4.2 The Quadratic Relation $y = ax^2 + k$

Student Text Pages
180–193

Suggested Timing
80 min

Tools
- grid paper
- graphing calculators
- computers
- *The Geometer's Sketchpad®*

Related Resources
BLM 4-5 Section 4.2 The Quadratic Relation $y = ax^2 + k$
BLM G-1 Grid Paper
BLM G-3 Four Quadrant Grids
BLM T-2 *The Geometer's Sketchpad®* 3
BLM T-3 *The Geometer's Sketchpad®* 4
BLM A-6 Knowledge and Understanding General Scoring Rubric

Link to Prerequisite Skills

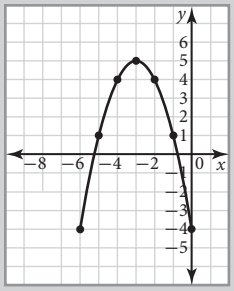
Students should complete **question 12** before proceeding with this section.

Warm-Up

1. Evaluate.
- **a)** $3(5)^2 - 8(5) + 4$
- **b)** $-6(-3)^2 + 9(-3) - 10$
- **c)** $3.2(-1.1)^2 - 2.3(-1.1) + 8.2$

2. Which of these relations are quadratic? How do you know?

a)



b)

x	y
5	4015
10	2265
15	1015
20	265
25	15

c) $y = 3x - 15x^2$

Warm-Up Answers

1. a) 43 **b)** −89 **c)** 14.602

2. a) This relation is quadratic. The graph is symmetrical and has a parabolic shape.

b) This relation is quadratic. Second differences are constant.

c) This relation is quadratic. The right side of the equation is a polynomial with 2 as the greatest power.

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Engage students in a discussion about video games they are familiar with. Ask them to describe the motion of an object in a game.

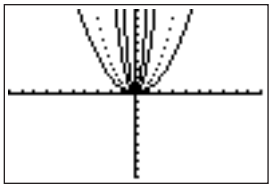
Investigate

- The curriculum expectation requires the use of technology to explore the transformations.
- If you use *The Geometer's Sketchpad®*, you may wish to assign the Investigate for homework as students can install the software at home. One advantage of using Method 2 is the random function generator on the third page of the sketch.
- For a basic introduction to *The Geometer's Sketchpad®*, supply students with **BLM T-2 *The Geometer's Sketchpad®* 3** or **BLM T-3 *The Geometer's Sketchpad®* 4**. This software is used throughout Chapter 4. Students completing Method 2 may benefit from referring to the BLMs.

Investigate Answers (pages 180-185)

Method 1: Use a Graphing Calculator

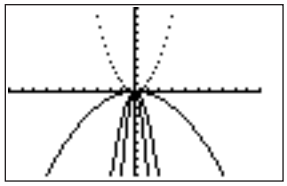
2.



b) All the graphs are parabolas with vertex at (0, 0). The graphs of $y = 4x^2$ and $y = 12x^2$ are vertically stretched compared to the graph of $y = x^2$. The graph of $y = 0.5x^2$ is vertically compressed compared to the graph of $y = x^2$.

c) The y-values for $y = 4x^2$ are 4 times the corresponding y-values for $y = x^2$. The y-values for $y = 0.5x^2$ are half the corresponding y-values for $y = x^2$. The y-values for $y = 12x^2$ are 12 times the corresponding y-values for $y = x^2$.

3. a)

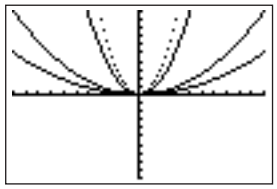


b) All the graphs are parabolas with vertex at (0, 0). The graphs of $y = -3x^2$, $y = -0.2x^2$, and $y = -10x^2$ open downward. The graphs of $y = -3x^2$ and $y = -10x^2$ are vertically stretched compared to the graph of $y = x^2$. The graph of $y = -0.2x^2$ is vertically compressed compared to the graph of $y = x^2$.

4. Answers may vary. Sample answers:

a) $y = -2x^2$ **b)** $y = 6x^2$

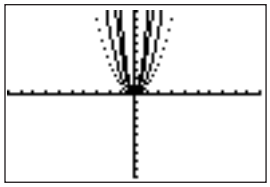
5. a)



b) All the graphs are parabolas with vertex at (0, 0). The graphs are all vertically compressed compared to the graph of $y = x^2$.

6. As a decreases, the parabola becomes more compressed vertically.

7. a)



b) All the graphs are parabolas with vertex at (0, 0). The graphs are all vertically stretched compared to the graph of $y = x^2$.

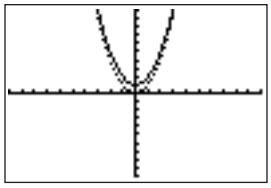
8. As the coefficient on x^2, a, increases, the parabola becomes more stretched vertically.

9. a) $a > 1$ **b)** $0 < a < 1$

10. Answers may vary. Sample answers:

a) $y = 2x^2$ **b)** $y = -2x^2$ **c)** $y = 0.5x^2$ **d)** $y = -0.5x^2$

11. a)



b) Shifting the graph up 1 unit will give the graph of $y = x^2 + 1$. It has the same shape as the graph of $y = x^2$, but the parabola has shifted up so the vertex is at (0, 1).

12. As the value of k increases, the parabola shifts up. The vertex of the parabola is at $(0, k)$.

13. As the value of k decreases, the parabola shifts down. The vertex of the parabola is at $(0, k)$.

14. a) When $k > 0$, the vertex of the parabola is above the x-axis.
b) When $k < 0$, the vertex of the parabola is below the x-axis.
c) When $k = 0$, the vertex of the parabola is on the x-axis.

15. Answers may vary. Sample answers:
a) $y = x^2 + 3$ **b)** $y = x^2 - 1$ **c)** $y = x^2$

16. Answers may vary. Sample answers:
a) $y = 2x^2 - 4$ **b)** $y = -0.5x^2 + 2$ **c)** $y = -2x^2$

17. The value of k is easier to determine. It can be found by substituting $x = 0$.

Method 2: Use *The Geometer's Sketchpad*®

1. Increasing the value of a makes the parabola narrow. Decreasing the value of a makes it wider.

2. a) When a is positive, the graph of $y = ax^2$ opens upward, the same direction as the graph of $y = x^2$.
b) When a is negative, the graph $y = ax^2$ opens downward, the opposite direction as the graph of $y = x^2$.

3. Answers may vary. Sample answers:
a) $y = -4x^2$ **b)** $y = 0.6x^2$

4. a) When $0 < a < 1$, the graph opens upward and is wider than the graph of $y = x^2$.
b) When $a > 1$, the graph opens upward and is narrower than the graph of $y = x^2$.
c) When $-1 < a < 0$, the graph opens downward and is wider than the graph of $y = x^2$.
d) When $a < -1$, the graph opens downward and is narrower than the graph of $y = x^2$.

5. Answers may vary. Sample answers:
a) $y = 3x^2$ **b)** $y = -3x^2$ **c)** $y = 0.2x^2$ **d)** $y = -0.2x^2$

6. As the value of k increases, the parabola shifts up. As the value of k decreases, the parabola shifts down. The vertex of the parabola is at $(0, k)$.

7. a) When $k > 0$, the vertex of the parabola is above the x-axis.
b) When $k < 0$, the vertex of the parabola is below the x-axis.
c) When $k = 0$, the vertex of the parabola is on the x-axis.

8. Answers may vary. Sample answers:
a) $y = x^2 + 2$ **b)** $y = x^2 - 1$ **c)** $y = x^2$

10. The value of k is easier to determine. Explanations may vary.

Examples

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- One way to develop students' understanding of the parameters of a quadratic relation is to do "relation aerobics". Have students stand next to their desk and raise their hands to make a "U" shape. Write equations and have students use their arms to model the parabola. For example, if a is a positive number that is less than 1, students spread their arms wider; if k gets decreases, students crouch to indicate a shift down.
- In Example 3, a quadratic relation is used to model motion. The Example demonstrates a paper-and-pencil method to match an equation to a set of points. This could be used if technology is limited.

Key Concepts

- Ensure students understand that when they describe the shape of a parabola, they do so in comparison to the graph of $y = x^2$.

Discuss the Concepts

- Have students work in pairs to answer these questions. Alternatively, discuss the answers as a class.

Discuss the Concepts Suggested Answers (page 190)

D1. The coordinates of the vertex are $(0, k)$, which can be determined by substituting $x = 0$ into the equation and solving for y.

D2. a) $a > 1, k = 0$ **b)** $-1 < a < 0, k > 0$

Practise (A)

- Encourage students to refer to the Investigate and the Examples before asking for assistance.
- For **questions 1, 2, and 5**, remind students that the dotted curve is the graph of $y = x^2$.
- Have copies of **BLM G-1 Grid Paper** and **BLM G-3 Four Quadrant Grids** available for students to use.
- For **question 6**, students work backward to identify the parameters of the relation from the graph.

Apply (B)

- **Question 8** links to the Chapter Problem. Remind students to keep the solution to this question handy as their solution may help them complete the Chapter Problem Wrap-Up.
- For **question 9**, you may wish to remind students how to choose appropriate scales for a graph. **Part c)** is a Literacy Connect question, which gives students an opportunity to explore literacy issues in the context of mathematics
- For **question 10**, you may wish to have students use the file **4.2 Water.gsp** to check their answers. Go to *www.mcgrawhill.ca/books/foundations11* and follow the links to download the file.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- Students may be interested in watching a video of the skier from **question 11**. Go to the McGraw-Hill Ryerson Web-site at *www.mcgrawhill.ca/books/foundations11* and follow the links. For **part a)**, the intent is for students to see that, for the same time interval (1 s), a much greater distance is travelled between 2 s and 3 s compared to between 0 s and 1 s.
- In **question 12**, students may be confused because there are more than two variables in the equation. Suggest that students substitute the values for the initial velocities into the general equation to create the two equations.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	12
Reasoning and Proving	1, 2, 4–7
Reflecting	4, 7
Selecting Tools and Computational Strategies	3, 8, 9–12
Connecting	11
Representing	4, 6, 9
Communicating	1, 2, 5

Common Errors

- Some students may have trouble completing tables of values because they do not follow the order of operations.

R_x Have students use the Warm-Up questions to refresh their memory of the order of operations.

- Some students may have trouble choosing appropriate scales for their graphs.

R_x Have students determine the greatest and least values for each variable, and then use these values to determine the vertical and horizontal scales.

Accommodations

Visual—provide pipe cleaners and enlarged copies of the graphing calculator screens. Encourage students to place the pipe cleaners over the graph and to move them according to the transformation.

Language—allow student to work with a partner to facilitate understanding of instructions

Motor—provide large grid paper for all graphing exercises

Ongoing Assessment

- You may wish to use **BLM A-6 Knowledge and Understanding General Scoring Rubric** to assess students' responses to **question 7**.

Extra Practice

- You may wish to use **BLM 4-5 Section 4.2 The Quadratic Relation $y = ax^2 + k$** for remediation or extra practice.