.52
45	2587.28
48	2040.27
49	1885.57
50	1742.86

10. When examining the values in the annual payment column, it can be seen that the more years you have until retirement, the less you need to invest per year to reach your goal. However, this is just common sense. A larger number of payments directed to the same goal must have a smaller value per payment. What should also be examined is the total amount invested over the time interval.

Years Until Retirement	Annual Payment Amount ($)	Total Amount Invested ($)
38	4538.94	172 480
39	4185.13	163 220
40	3860.16	154 406
43	3034.14	130 468
44	2801.52	123 267
45	2587.28	116 428
48	2040.27	97 933
49	1885.57	92 393
50	1742.86	87 143

As the time period increases, the total number of the payments required becomes much smaller. This is because the early payments earn progressively larger amounts of interest, which replaces some of the money that would otherwise have to be directly invested.

Investigate B

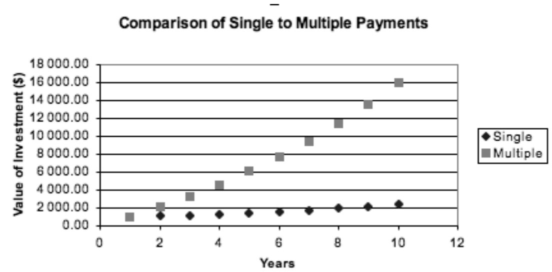
1. to 5., 7. to 13.

◇	A	B	C	D	E	F	G	H	I	J	K	L
1	Year	Value at Start of Year ($)	Interest Paid at End of Year ($)	Value at End of Year ($)		Year	Value at Start of Year ($)	Interest ($)	Value with Interest ($)	End of Year Investment ($)	Value at End of Year ($)	
2	1	0.00	0.00	1000.00		1	0.00	0.00	0.00	1000.00	1000.00	
3	2	1000.00	100.00	1100.00		2	1000.00	100.00	1100.00	1000.00	2100.00	
4	3	1100.00	110.00	1210.00		3	2100.00	210.00	2310.00	1000.00	3310.00	
5	4	1210.00	121.00	1331.00		4	3310.00	331.00	3641.00	1000.00	4641.00	
6	5	1331.00	133.10	1464.10		5	4641.00	464.10	5105.10	1000.00	6105.10	
7	6	1464.10	146.41	1610.51		6	6105.10	610.51	6715.61	1000.00	7715.61	
8	7	1610.51	161.05	1771.56		7	7715.61	771.56	8487.17	1000.00	9487.17	
9	8	1771.56	177.16	1948.72		8	9487.17	948.72	10435.89	1000.00	11435.89	
10	9	1948.72	194.87	2143.59		9	11435.89	1143.59	12579.48	1000.00	13579.48	
11	10	2143.59	214.36	2357.95		10	13579.48	1357.95	14937.42	1000.00	15937.42	
12												
13												
14												
15												

6. The value in cell D11 is $2357.95. This represents the future value of a single investment of $1000 at the end of 10 years. Based on the entry in cell C3, the annual rate of interest is 10% compounded annually.

14. The value in cell K11 is $15 937.42. This represents the future value of 10 investments of $1000 made at the end of each of 10 years.

16.



17. The second graph increases much more quickly because a total of $10 000 was invested over the 10 years compared to a single investment of $1000 at the end of the first year.

Examples

- Ensure that students are competent using a TVM Solver.
- Determine students' competence with calculating compound interest.
- If each method is not thoroughly worked through, students should at least read each method thoroughly.

Communicate Your Understanding

- These can be done as a whole group discussion.
- Some students may need a hands-on example in order to obtain a clear understanding of the concept in **question C1**.
- For **question C2**, discuss the effects of division by a number less than 1 and by a number greater than 1.
- You may wish to use **BLM 8–3 Section 8.1 Future Value of an Ordinary Simple Annuity** for remediation or extra practice.

Communicate Your Understanding Responses (page 387)

C1 The variable n in the annuity formula represents the number of payments and the number of compounding periods over the life of the annuity. This corresponds directly to the value of the variable N in the TVM Solver. Both represent the number of payments. This differs from the variables for a one-time compound interest calculation. In that case, the n in the formula represents the number of payments over the term of the investment but N is used in the TVM Solver to represent the number of years rather than the number of payments.

C2
$$FV = 5000\left(\frac{(1.0625)^{20} - 1}{1.0625}\right)$$
$$\doteq 11\ 114.60$$

Without earning any interest, 20 payments of $5000 will have a value of $100 000. The future value shown is far too small. The denominator in the formula takes on the value of i, which in this case is 0.0625. Eric mistakenly entered the value of $1 + i$.

C3 **a)** If $10 000 is invested now, interest will be earned on the full amount for 10 years. If the investment is made in increments of $1000 then interest earned will be on smaller amounts of principal over the entire time. The single large investment will have a larger value at the end of the 10 years.

b) It is quite likely that a person would not have a single large amount available to invest. If long-term savings is the goal, it is wiser to invest as much as you can afford on a regular basis.

Practise, Connect and Apply, Extend

- While most of the questions require the use of a calculator, encourage students to reflect on the reasonableness of the answer provided by the calculator.
- For some students, not all parts of all questions need to be assigned. However, as this is the first section of the chapter, students need to be able to perform the calculations or operations in this section for the sections that follow.
- **Questions 1 to 5** can be easily solved using a TVM Solver or an on-line calculator, if available. Ensure students understand that questions 2 to 4 relate to question 1. It may be necessary to demonstrate question 1, part a) to the whole class and then work through questions 2 to 4.
- For **questions 6 to 10**, students should be encouraged to refer to the worked Example if any difficulties arise. If students are using a combination of approaches to solve questions, as in question 8, ensure that they understand the difference between the variables used in each formula, such as the variable n in the annuity formula and the variable N in a TVM Solver.
- Some students may benefit from starting **questions 9** and **10** by constructing a time line. Student should be encouraged to keep their notes and calculations separate for question 9, the Chapter Problem. Question 10 is an Achievement Check question. Provide students with **BLM 8–4 Section 8.1 Achievement Check Rubric** to help them understand what is expected.

Common Errors

- Some students might confuse variables in the various formulas, such as the variable *n* in the annuity formula and the variable N in a TVM Solver.

R_x Have students refer to their notes for clarification or to previously solved problems for models of correct values.

Ongoing Assessment

- While students are working, circulate to observe how well each works. This is an opportunity to observe and record individual student's learning skills.
- **Question 10** is an Achievement Check question. Use **BLM 8–4 Section 8.1 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Motor–encourage students to use technology for making calculations and for graphing

Language–encourage students to work in pairs for reading support

Student Success

- Throughout this chapter, the use of technology for making calculations is strongly recommended in many places for speed and accuracy. Ensure that students are familiar with the calculator sequences required.

- **Question 11** requires an annuity calculation followed by a compound interest calculation as seen in Chapter 7.
- For **question 13**, students might also make notes on the user-friendliness of the different on-line calculators. A list of calculator links and reviews can be generated by the class for future reference.

Achievement Check Sample Solution (page 389, question 10)

a) Krystyne's investment is $872 751.96. This is an annuity of $250/month for 40 years. Mychal's investment is $294 510.21. This is an annuity of $500/month for 20 years.

b) The doubled monthly payment cannot make up for all the compound interest earned over the 20 years.

c) By using a TVM Solver, Mychal would have to invest $1481.70/month for 20 years to receive the same amount as his sister.

Literacy Connections

- Encourage students to create their own dictionary or graphic organizer of key terms to use for reference. Examples should accompany the definitions.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 7, 9–12
Reasoning and Proving	6, 9, 10
Reflecting	8, 9, 12
Selecting Tools and Computational Strategies	2–13
Connecting	6–9
Representing	1, 2, 7
Communicating	4, 8–10