

# 4.3

## The Quadratic Relation $y = a(x - h)^2$

### Student Text Pages

194–203

### Suggested Timing

80 min

### Tools

- grid paper
- graphing calculators
- computers
- *The Geometer's Sketchpad*®

### Related Resources

BLM 4-6 Section 4.3 The Quadratic Relation  $y = a(x - h)^2$   
BLM G-1 Grid Paper  
BLM G-3 Four Quadrant Grids  
BLM A-9 Communication General Scoring Rubric

### Link to Prerequisite Skills

Students should complete all questions in the Prerequisite Skills prior to starting this section.

### Warm-Up

1. Write an equation for a parabola that is
  - a) vertically compressed and opens upward
  - b) shifted up 3 units on the  $y$ -axis and opens downward
2. Describe how each parabola compares to  $y = x^2$ .
  - a)  $y = 0.5x^2 + 8$
  - b)  $y = -3x^2 - 10$

### Warm-Up Answers

1. Answers may vary. Sample answers: a)  $y = 0.1x^2$       b)  $y = -x^2 + 3$
2. a) opens upward, vertically compressed, shifted up 8 units  
b) opens downward, vertically stretched, shifted down 10 units

### Teaching Suggestions

#### Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

#### Section Opener

- Read the opening paragraph aloud. Discuss with the class the importance of accurate modelling when designing and building bridges.

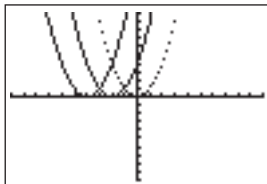
#### Investigate

- You may wish to have half the class complete Method 1 and the other half complete Method 2, and then compare their results.
- Consolidate students' understanding by discussing the results as a class.

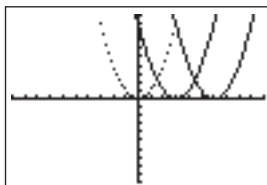
### Investigate Answers (pages 194–196)

#### Method 1: Use a Graphing Calculator

2. a)



3. a)



b) All the graphs have the same shape as the graph of  $y = x^2$ .

c) The vertices of all the graphs are on the  $x$ -axis. Each vertex is on, or to the left of, the  $y$ -axis.

d) The constant term,  $h$ , determines how many units the vertex translates to the left.

b) The vertices of all the graphs are on the  $x$ -axis. Each vertex is on, or to the right of, the  $y$ -axis.

c) The constant term  $h$ , determines how many units the graph translates to the right.

- 4. **a)** When  $h > 0$ , the vertex of the parabola is to the right of the  $y$ -axis.  
**b)** When  $h < 0$ , the vertex of the parabola is to the left of the  $y$ -axis.  
**c)** When  $h = 0$ , the vertex of the parabola is on the  $y$ -axis.
- 5. Answers may vary. Sample answers:  
**a)**  $y = (x - 1)^2$       **b)**  $y = (x + 1)^2$       **c)**  $y = x^2$
- 6. Answers may vary. Sample answers:  
**a)**  $y = 0.5(x - 5)^2$       **b)**  $y = -2(x + 3)^2$       **c)**  $y = 2x^2$
- 7. The value of  $h$  is easier to determine. Substitute  $y = 0$  to find  $h$ .

**Method 2: Use *The Geometer's Sketchpad*<sup>®</sup>**

- 1. Increasing the value of  $h$  makes the parabola shift to the right.  
Decreasing the value of  $h$  makes the parabola shift to the left.
- 2. **a)** When  $h > 0$ , the vertex of the parabola is to the right of the  $y$ -axis.  
**b)** When  $h < 0$ , the vertex of the parabola is to the left of the  $y$ -axis.  
**c)** When  $h = 0$ , the vertex of the parabola is on the  $y$ -axis.
- 3. Answers may vary. Sample answers:  
**a)**  $y = (x - 1)^2$       **b)**  $y = (x + 7)^2$       **c)**  $y = x^2$
- 4. Answers may vary.
- 5. The value of  $h$  is easier to determine. Substitute  $y = 0$  to find  $h$ .

**Examples**

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- Concentrate on the language of transformations. For example: shifted, vertically stretched, vertically compressed.

**Key Concepts**

- Give students the opportunity to consolidate their understanding of the effects of changing  $a$ ,  $h$ , and  $k$  on the graph of a quadratic relation.

**Discuss the Concepts**

- Give students time to formulate their answers before discussing the questions as a class.

**Discuss the Concepts Suggested Answers (page 199)**

- D1.** The value of  $h$  is easier to determine. It can be found by substituting  $y = 0$ .
- D2. a)**  $a = -1, h = 0$       **b)**  $a = -2, h = -1$

**Practise (A)**

- Encourage students to refer to the Investigate and the Examples before asking for assistance.
- Have copies of **BLM G-1 Grid Paper** and **BLM G-3 Four Quadrant Grids** available for students to use.
- In **questions 1 and 3**, remind students that the dotted curve is the graph of  $y = x^2$ .
- For **question 4**, students work backward to write an equation for the quadratic relation from the graph.

**Apply (B)**

- **Question 6** is a Literacy Connect question. It allows students an opportunity to explore literacy issues in the context of mathematics.
- **Question 8** links to the Chapter Problem. This is quite a challenging problem. Let students work on it in small groups so they can help each other. Ensure that students understand that the dotted line is the parabola

### Common Errors

- Some students may interpret the value of  $h$  incorrectly. For example, given the equation  $y = (x + 7)^2$ , some students may say that  $h = 7$  instead of  $h = -7$ .

R, Have students refer to the form  $y = a(x - h)^2$  to remind them that the sign is opposite.

### Accommodations

**Memory**—provide a graphic organizer suitable for the transformations and have students begin to organize the information

**Perceptual**—provide a handout of graphs needed for exercises; where there is more than one graph on a grid, use colour to help distinguish them

**Gifted and Enrichment**—have students explain why negative values for  $x$  are not used. Have them find examples of other graphs with this property on the Internet

**Language**—encourage verbal responses for explanations or have a partner record the answers

(or part of it) that the equation models. The black part at the bottom of the diagram is the ramp. Explain to students that the dotted line represents a parabola that passes through  $(-10, -10)$  and has a vertex where the rider jumps off the cliff. In **part a**), students should substitute  $x = -10$ ,  $y = -10$ , and  $v = 5$  into the equation and solve for  $h$ .

Another method of tackling the problem, for those who like using *The Geometer's Sketchpad*<sup>®</sup>, is to use the sketch **4.3 Investigation.gsp** (which students will have used if they did Method 2 of the Investigate on p.195 of the student text). They will first need to substitute  $v = 5$  to obtain the value of  $a$ ,  $a = -0.2$ . Then they can manipulate the slider for  $h$  and the blue parabola until it passes through  $(-10, -10)$ .

Remind students to keep their solution to this question handy so they can refer to it when they complete the Chapter Problem Wrap-Up.

- For **question 9**, you may want to make the file **4.3 Bridge.gsp** available to students or use it yourself when taking up the homework. Go to [www.mcgrawhill.ca/books/foundations11](http://www.mcgrawhill.ca/books/foundations11) and follow the links to download the file.

### Extend (C)

- Assign the Extend question to students who are not being challenged by the questions in Apply.
- In **question 10**, the second and third graphs are the reflection of the first graph in the line  $y = x$ . The content of this question is not an expectation of the course but it can be used to start a discussion regarding the fact that parabolic curves do not always open upwards or downwards but may open to the right or left.

### Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9
Reasoning and Proving	1–5, 8, 9
Reflecting	2, 5, 10
Selecting Tools and Computational Strategies	7, 8
Connecting	7, 9
Representing	2, 4, 7, 10
Communicating	1, 3, 6–8, 10

### Ongoing Assessment

- While students are working, circulate and see how well each person works. This may be an opportunity to continue observing and recording individual students' learning skills.
- Assess students' ability to communicate mathematically, and to justify their thinking. You may wish to use **BLM A-9 Communication General Scoring Rubric** to assist you in assessing your students.

### Extra Practice

- You may wish to use **BLM 4-6 Section 4.3 The Quadratic Relation  $y = a(x - h)^2$**  for remediation or extra practice.