

**CHAPTER
4**

Quadratic Relations I

Get Set

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 166–167 of the *Foundations for College Mathematics 11* textbook.

Number Skills

1. Calculate.

a) $4.4 + 9.7$

b) $-16.3 + 0.6$

c) $-1.5 - (-8.3)$

d) $52.4 - (-2.8)$

e) $(1.5)(-42)$

f) $(-14)(-3.3)$

Algebraic Expressions

2. Simplify.

a) $3x^2 - 10x + 2x^2 - (-4)$

b) $5x^2 - 3x + 2 - 7x + 3x$

3. Find the value of y when $x = 3$.

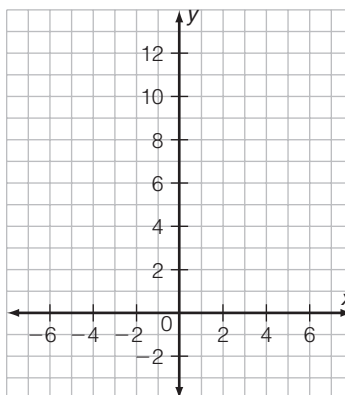
a) $y = 2x^2 + 4x$

b) $y = -2(x - 1)^2$

Linear Relations

4. Complete the table of values for the linear relation $y = 4x + 1$. Graph the relation. Identify the slope and the y -intercept.

x	y
-1	
0	
1	
2	
3	



Transformations

5. What point is the image of $(3, -5)$ after each transformation?

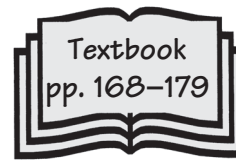
a) a translation 10 units to the left and 10 units down

b) a rotation of 180° clockwise about the point $(0, -4)$

c) a reflection in the line $y = x$

4.1

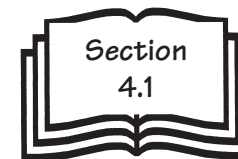
Modelling With Quadratic Relations



Warm-Up

1. Number Skills	2. Algebra															
Evaluate. a) $\sqrt{0.01}$ b) $\sqrt{225}$ c) $\sqrt{169}$	Solve. a) $3x + 4 = 11$ b) $2(x + 2) = -2(3x - 1)$															
3. Relations	4. Geometry/Masurement															
Solve the linear system. $4x + 3y = -5$ $3x + 8y = 2$	$\triangle ABC \sim \triangle DEF$ $AB = 4 \text{ cm}, AC = 6 \text{ cm}, BC = 7 \text{ cm}$ If $DF = 12 \text{ cm}$, find a) DE b) EF															
5. Data/Probability	6. Problem Solving															
What is probability of rolling a sum greater than 8 on a pair of dice?	Which figure has a greater perimeter? A a circle with radius 24 cm B a square with side length 24 cm															
7. Math Literacy	8. Previous Section															
Which term describes a slide transformation? A translation B rotation C dilatation D reflection	Complete the table. <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> <th>First Differences</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>11</td> <td></td> </tr> <tr> <td>4</td> <td>15</td> <td></td> </tr> <tr> <td>5</td> <td>19</td> <td></td> </tr> <tr> <td>6</td> <td>23</td> <td></td> </tr> </tbody> </table>	x	y	First Differences	3	11		4	15		5	19		6	23	
x	y	First Differences														
3	11															
4	15															
5	19															
6	23															

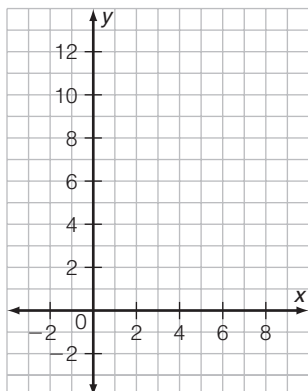
Practise



1. Graph each relation. Determine if the relation is linear, quadratic, or neither.

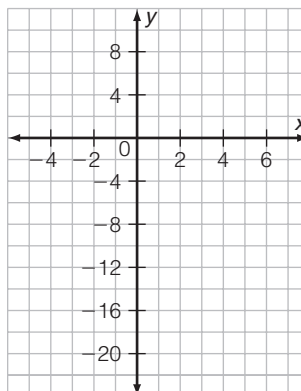
a)

x	y
0	6
1	3
2	0
3	3
4	6
5	9



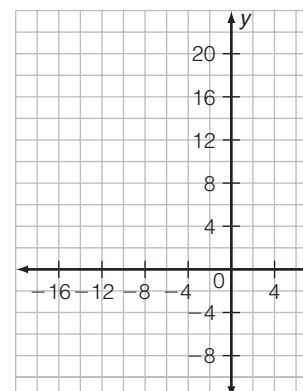
b)

x	y
-4	-5
-2	1
0	3
2	1
4	-5
6	-15



c)

x	y
-12	-10
-11	-4
-10	2
-9	8
-8	14
-7	20



2. Check your answers to question 1 by calculating the first and second differences for each relation.

a)

b)

c)

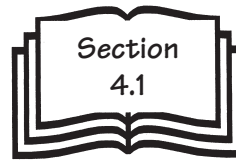
3. Predict which relations are quadratic. Explain your reasoning.

a) $y = x + 2$

b) $y = 0.5x^2 - 5x + 5$

c) $y = 3 - 2x$

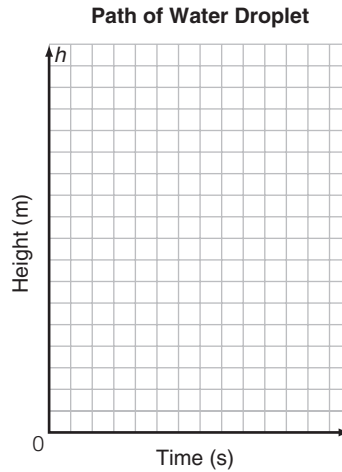
d) $y = 0.25 + 5x - 50x^2$



4. A fountain shoots a stream of water into the air. The path of a water droplet is modelled by the relation $h = -4.9t^2 + 30t + 1$, where h is the droplet's height above the ground, in metres, and t is the time, in seconds.

a) Complete the table.

Time (s)	Height (m)
0	
1	
2	
3	
4	
5	
6	



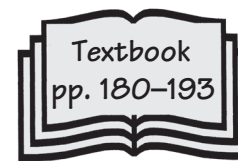
- b) After how many seconds will the water droplet reach its maximum height?
- c) Graph the relation.
- d) Use the graph to estimate the time at which the water droplet lands.

5. Two teams, one wearing red and one wearing blue, compete in a 500-m rowing race. The red team's speed is modelled by the relation $d = 0.04t^2$ and the blue team's speed is modelled by the relation $d = 4t$. For both, d is the distance from the starting line, in metres, and t is the time, in seconds.

a) Which team is in the lead after 90 s? after 110 s?

b) Which team wins the race? Describe two different methods of finding the answer.

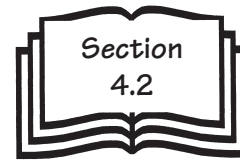
4.2 The Quadratic Relation

$$y = ax^2 + k$$


Warm-Up

<p>1. Number Skills</p> <p>Evaluate.</p> <p>a) $52 + 3 \times (4 + 7)$</p> <p>b) $12 \div (3 + 1) - 2^2$</p>	<p>2. Algebra</p> <p>Isolate x.</p> <p>a) $3y - x = 8$</p> <p>b) $2x + 3y = 5$</p>
<p>3. Relations</p> <p>Solve the linear system.</p> $3x - y = 17$ $2x + 3y = -7$	<p>4. Geometry/Measurement</p> <p>In a right triangle, the side adjacent to an angle of 32° is 13 cm. How long is the side opposite the 32° angle, to the nearest tenth of a centimetre?</p>
<p>5. Data/Probability</p> <p>Calculate the mean, the median, and the mode for the set of data.</p> <p style="text-align: center;">3, 9, 4, 7, 5, 6, 9, 12</p>	<p>6. Problem Solving</p> <p>The volume of a square-based prism is 400 m^3. Calculate the volume of a prism with the same base, that is half the height of the first prism.</p>
<p>7. Math Literacy</p> <p>Which is more?</p> <p>A six squared</p> <p>B the product of eight and nine</p>	<p>8. Previous Section</p> <p>Predict which relation is quadratic. Explain your reasoning.</p> <p>a) $y = 5x - 1$ b) $y = 3x^2 + 2x - 1$</p>

Practise



1. Suppose each pair of relations is graphed on the same set of axes.
- Which parabola will be the widest (most vertically compressed)?
 - Which parabola will have its vertex farther from the x -axis?

Justify your answers.

a) $y = 7x^2 + 2$
 $y = 2x^2 + 7$

b) $y = -0.5x^2 + 2$
 $y = x^2 - 1.5$

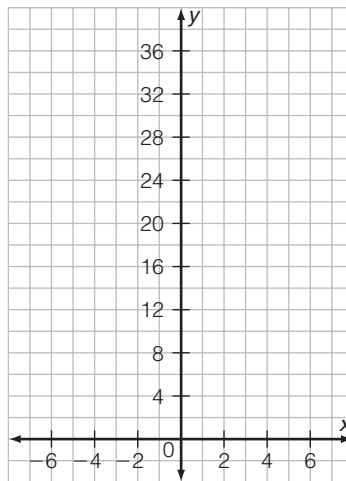
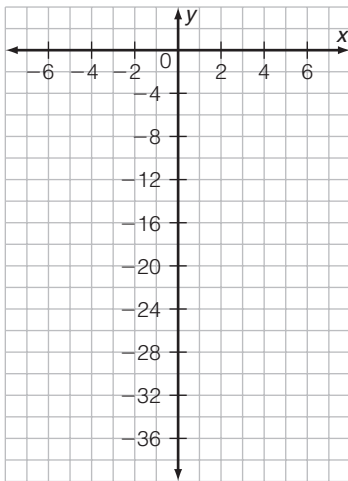
c) $y = 0.1x^2 - 9$
 $y = -0.01x^2 - 0.9$

d) $y = 10x^2 - 10$
 $y = 11x^2 + 1$

2. Describe the shape and position of each parabola relative to the graph of $y = x^2$. Sketch each graph.

a) $y = -2x^2$

b) $y = 6x^2 + 3$



3. Represent each relation with an equation of the form $y = ax^2 + k$.

a)

x	y
-6	-1
-4	4
-2	7
0	8
2	7
4	4
6	-1

b)

x	y
-4	27
-3	13
-2	3
-1	-3
0	-5
1	-3
2	3

c)

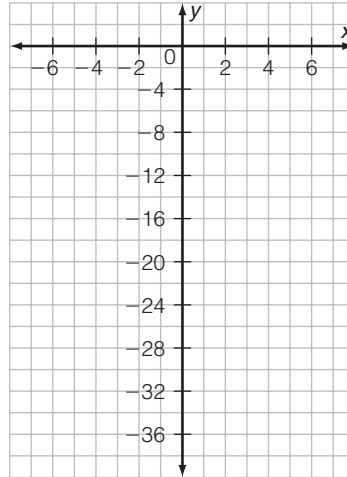
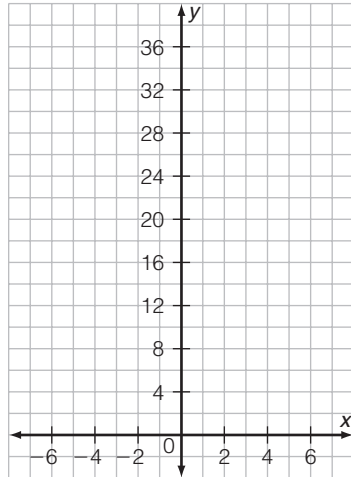
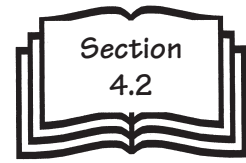
x	y
-1	2.5
0	1
1	2.5
2	7.0
3	14.5
4	25.0
5	38.5

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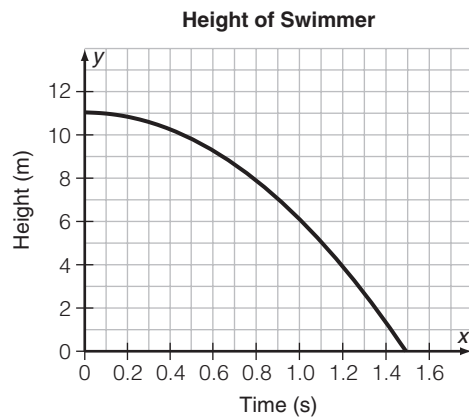
4. Describe the transformations needed to transform the graph of $y = x^2$ to graph each given relation. Sketch the graph.

a) $y = 6x^2 + 2$

b) $y = -x^2 + 1$



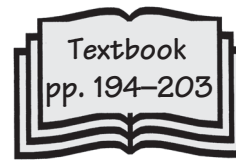
5. The graph shows the height over time of a swimmer after diving.



- a) Determine the coordinates of the vertex of this parabola.
- b) Write an equation to model the parabola.

4.3

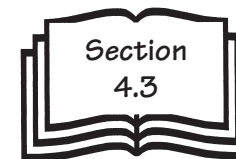
The Quadratic Relation

$$y = a(x - h)^2$$


Warm-Up

<p>1. Number Skills</p> <p>Write as a single power.</p> <p>a) 2×2^6</p> <p>b) $2^4 \times 2^2$</p> <p>c) $2^{-3} \div 2^{-4}$</p>	<p>2. Algebra</p> <p>Isolate y.</p> <p>a) $2x + y = 3$</p> <p>b) $x - y = 2$</p>
<p>3. Relations</p> <p>Solve the linear system.</p> $0.4x + 0.5y = 1.1$ $0.8x - 0.2y = -1.4$	<p>4. Geometry/Masurement</p> <p>A ladder rests against a building with the foot of the ladder 0.73 m from the base of the building, forming a 61° angle with the ground. How far up the building does the ladder reach?</p>
<p>5. Data/Probability</p> <p>Which event has a greater probability?</p> <p>A rolling an even number on a die</p> <p>B drawing a black face card from a deck of cards</p>	<p>6. Problem Solving</p> <p>Calculate the radius of a cylinder with volume 130 mL and height 11 cm.</p>
<p>7. Math Literacy</p> <p>A quadrilateral has four equal sides. Which shape might it be?</p> <p>A rectangle</p> <p>B square</p> <p>C kite</p> <p>D trapezoid</p>	<p>8. Previous Section</p> <p>Suppose each relation is graphed on the same set of axes.</p> <p>a) Which parabola would be the widest (most vertically compressed)?</p> <p>b) Which parabola would have its vertex farther from the x-axis?</p> <p>A $y = 5x^2 - 3$ B $y = -2x^2 + 1$</p>

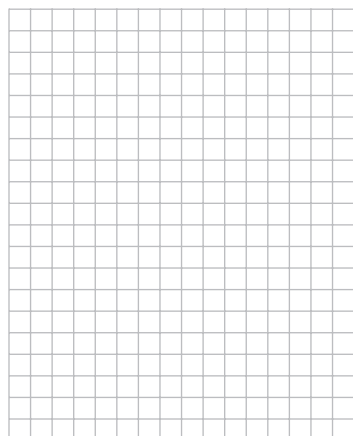
Practise



1. Describe the graph of each parabola relative to the graph of $y = x^2$. Sketch each graph.

a) $y = (x + 2)^2$

b) $y = -0.5(x - 5)^2$



2. Represent each relation with an equation of the form $y = a(x - h)^2$.

a)

x	y
-3	32
-2	18
-1	8
0	2
1	0
2	2
3	8

b)

x	y
-4	-1
-3	0
-2	-1
-1	-4
0	-9
1	-16
2	-25

c)

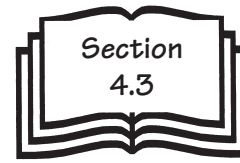
x	y
0	18
2	8
4	2
6	0
8	2
10	8
12	18

3. Suppose each pair of relations were graphed on one set of axes. Which parabola would have its vertex farther from the y-axis? Justify your answers.

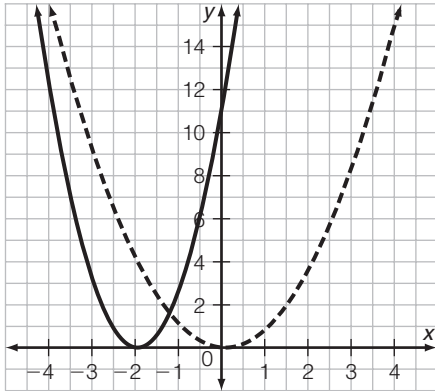
a) $y = 10(x + 7)^2$
 $y = 3(x - 3)^2$

b) $y = 0.375(x - 10)^2$
 $y = (x + 9)^2$

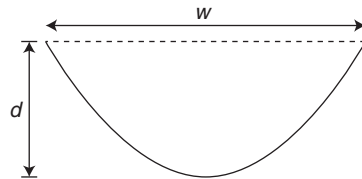
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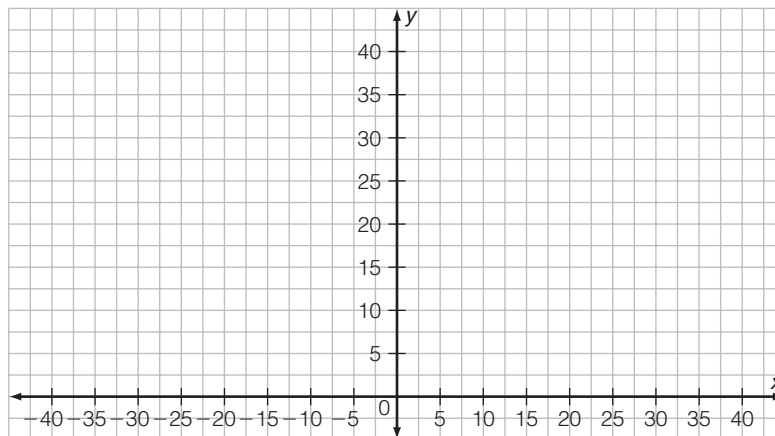
4. In the graph, $y = x^2$ is shown as a dotted line parabola. Describe the position of the solid parabola relative to the graph of $y = x^2$ in terms of a and h .



5. A parabolic heater is one that uses a dish with a side view in the shape of a parabola to direct heat. A certain brand of parabolic heater comes in varying sizes, with each dish modelled by the relation $d = 0.02(w - 4)^2$, where d is the depth and w is the width.



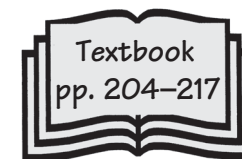
- a) Draw a graph to represent the side view of this parabolic heater.



- b) How wide is the parabola when the heater is 25 cm deep? How much wider is it when the heater is 35 cm deep?

4.4

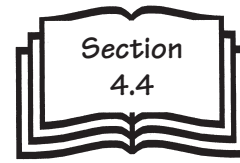
The Quadratic Relation

$$y = a(x - h)^2 + k$$


Warm-Up

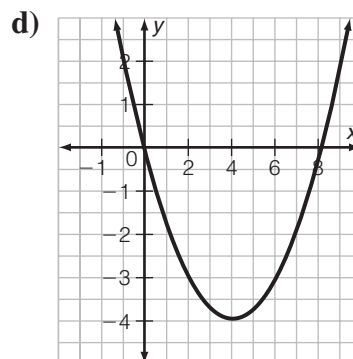
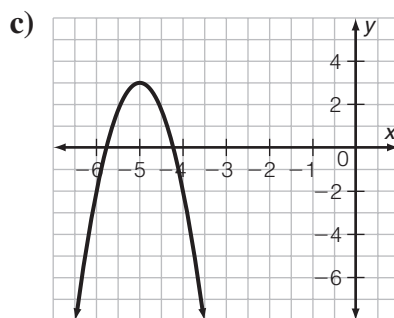
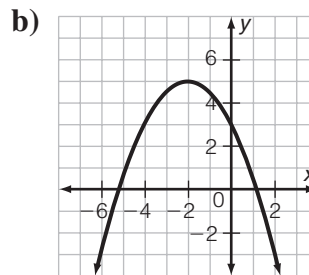
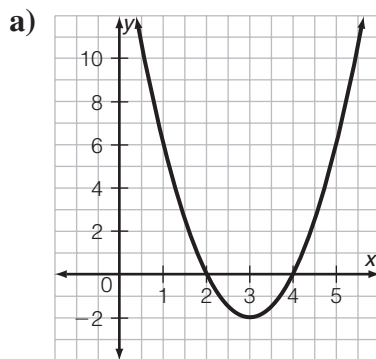
1. Number Skills	2. Algebra
Evaluate. a) $\left(\frac{3}{4}\right)^{-2}$ b) $\frac{(5)^0}{2^{-2}}$	Simplify. a) $4x + 3y + 6 + 5x - 7y - 5$ b) $8x - 5y + 2 - 9x + 2y - 12$
3. Relations	4. Geometry/Measurement
How many solutions are there for this linear system? $x + 4y = 8$ $y + 2x = 0$	A kite is 26 m above the ground. The angle the string makes with the ground is 34° . How long is the kite string?
5. Data/Probability	6. Problem Solving
What is the probability of each event? a) drawing a black queen from a deck of cards b) drawing a red king from a deck of cards	The volume of a cylinder is 333 cm^3 . Calculate the volume of a cone with the same height and the same base as the cylinder.
7. Math Literacy	8. Previous Section
Which number is not a perfect square? A 16 B 140 C 121 D 225	Suppose these relations are graphed on one set of axes. Which parabola will have its vertex farther from the y-axis? Justify your answer. a) $y = -2(x - 3)^2$ b) $y = 4(x + 2)^2 + 1$

Practise



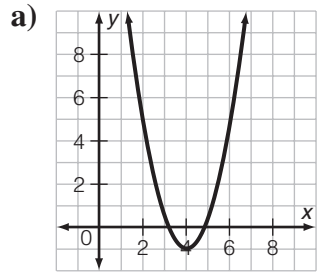
1. For each parabola
- identify the coordinates of the vertex
 - determine if the parabola opens upward or downward
 - determine if the parabola is vertically stretched or vertically compressed.
- a) $y = 3(x - 4)^2 - 2$ b) $y = -(x + 1)^2 - 5$
- c) $y = 0.2(x - 5)^2 - 1$ d) $y = -0.33(x + 9)^2 + 3$

2. For each parabola
- identify the coordinates of the vertex
 - determine whether a is positive or negative
 - write an equation for the relation in the form $y = a(x - h)^2 + k$

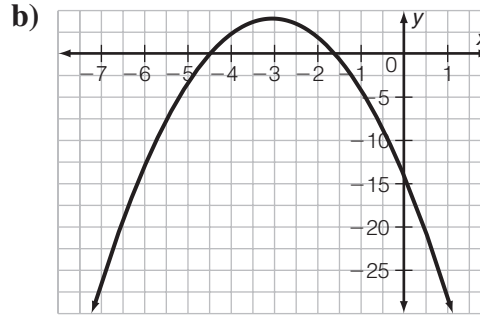


Section
4.4

3. Write an equation for each parabola in the form $y = a(x - h)^2 + k$.



vertex: (4, -1)
passes through: (6, 5)

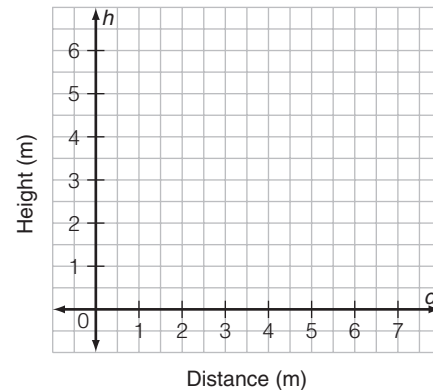


vertex: (-3, 4)
passes through: (-5, -4)

4. A volleyball was bumped over the net. Its path can be modelled by the relation $h = -0.42(d - 3.3)^2 + 5.1$, where h is the volleyball's height over the ground and d is the horizontal distance from where the ball was bumped, both in metres.

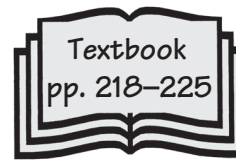
- a) What is the vertex of the parabola?
- b) What was the volleyball's initial height?
- c) Sketch a graph of the volleyball's path.
- d) What does the vertex represent in this situation?
- e) The ball was bumped directly towards the net. If the net is 2.4 m high and 1 m away from the player, will the ball hit the net?

Path of Volleyball



4.5

Interpret Graphs of Quadratic Relations

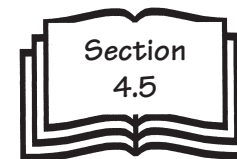


Warm-Up

<p>1. Number Skills</p> <p>Evaluate. $4^{-1} + 3^{-2}$</p>	<p>2. Algebra</p> <p>Solve and check. $4(x + 3) = 2(x - 1)$</p>
<p>3. Relations</p> <p>A triangle is formed by three lines: $y = 2x$, $y = -x + 6$, and $y = -2$. Find the coordinates of the vertices.</p>	<p>4. Geometry/Measurement</p> <p>Calculate the volume of a cylinder with radius 12 cm and height 9 cm. Round your answer to the nearest cubic centimetre.</p>
<p>5. Data/Probability</p> <p>A pencil case holds 4 red pens, 3 blue pens, and 1 green pen. You reach in and randomly pull out a pen. What is the probability that the pen is red or green?</p>	<p>6. Problem Solving</p> <p>Two guy wires supporting a flagpole are each anchored 8 m from the flagpole and each form an angle of 49° with the ground. What is the total length of guy wire, to the nearest metre, needed to support the flagpole?</p>
<p>7. Math Literacy</p> <p>Which term means to estimate values lying outside the range of data?</p> <p>A interpolate B extrapolate C estimate D indicate</p>	<p>8. Previous Section</p> <p>A soccer ball is kicked from ground level and follows a parabolic path. The path can be modelled by the relation $h = -0.4(t - 20)^2 + 6$, where h is the height of the ball, in metres, and t the time in seconds after it was kicked.</p> <p>a) What is the maximum height of the ball?</p> <p>b) At what time did the ball reach the maximum height?</p>

Date: _____

Practise



1. Find the y-intercept for each relation.

a) $y = x^2 - x + 1$

b) $y = -0.025x^2 + 0.15x - 75$

c) $y = -2(x + 7)^2 + 50$

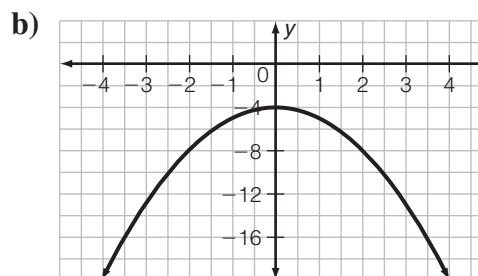
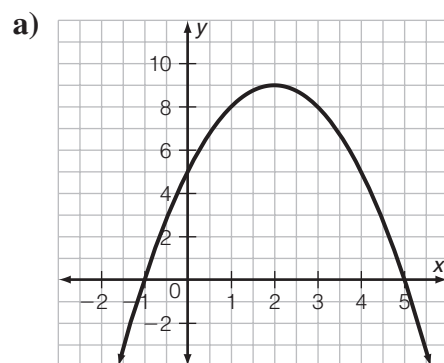
d) $y = 0.25(x - 4)^2 - 4$

e) $y = 4(x + 6)^2$

f) $y = -x^2$

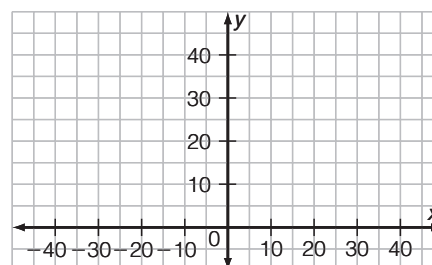
2. For each parabola, identify

- the x-intercepts
- the y-intercept
- the maximum or minimum value
- the coordinates of the vertex



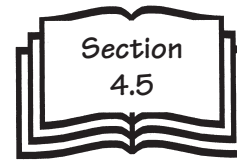
3. A parabola is 10 cm deep and 100 cm wide. The parabola opens upward.

a) Sketch a graph of the parabola.



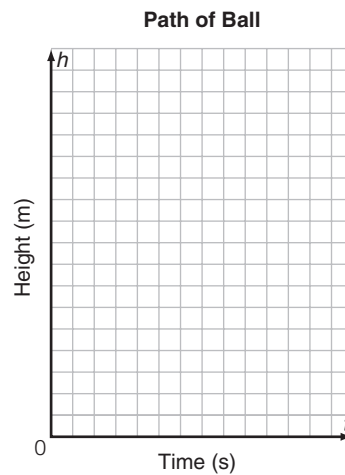
b) Determine the equation for the parabola.

Date: _____



4. The path of a ball thrown from a ledge can be modelled by the relation $h = -4.9t^2 + 19.6t + 11$, where h is the ball's height, in metres, and t is the time, in seconds.
- a) Complete the table. Graph the relation.

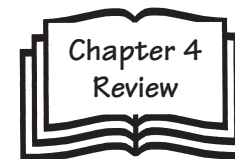
t	h
0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	



- b) Model the ball's path with a relation in the form $y = a(x - h)^2 + k$.
- c) Estimate the two times when the ball is 28 m above the ground.

5. A projectile is launched from ground level and reaches a maximum height of 122.5 m after 5 s.
- a) Write a relation that models this situation.
- b) What is the projectile's height after 3 s?

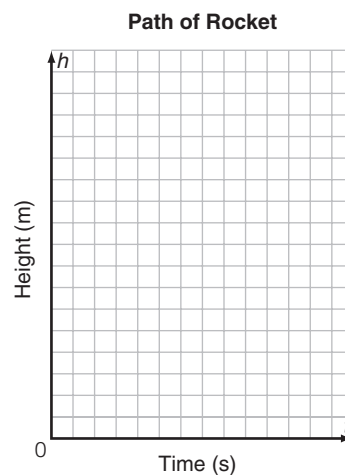
Chapter 4 Review



4.1 Modelling With Quadratic Relations, textbook pages 168–179

1. The table shows the height of a toy rocket above the ground over time after it was fired.

Time (s)	Height (m)
0	0.5
1	25.6
2	40.9
3	46.4
4	42.1
5	28.0
6	4.1



- a) Graph the data. Draw a smooth curve through the points.
 b) For about how many seconds was the rocket in the air?
- c) Determine which relation best represents this situation (without graphing the relations).
 A $h = -4.9t^2 + 46.4$
 B $h = -4.9t^2 + 30t + 0.5$
 C $h = 4.9t^2 + 0.5$

4.2 The Quadratic Relation $y = ax^2 + k$, textbook pages 180–193

3. Represent each relation with an equation of the form $y = ax^2 + k$.

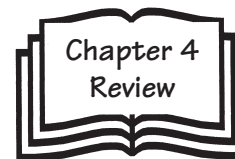
a)

x	y
-3	15.0
-2	7.5
-1	3.0
0	1.5
1	3.0
2	7.5
3	15.0

b)

x	y
-4	0
-2	-6
0	-8
2	-6
4	0
6	10
8	24

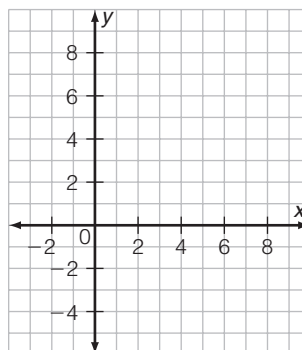
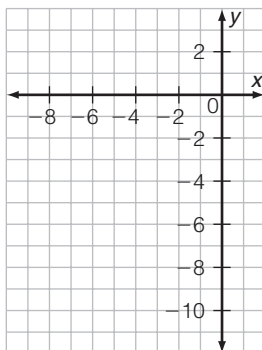
4.3 The Quadratic Relation $y = a(x - h)^2$, textbook pages 194–203



4. Sketch the graph of each parabola. Then, determine its equation.

a) opens downward, vertex is $(-5, 3)$, passes through point $(-1, -5)$

b) opens upward, vertex is $(3, -5)$, passes through point $(-3, 4)$



4.4 The Quadratic Relation $y = a(x - h)^2 + k$, textbook pages 204–217

5. Describe the shape and position of each parabola relative to the graph of $y = x^2$.

a) $y = -9(x - 3)^2 - 2$

b) $y = 0.9(x - 1)^2 + 1$

4.5 Interpret Graphs of Quadratic Relations, textbook pages 218–225

6. A football was kicked upwards. The football's path can be modelled by the relation $h = -3d^2 + 7.5d + 1.5$, where h is the football's height above the ground and d is the football's horizontal distance from the point it was kicked, both in metres.

a) Complete the table. Graph the relation.

d	0	0.5	1.0	1.5	2.0	2.5
h						

b) What was the football's initial height above the ground?

c) Write an equation in the form $y = a(x - h)^2 + k$ to model the football's path.

