8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity

Student Text Pages
405–411

Suggested Timing
75–150 min

Materials and Technology Tools
- computer with Internet access
- TVM solver

Related Resources
- BLM A–13 Self Assessment Recording Sheet
- BLM 8–7 Section 8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity
- BLM 8–8 Section 8.4 Achievement Check Rubric

Teaching Suggestions

- This section brings the concepts of the chapter together. Stress that with financial applications there are generally no right and wrong answers but that the mathematical calculations required assists with decision-making in life. Have students continually reflect on situations, calculations, options, and the effects of changing one or more of the conditions of annuities, be they investments or borrowed money. The correct answer must be the one that fits into an individual's goals and current financial situation.

Investigate

- Students may work individually or in pairs.
- Ensure that instructions are read clearly and followed in detail.
- Discuss as a whole group to ensure big ideas have been grasped by students. Stress the connection to the concepts of the previous chapter or the previous sections of this chapter. Have students continually reflect upon each calculation and the pros and cons about a decision.
- For **Investigate B**, ensure students understand that the same amount is invested in either case.

Investigate Responses (pages 405-407)

Investigate A

Values may vary slightly depending on the on-line calculator used.

2. e) The payment amount is approximately $348.86. The interest cost for the term is $3931.60.

3.

Principal ($)	Term (years)	Rate (%)	Monthly Payment ($)	Interest Cost for Term ($)
17 000	5	8.5	348.86	3931.60
17 000	5	8	344.77	3686.20
17 000	5	7.5	340.72	3443.20
17 000	5	7	336.68	3200.80

4. As the interest rate increases, the interest cost for the term also increases. The overall amount paid for the vehicle will increase if the interest rate charged is increased.

5. b)

Principal ($)	Term (years)	Rate (%)	Monthly Payment ($)	Interest Cost for Term ($)
17 000	5	8.5	348.86	3931.60
17 000	4	8.5	419.12	3117.76
17 000	3	8.5	536.77	2323.72

6. As the length of a loan is decreased, the interest cost for the term also decreases. As a result the overall amount paid for the vehicle decreases as the term of the loan decreases. The monthly payment must increase in order to achieve a shorter term.

7. The table is based on a principal of $17 000 paid over 5 years at a rate of 8.5% per year.

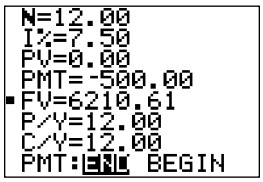
Frequency of Payments	Payment ($)	Interest Cost for Term ($)
monthly	348.78	3926.80
semi-monthly	174.14	3896.80
bi-weekly	160.73	3894.90
weekly*	80.31	3880.60

* 260 payments

8. By making payments at shorter time intervals, there is a slight decrease in the overall amount paid for the vehicle.

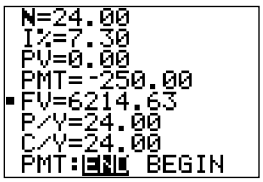
Investigate B

1. Option 1



The future value of his savings is $6210.61.

Option 2



The future value of his savings is $6214.63.

2. The second option has a slightly higher return in spite of the nominal interest rate being lower. You would expect the difference in the future value to be greater after 2 years than after 1 year. Examining the growth factor per month alone will not explain why the lower rate outperforms the higher rate.

For Option 1, $i = 0.075 \div 12$
$= 0.006\ 25$

The growth factor for one month is 1.006 25.

For Option 2, $i = 0.073 \div 12\left(1 + \frac{0.073}{12}\right)^2$
$\doteq 1.006\ 092\ 6$

The growth factor for one month is 1.006 092 6 which is still slightly less than for the 7.5% compounded monthly.

The difference is due to the fact that in the first month Option 2 starts earning interest on $250 at the beginning of week 3 while Option 1 does not start earning interest until the beginning of the second month. This is repeated every month. The interest earned by the early deposits outweighs the higher rate being paid on the monthly plan.

Time (months)	Option 1 Accumulated Value ($)	Option 2 Accumulated Value ($)
0.5		250.00
1.0	500.00	500.76
1.5		752.28
2.0	1003.13	1004.57
2.5		1257.63
3.0	1509.39	1511.45
3.5		1766.05
4.0	2018.83	2021.42

From the table, Option 2 will continue to outperform Option 1 over time.

3. The TVM solver gives a future value of $6213.48 for 12 monthly payments of $500 earning 7.6% per year. This is still slightly less than the semi-monthly deposits of $250 at 7.3%.
4. In all cases the total value invested will be the same. Karl is investing $6000 per year.
5. For a fixed rate of interest and a constant value being deposited per year, the future value of an annuity will increase as the frequency of the deposits increases.

Example

- The **Example** demonstrates how changing one variable has an effect on potential decision-making. This is a key theme to this section.
- Discuss the advantages and disadvantages of each option.

Communicate Your Understanding

- Students will likely need to back up their opinion in **questions C1 to C3** with examples.
- **Question C4** includes a common error, that is, letting N equal 4 rather than 48. The size of the monthly payment should indicate to students the error if they reflect on the appropriateness of a monthly payment of that size.
- You may wish to use **BLM 8–7 Section 8.4 Effects of Changing the Conditions of an Ordinary Simple Annuity** for remediation or extra practice.

Communicate Your Understanding Responses (page 409)

For a concrete example, use a principal of $5000.

Rate (%)	Term (years)	Monthly Payment ($)	Total Cost ($)
5	5	94.36	5661.60
4	5	92.08	5524.80
4	4	112.90	5419.20

C1 If the interest rate increases from 4% to 5%, this is a rate increase of 25%. If the term of the loan decreases from 5 years to 4 years this is a decrease of 20% in the term. The rate increase is likely to have a slightly larger impact. From the table, the rate increase costs an extra $136.80. The reduction of one year results in a savings of $105.60.

C2 This question is much easier to predict. Most of a monthly payment goes to repaying principal while the smaller amount pays the interest due. You can expect a small comparative change in monthly payment when a rate changes. From the table, an increase from 4% to 5% interest causes an increase of $2.28 in the monthly payment. If the term of a loan is reduced, you have to pay a greater principal per payment so you would expect about a 20% increase. From the table, this is an increase of $20.82 per payment, which is much greater than the effect of the interest rate increase.

C3 It is possible for a slightly lower rate to have a larger future value if the payments are made more frequently. If Claire makes her payments less frequently than Sonya and if the rates are close, it is possible that Claire will not earn more interest in spite of the higher nominal rate.

C4 Justin entered the number of years for N rather than the number of payments required. He should have entered N = 48. The payment shown on the screen would be sufficient to repay the loan in four monthly payments.

Practise, Connect and Apply, Extend

- Discuss the advantages and disadvantages of the decisions made by or the options available to the people in each question.
- Encourage students to use a TVM Solver for calculations.
- Fro **questions 6 to 11**, students should reflect upon the various scenarios as they make the calculations in order to have a greater understanding of how changing each variable changes amounts earned or paid.

Common Errors

- Students make calculation errors using the TVM Solver.

R_x Have students refer to their notes for clarification or to previously solved problems for models of correct use of the technology.

Ongoing Assessment

- **Question 11** is an Achievement Check question. Use **BLM 8–8 Section 8.4 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Motor–encourage students to use technology for calculations

Language–simplify instructions and provide additional scaffolding for problems in Connect and Apply

- **Question 11** investigates debt and the financial problems relating to debt. It is an Achievement Check question. Provide students with **BLM 8–8 Section 8.4 Achievement Check Rubric** to help them understand what is expected.
- If **question 14** is assigned, have students consider the ramifications of Owen missing 8 payments (i.e., credit rating, accrued interest, financial penalties, etc.).

Achievement Check Sample Solution (page 410, question 11)

a) Matt owes $15 000 over all. If he takes a 5-year loan at 9% compounded monthly, his monthly payments would be $311.38.

b) Matt would pay $18 682.80 in total for the loan.

c) The interest paid would be $3 682.80.

d) Matt would pay $373.28/month for a 4-year loan, $17 917.44 by the end of the loan so the interest paid would be $2 917.44. This would be a savings of $765.36.

e) Matt would pay $241.34/month for a 7-year loan.

f) Advantage: monthly payment is lower on the 7-year loan so may be more manageable. Disadvantage: overall amount paid is higher so you pay more interest on a 7-year loan.

g) Debt service ratio: the ratio of net operating income to debt payments to determine whether or not an investment is keeping up its interest payments. Matt needs to take a look at this ratio to determine whether or not he can afford to repay the loan quickly (4-year loan) or more slowly (7-year loan). The ratio needs to be greater than 1 but that would leave no money for anything else.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 7, 9–12, 14
Reasoning and Proving	6, 10, 11, 13
Reflecting	7, 9, 11, 13, 14
Selecting Tools and Computational Strategies	1–12, 14
Connecting	6–13
Representing	n/a
Communicating	10, 11, 13