4.4 The Quadratic Relation $y = a(x - h)^2 + k$

Student Text Pages
204–217

Suggested Timing
80 min

Tools
- grid paper
- graphing calculators
- computers
- *The Geometer's Sketchpad®*

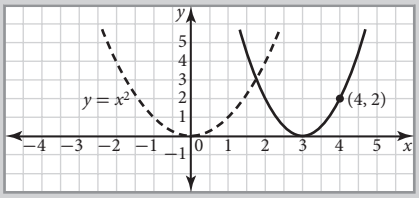
Related Resources
BLM 4-7 Section 4.4 The Quadratic Relation $y = a(x - h)^2 + k$
BLM 4-8 Section 4.4 Achievement Check Rubric
BLM G-1 Grid Paper
BLM G-3 Four Quadrant Grids

Link to Prerequisite Skills

Students should have completed all questions prior to beginning this section.

Warm-Up

1. Write an equation for the solid-line parabola.



2. Compare the graph of each relation to the graph of $y = x^2$.
a) $y = -0.4(x + 5)^2$
b) $y = 10(x - 8)^2$

Warm-Up Answers

1. The solid parabola opens upward and is stretched vertically compared to the graph of $y = x^2$, so $a > 1$. The vertex is shifted 3 units to the right so $h = 3$. So, the equation has the form $y = a(x - 3)^2$. Then, use the fact that the parabola passes through (4, 2). Substitute $x = 4$ and $y = 2$, to find the value of a. The equation is $y = 2(x - 3)^2$.

2. a) The value of a is –0.4, so the parabola opens downward and is compressed vertically. The vertex is shifted 5 units to the left.

b) The value of a is 10, so the parabola opens upward and is stretched vertically. The vertex is shifted 8 units to the right.

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Discuss with students the fact that real data often does not fit a model perfectly. The relationship between the number of infants with HIV born each year and the year is approximately quadratic.

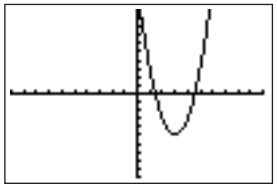
Investigate

- You may wish to have half the class complete Method 1 and the other half complete Method 2, and then compare their results.
- Consolidate students' understanding by discussing the results as a class.

Investigate Answers (pages 205-206)

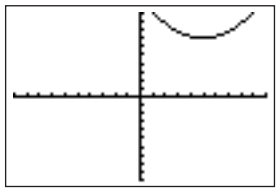
Method 1: Use a Graphing Calculator

1.



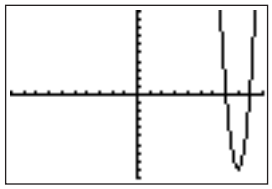
2. The minimum value is –5 and the coordinates of the vertex are (3, –5).

3. a)



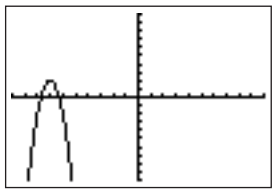
The coordinates of the vertex are (5, 7) and the minimum value is 7.

b)



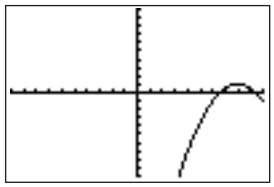
The coordinates of the vertex are (8, –9) and the minimum value is –9.

4. a)



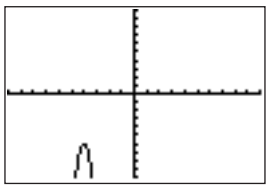
The coordinates of the vertex are (–7, 2) and the maximum value is 2.

b



The coordinates of the vertex are (8, 1) and the maximum value is 1.

c)



The coordinates of the vertex are (–4, –6) and the maximum value is –6.

5. For each quadratic relation, the minimum or maximum value is the y-coordinate of the vertex.

6. No. You also need to determine the value of a.

Method 2: Use *The Geometer's Sketchpad*®

1. The values of h and k relate directly to coordinates of the vertex.

2. Answers may vary. Sample answers:

a) $y = x^2$, vertex at (0, 0); $y = 2(x + 2)^2 + 1$, vertex at (–2, 1); $y = 0.5(x - 1)^2 - 1$, vertex at (1, –1)

b) $y = -x^2$, vertex at (0, 0); $y = -2(x + 2)^2 + 1$, vertex at (–2, 1); $y = -0.5(x - 1)^2 - 1$,vertex at (1, –1)

3. For each quadratic relation, the minimum or maximum value is the y-coordinate of the vertex.

4., 5. Answers may vary.

Examples

- Have students work through the Examples as a class before proceeding to the Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- In Example 1, ensure students understand the concept of vertical stretches and vertical compressions. If the terms *vertically stretched* and *vertically compressed* are confusing to students, you may want to describe the graphs as *wider than* or *narrower than* $y = x^2$. But be sure to always compare the graphs to the graph of $y = x^2$. Since students are asked to sketch the parabolas, the only important point to plot is the vertex.
- In Example 2, no knowledge of the description of the parabola is needed. The general shape of the parabola can be determined using the coordinates of the vertex and two other points on the curve.
- In Example 3, students determine the value of a using the coordinates of the vertex and a known point. Some students will be unsure of what values to substitute for x and y in the equation.

Key Concepts

- Have students review the meanings of the parameters of a quadratic relation of the form $y = a(x - h)^2 + k$.

Discuss the Concepts

- Students may think that the a-values of the red and green parabolas are the same since they appear to be "parallel". You may wish to use dynamic geometry software to show that the green parabola is not a translation image of the red parabola.

Discuss the Concepts Suggested Answers (page 211)

D1. a) Method A. I can find the direction of the graph (upward or downward), the vertical stretch or compression, and the coordinates of the vertex from the equation.

b) Method A is more accurate, since it can generate all the points on the curve while Method B only gives information about three points.

D2. a) B **b)** C **c)** A

D3. The y-coordinates stayed the same. This will also be true for points 3 units to the right and the left of the vertex, because of the symmetry of the parabola about the axis of symmetry.

Practise (A)

- Encourage students to refer to the Investigate and the Examples before asking for assistance.
- Have copies of **BLM G-1 Grid Paper** and **BLM G-3 Four Quadrant Grids** available for students to use.

Apply (B)

- **Question 7** is a Literacy Connect question, which allows students the opportunity to communicate in the context of mathematics.
- **Question 9** links to the Chapter Problem. Remind students to keep their solution to this problem handy for reference when they complete the Chapter Problem Wrap-Up.

- **Question 10** is an Achievement Check question. All students, regardless of ability, should be able to answer some parts of this question. The first parts of the question require knowledge and understanding. The last parts of the question require reflecting, reasoning, and communication. For part d), most students will need to use systematic trial to find the coordinates (h, k) of the vertex, since it is between the plotted values. Some students may choose to make a scatter plot using a graphing calculator or other technology tool, find the quadratic curve of best fit and then its maximum. You may need to tell students that x represents the charge per lesson and y represents the weekly revenue, both in dollars. You may wish to use **BLM 4-8 Section 4.4 Achievement Check Rubric** to assist you in assessing your students.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- In **Question 11**, students compare the standard form of a quadratic relation to the vertex form. Students will work with equations in standard form in Chapter 5.

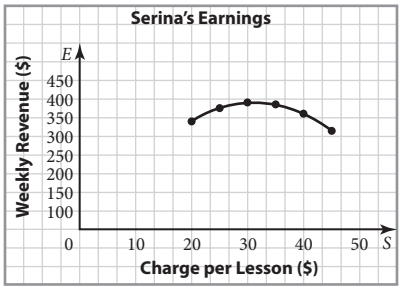
Achievement Check Answers (page 216)

10. a)

Charge per Lesson ($)	Expected Number of People per Week	Weekly Revenue ($)
45	**7**	**315**
40	9	360
35	11	**385**
30	**13**	**390**
25	**15**	**375**
20	**17**	**340**

b) $374.37

c)



d) The graph is a parabola that opens downward. The vertex is at (31.125, 390.625).

e) $y = -0.4(x - 31.125)^2 + 390.625$

f) Serina should charge $30 per lesson. This rate will generate the greatest weekly revenue.

g) The graph will still look like a parabola, with a maximum revenue $450 at $x = 30$ and 35.

Literacy Connections

- Have students use examples to explain the meaning of the parameters of the vertex form of a quadratic equation.

Common Errors

- Some students may have trouble identifying vertical compressions or vertical stretches for values of a close to 1 or –1.

R_x Have students use graphing calculators or dynamic geometry software to graph a variety of quadratic relations.

Accommodations

Memory—have students continue to work on the graphic organiser

Perceptual—provide grid paper with different coloured x- and y-axes

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9, 10–12
Reasoning and Proving	1–7, 8–11
Reflecting	6, 7, 10, 11
Selecting Tools and Computational Strategies	4, 5, 7–10
Connecting	12
Representing	3, 6–8, 10
Communicating	5, 8, 11

Extra Practice

- You may wish to use **BLM 4-7 Section 4.4 The Quadratic Relation $y = a(x - h)^2 + k$** for extra practice or remediation.