Chapter 4 TASK: Fore!

Level 4 Sample Response

1.a) The path of a golf ball is modeled using a quadratic equation. The golf drive has traveled 325 yd and reached a height of 65 ft. The following three points are on the path of the ball:

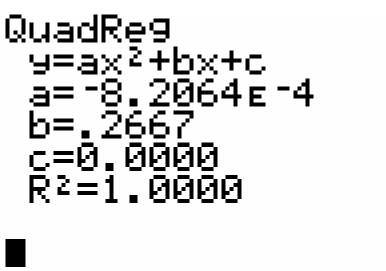
The starting point where the drive is struck is (0, 0).

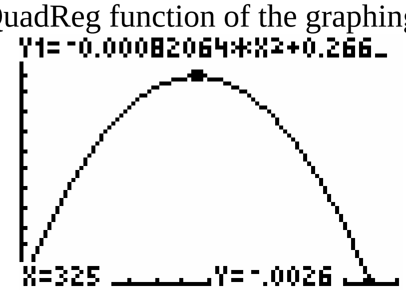
The point where the ball reaches its maximum height is the mid point of its path which is, in yards, (325 ÷ 2, 65 ÷ 3) or (162.5, 21.67).

The point where the ball reaches the ground again is given by (325, 0).

b) The three points are graphed using a graphing calculator.

c) Here is a curve of best fit using the QuadReg function of the graphing calculator.





d) The three points (0, 0), (325, 0), and (162.5, 21.67) are on the curve $y = a(x - h)^2 + k$. The coordinates of the point where the golf ball reaches its maximum are (h, k). Therefore h = 162.5 and k = 21.67.

So, the equation of the graph is $y = a(x - 162.5)^2 + 21.67$. Then, use the fact that the point (0, 0) is on this curve to find a.

$$0 = a(0 - 162.5)^2 + 21.67$$

$$a = \frac{-21.67}{162.5^2}$$

$$= -0.000\ 82$$

Therefore the equation of the curve is $y = -0.000\ 82(x - 162.5)^2 + 21.67$.

2. The equation which models the golf shot is $y = –0.1(0 – 15)^2 + 22.5$, where x and y are in feet.
a) The maximum height reached by the golf ball is 22.5 ft (it is k in the equation $y = a(x – h)^2 + k$).
b) The ball will be on the ground when it is struck, that is at (0, 0).

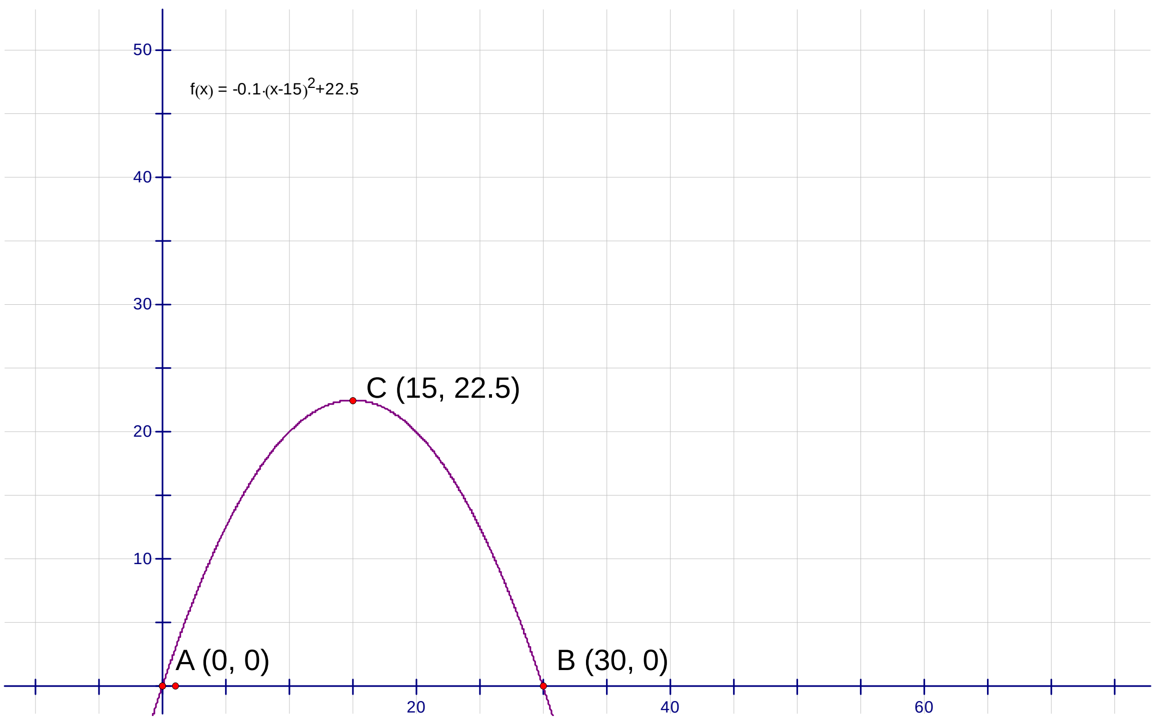
When the ball hits the ground again at the end of the shot, $y = 0$. Substitute $y = 0$ in $y = –0.1(0 – 15)^2 + 22.5$ to find the corresponding value of x.

$$0 = -0.1(x - 15)^2 + 22.5$$
$$0.1(x - 15)^2 = 22.5$$
$$(x - 15)^2 = 225$$
$$x - 15 = \pm\sqrt{225}$$

So, $x = 0$ or $x = 30$.
The value $x = 0$ represents the point where the golf ball was struck and the point $x = 30$ represents the point where it landed. So the golf ball traveled 30 ft.

c) When $x = 0$, $y = –0.1(0 – 15)^2 + 22.5 = 0$. This means that when $x = 0$, $y = 0$, therefore the golf ball is struck at the beginning of the shot from the point (0, 0).

d) Here is the GSP graph for the quadratic relation $y = –0.1(0 – 15)^2 + 22.5$.



Level 4 Notes

Look for the following:

- shows thorough understanding of the shape of the parabola and its relation to the equation $y = a(x - h)^2 + k$
- provides highly accurate calculations, algebraic manipulation and graphs
- demonstrates thorough ability to apply algebraic techniques in modeling involving a quadratic equation
- applies graphing technology in modeling involving a quadratic equation with a high degree of effectiveness
- finds a curve of best fit with a high degree of effectiveness
- provides highly organized solutions with clear justification of responses
- makes highly effective use of mathematical symbols and technology

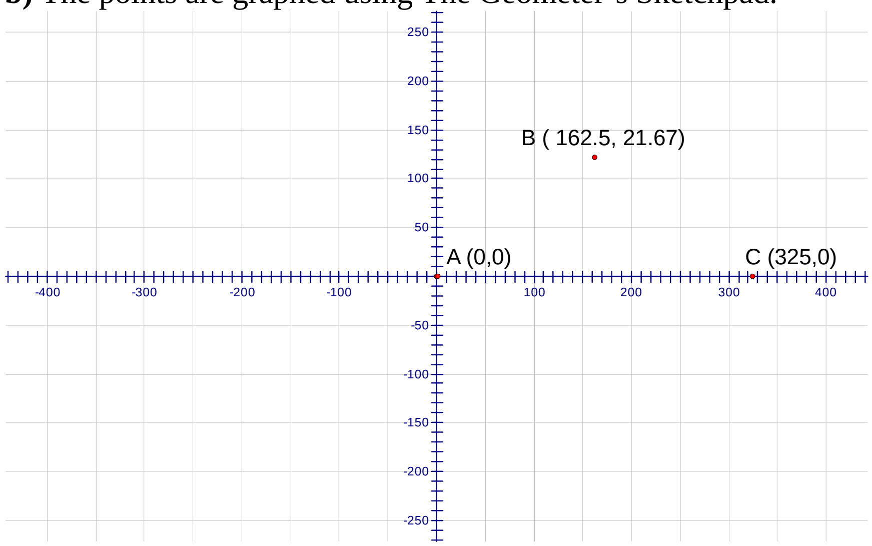
Chapter 4 TASK: Fore!

Level 3 Sample Response

1. a) Mei has been modeling the path of a golf ball using a quadratic equation. If a golf drive has traveled 325 yards and reached a height of 65 feet, or 21.67 yd, then:

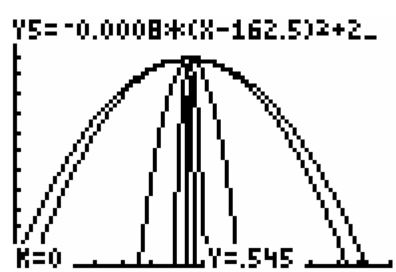
i) The point where the drive is struck is (0, 0)
ii) The point where the ball reaches the ground is given by (325, 0)
iii) The ball reaches its maximum height at the mid point and is 65 feet high—
(325 ÷ 2, 65 ÷ 3) which is (162.5, 21.67)

b) The points are graphed using The Geometer's Sketchpad.



c) Since Mei is modeling the path of the golf ball by using a quadratic relation, the path will be a parabola that opens downward and passes through the points (0, 0), (325, 0), and (162.5, 21.67).
If the equation of the quadratic relation is $y = a(x - h)^2 + k$, then $h = 162.5$ and $k = 21.67$.
I used trial and error to find a value for a that gives a curve of best fit. Since the parabola opens downwards, a must be negative.

Try $a = -1$, graph $y = -(x - 162.5)^2 + 21.67$.
The graph does not pass through (0, 0) or (325, 0).
Try $a = -0.1$, graph $y = -0.1(x - 162.5)^2 + 21.67$.
The graph does not pass through (0, 0) or (325, 0).
Try $a = -0.01$, graph $y = -0.01(x - 162.5)^2 + 21.67$.
The graph does not pass through (0, 0) or (325, 0).
Let $a = -0.001$, graph $y = -0.001(x - 162.5)^2 + 21.67$.
The graph does not pass through (0, 0) or (325, 0).
Let $a = -0.0008$, graph $y = -0.0008(x - 162.5)^2 + 21.67$.
The graph passes through (0, 0) and (325, 0).



d) From part c), the curve of best fit is $y = -0.0008(x - 162.5)^2 + 21.67$.

2. The equation which models the golf shot is $y = –0.1(0 – 15)^2 + 22.5$ where x and y are in feet.

a) The maximum height reached by the golf ball is 22.5 ft (it is k in the equation $y = a(x – h)^2 + k$).

b) The ball will be on the ground when it is struck, that is at (0, 0).

When it reaches the ground again after being hit, $y = 0$.

$$0 = -0.1(x-15)^2 + 22.5$$

$$0.1(x-15)^2 = 22.5$$

$$(x-15)^2 = 225$$

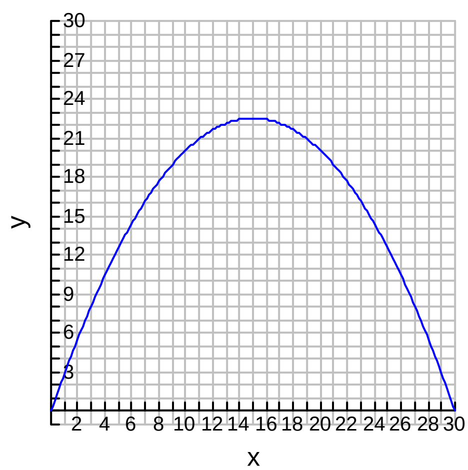
$$x-15 = \pm\sqrt{225}$$

So, $x = 0$ or $x = 30$.

The first value, 0, is when the ball is struck and the second value, 30, is when it reaches the ground again. So the golf ball traveled 30 ft.

c) When $x = 0$, $y = –0.1(0 – 15)^2 + 22.5 = 0$. This means that when $x = 0$, $y = 0$, therefore the golf ball is struck at the beginning of the shot from the point (0, 0).

d) $y = –0.1(0 – 15)^2 + 22.5 = 0$



Level 3 Notes:

Look for the following:

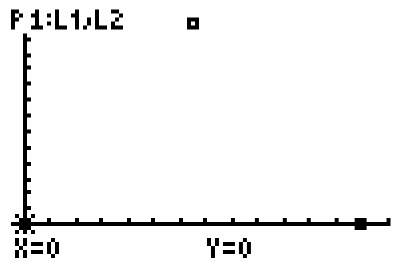
- shows considerable understanding of the shape of the parabola and its relation to the equation $y = a(x – h)^2 + k$
- provides mostly accurate calculations, algebraic manipulations and graphs
- applies algebraic techniques in modeling involving a quadratic equation with a considerable degree of effectiveness
- applies graphing technology in modeling involving a quadratic equation with a considerable degree of effectiveness
- finds a curve of best fit with a considerable degree of effectiveness
- organizes solutions and justifies responses with considerable clarity
- makes effective use of mathematical symbols and technology

Chapter 4 TASK: Fore!

Level 2 Sample Response

1. a) The path of a golf ball is being modeled with a quadratic equation. The ball has traveled 325 yards and reached a height of 65 feet, so the three points that would lie on the path of the ball are the starting point (0, 0), the ending point (325, 0), and the point where is reaches its maximum height is (325 ÷ 2, 65).

b) Here are the three points (0, 0), (162.5, 65), and (325, 0).



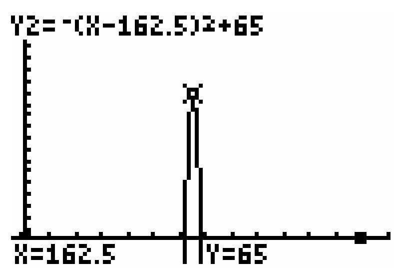
c) If the path of the golf ball is modeled by a quadratic relation, $y = a(x - h)^2 + k$, it passes through the points (0, 0), (325, 0), and (162.5, 65). The coordinates of the point where the golf ball reaches its maximum are (h, k). So h = 162.5 and k = 65. I can try values for a that give a curve of best fit using the TI-83.

Try a = 1, it doesn't work because the graph is open upwards.

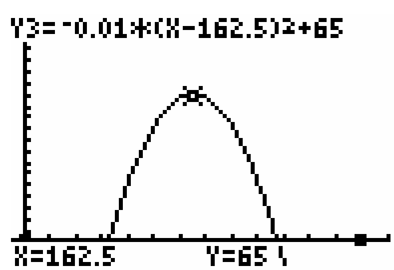
Try a = –1, the graph is too thin and does not pass through (0, 0) or (325,0).

Try a = –5, the graph is even thinner.

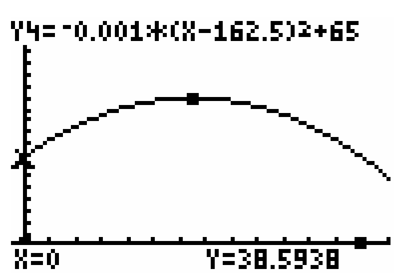
Try a = –0.1, the graph is wider.



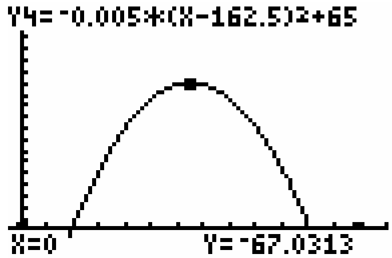
Try a = –0.01, the graph is wider.



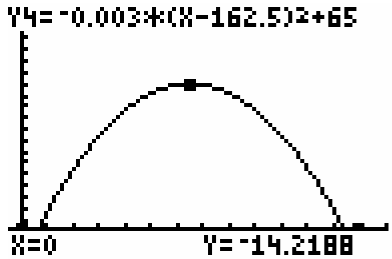
Try a = –0.001, the graph has gone past the point (0, 0).



Try $a = -0.005$, the graph is closer to (0, 0).



Try $a = -0.003$, the graph is very close to (0, 0).

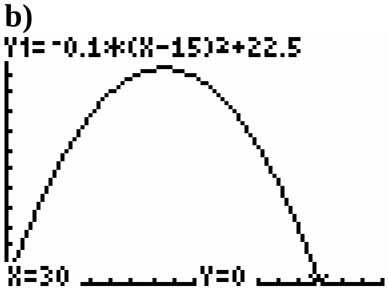


d) The is $y = -\,0.003(x - 162.5)^2 + 65$.

2. The equation which models the golf shot is $y = -0.1(0 - 15)^2 + 22.5$ where x and y are in feet.

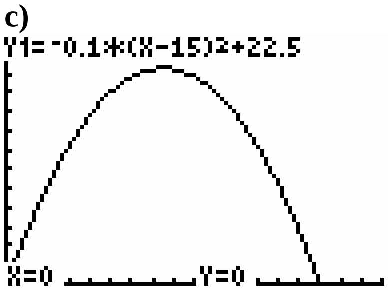
a) The maximum height reached by the golf ball is 22.5 feet.

b)



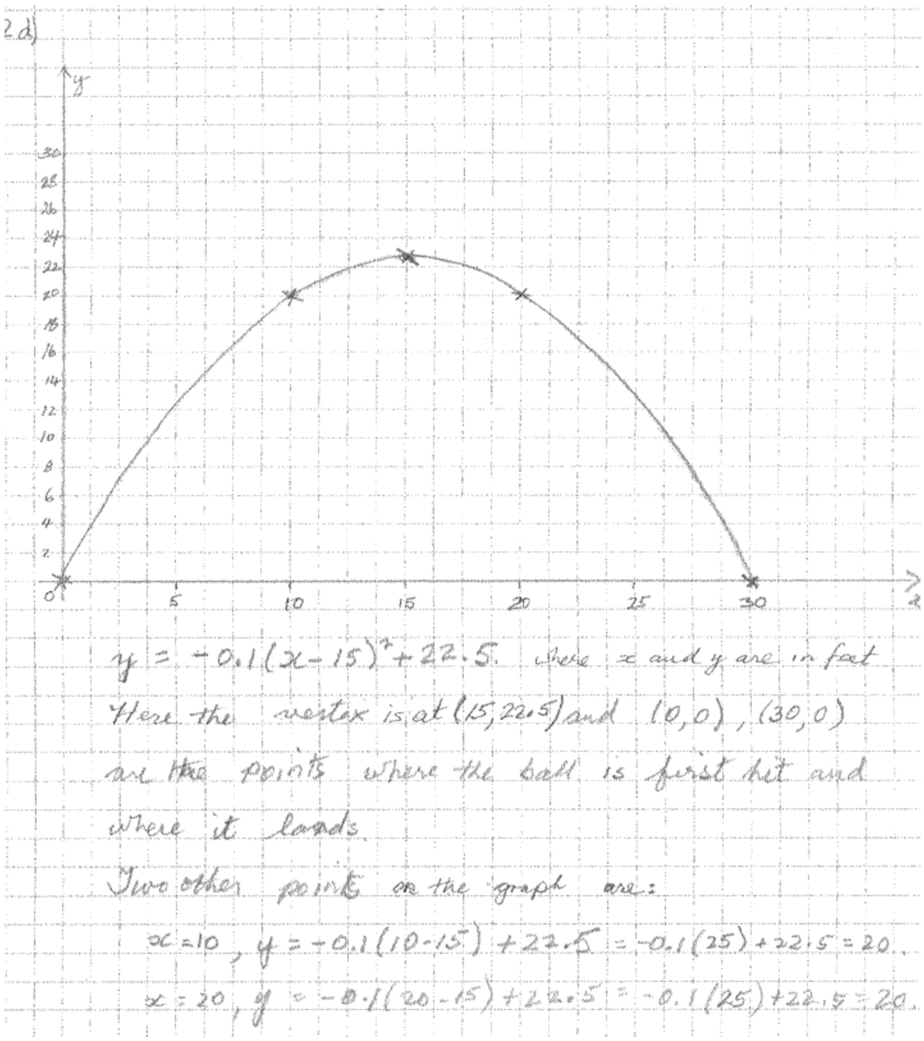
From the graph I can see that the golf ball travelled 30 feet.

c)



Explanation: This shows that the ball was at (0, 0) when it was hit.

d)



<u>Level 2 Notes:</u>

Look for the following:

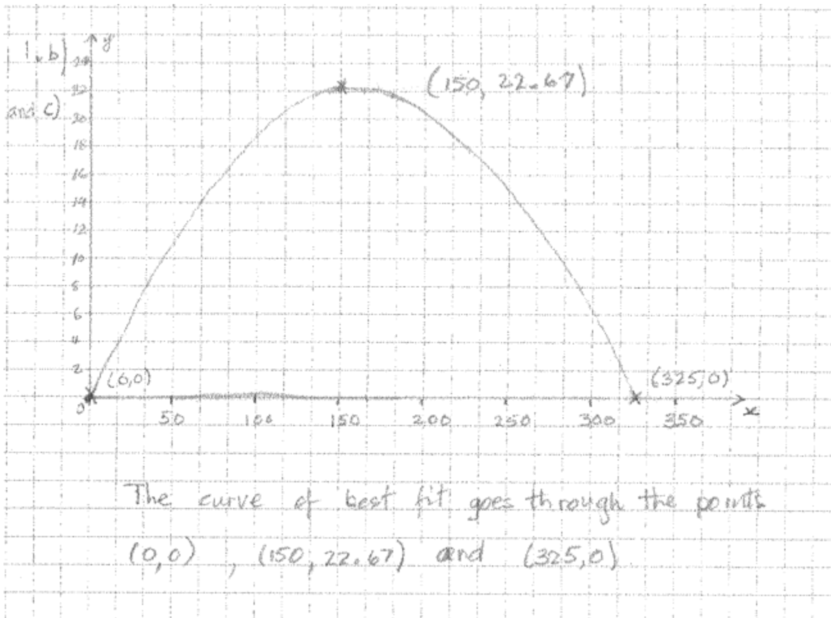
- shows some understanding of the shape of the parabola and its relation to the equation $y = a(x - h)^2 + k$
- provides somewhat accurate calculations, algebraic manipulation and graphs
- demonstrates some ability to apply algebraic techniques in modeling involving a quadratic equation
- applies graphing technology in modeling involving a quadratic equation with a some degree of effectiveness
- finds a curve of best fit with some degree of effectiveness
- provides solutions that are somewhat organized and responses that have some justification
- makes some effective use of mathematical symbols and technology

Chapter 4 TASK: Fore!

Level 1 Sample Response

1. a) The coordinates of the points that would lie on the graph of the flight of the golf ball are the place where it is hit, which is (0, 0). The other points are the place where it lands, which is (325, 0) and the place where it reaches its maximum height of 65 feet. The y – coordinate of the maximum height is 65 and the x-coordinate is about 150.

b) c)



d) The equation for the curve is $y = -0.1(x - 325)^2 + 65$.

2. a) The maximum height of the golf ball is 22.5 feet.

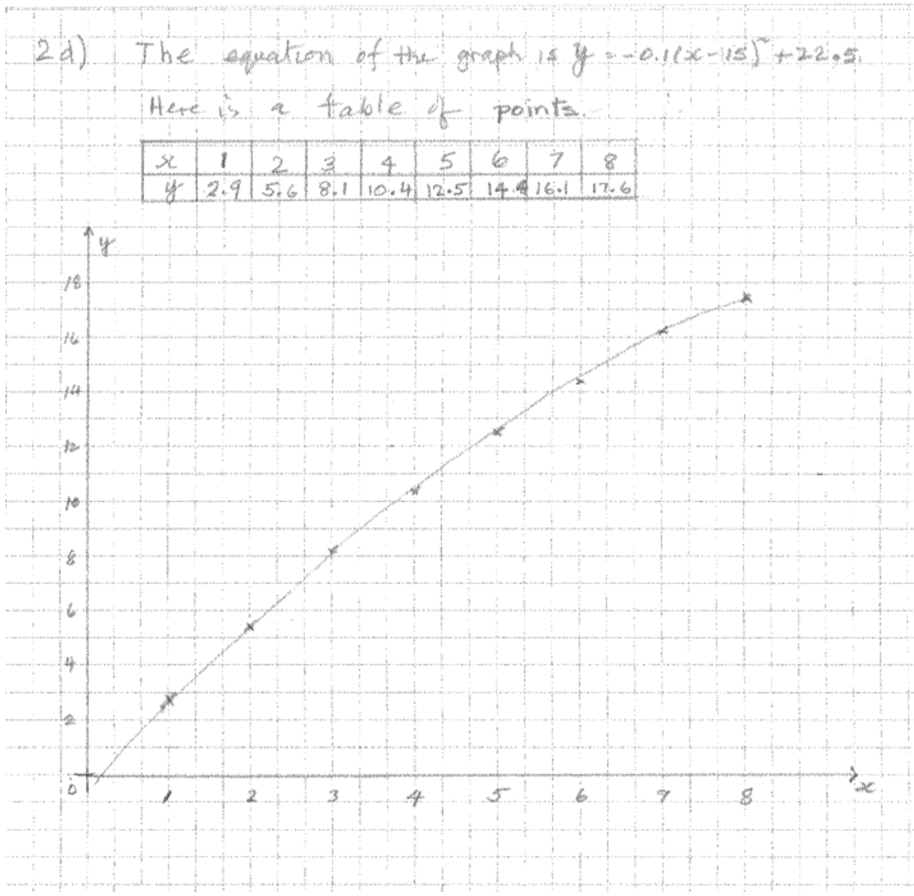
b) The golf ball travelled 15 feet.

c) When $x = 0$, $y = 0.1(0 - 15)^2 + 22.5$

$y = -0.1 \times (-225) + 22.5$

$= 22.5 + 22.5$

$= 45$ feet. This is where the ball landed.



Level 1 Notes:

Look for the following:

- shows a little understanding of the shape of the parabola and its relation to the equation $y = a(x - h)^2 + k$
- provides a little accuracy in calculations, algebraic manipulation and graphs
- demonstrates little ability to apply algebraic techniques in modeling involving a quadratic equation
- draws graphs by hand with a little degree of effectiveness
- guesses a curve of best fit with little degree of effectiveness
- provides solutions that have little organization and partial responses that have very little justification
- makes little effective use of mathematical symbols and technology