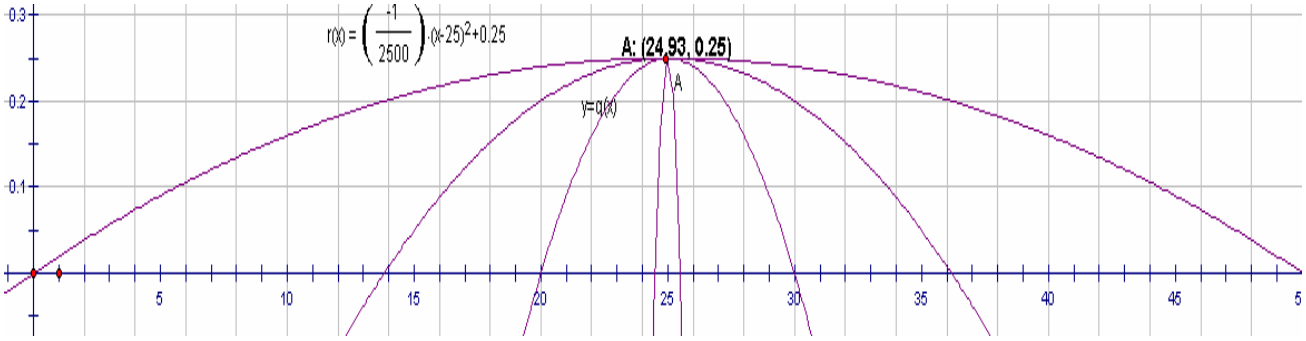
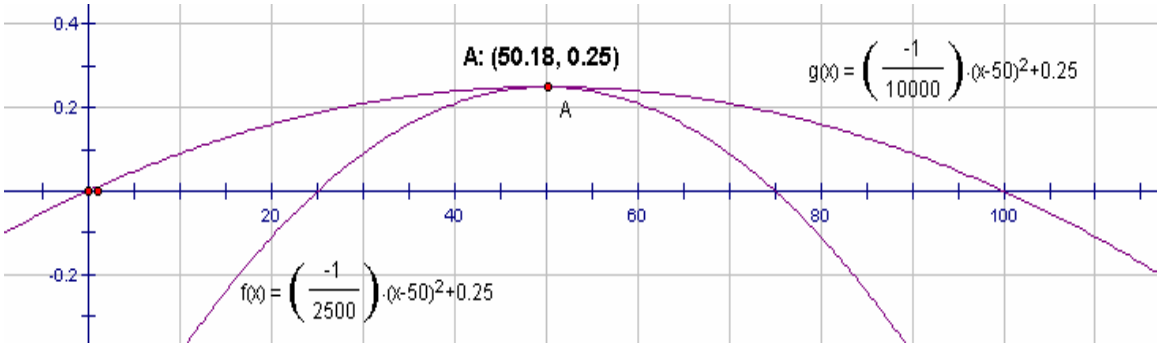
TASK: Design a Soccer Field

Level 4 Sample Response

1. Equation: $h = -\frac{1}{2500}(d-25)^2 + 0.25$ or $h = -0.0004(d-25)^2 + 0.25$.



2. Equation: $h = -\frac{1}{10\,000}(d-5)^2 + 0.25$ or $h = -0.0001(d-50)^2 + 0.25$.



3. Need to find h when $d = 11$. Substitute $d = 11$ into the equation from qu.2.

$h = -0.0001(d-50)^2 + 0.25$

$h = -0.0001(11-50)^2 + 0.25$

$= 0.0979$

The ball is kicked 0.0979 m, or 9.8 cm, above the goal line.

4. In the equation from Qu 1, need to find d when $h = 0.16$.

$h = -0.0004(d-25)^2 + 0.25$

$0.16 = -0.0004(d^2 - 50d + 625) + 0.25$

$0 = -0.0004d^2 + 0.02d - 0.25 + 0.25 - 0.16$

$0 = -0.0004d^2 + 0.02d - 0.16$

$0 = -0.0004(d^2 + 50d - 400)$

$0 = -0.0004(d-40)(d-10)$

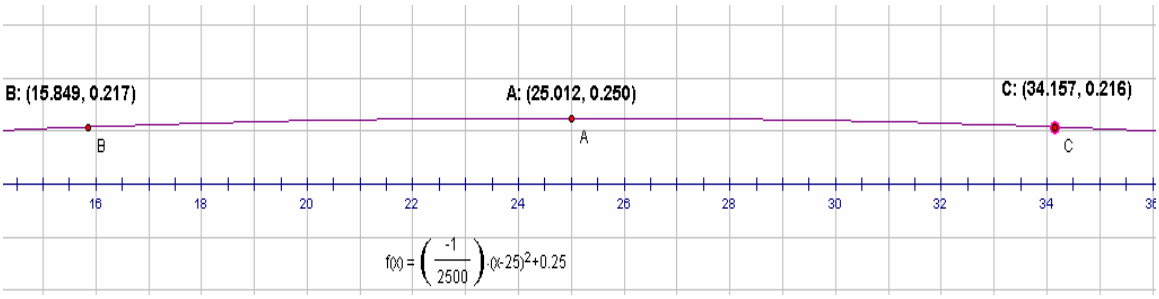
So, d = 10 or 40.

The sprinkler head on the halfway line should be placed 10 m from the sidelines.

5. Minimum points of the circle will be at the midpoint, 25 ± 9.15
So, minimum points will be (15.850, ?) (34.15, ?)

Maximum point will be at the centre of the circle: (25, ?)

GSP Solution:



Therefore the minimum heights above the sidelines are 21.7 or 21.6 cm and the maximum heights above the sidelines are 25.0 cm.

Algebraic method:

Substitute d = 15.85
$h = -0.0004(d - 25)^2 + 0.25$
$h = -0.0004(15.85 - 25)^2 + 0.25$
$= 0.2165$

Substitute d = 25
$h = -0.0004(d - 25)^2 + 0.25$
$h = -0.0004(25 - 25)^2 + 0.25$
$= 0.25$

So, the minimum heights, on the centre circle, are 21.7 cm above the sidelines and the maximum height is 25.0 cm.

Level 4 Notes
Look for the following:

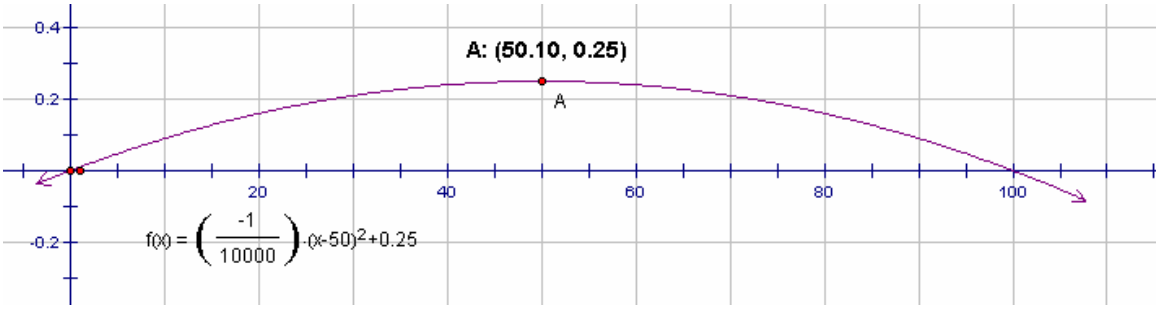
- very few or no erros in solutions
- thorough understanding of the problem and how it realtes to quadratic relations
- highly organised solution and clear, accurate, and detailed justifications for responses
- some use of technology, either a graphing calculator or GSP to show evidence of a trial and error method to find the equations. Students may also use the algebraic method to solve for a (see page 210 in the text)

Level 3 Sample Response

1. I used a trial and error method using a graphing calculator.

Equation: $h = -0.0004(d - 25)^2 + 0.25$

2. Equation: $h = -\frac{1}{10\,000}(d-5)^2 + 0.25$ or $h = -0.0001(d - 50)^2 + 0.25$.



3. Point: (d, h) $(11, h)$ $h = ?$

$h = -0.0001(11 - 50)^2 + 0.25$
$= -0.0001(-39)^2 + 0.25$
$= -0.0001(1521) + 0.25$
$= -0.1521 + 0.25$
$= 0.0979$ m or 9.8 cm

Therefore the ball is kicked 9.8 cm above the goal line.

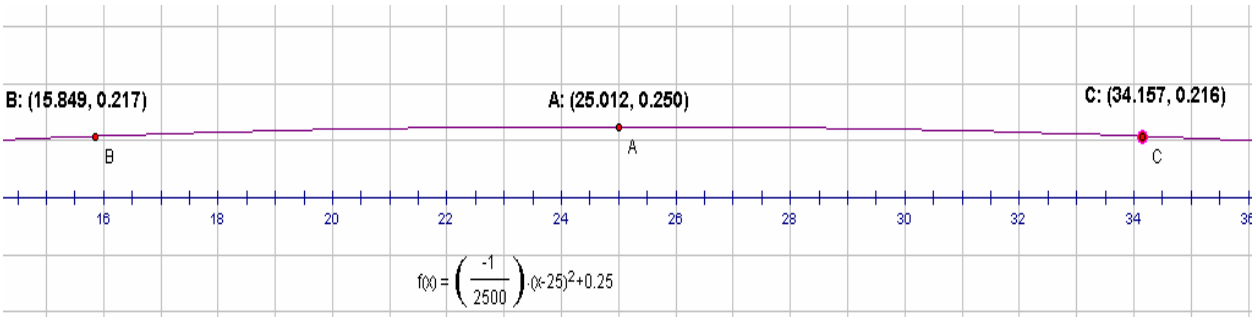
4. $(d, 0.16)$ $(?, 0.16)$

$h = -0.0004(d - 25)^2 + 0.25$
$0.16 = -0.0004(d - 25)^2 + 0.25$
$0 = -0.0004(d - 25)^2 + 0.09$
$0 = -0.0004(d^2 - 50d + 625) + 0.09$
$0 = -0.0004d^2 + 0.02d - 0.25 + 0.09$
$0 = -0.0004d^2 + 0.02d - 0.16$
$d = ?$

5. Minimum points of the circle: 25 ± 9.15 (15.850, ?) (34.15, ?)

Maximum point of the circle: (25, ?)

Solution using GSP:



Therefore the minimum heights above the sidelines are 21.7 or 21.6 cm and the maximum height above the sidelines is 25.0 cm

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Level 3 Notes

Look for the following:

- Students will find the equation but not provide clear evidence as to how they determined the equation modeling the parabola. They will probably indicate which type of technology was used (ie. graphing calculator or GSP).
- In question 4, students may not know how to solve for *d*, they may attempt to use the quadratic formula and substitute correctly but make some mistakes or they may attempt to use a graphing calcuator to find the zeros, but fail to determine the correct value for *d*
- For question 5, students will provide either the graphical solution or the algebraic solution as shown in the level four response. The difference between a level three solution and a level four solution is that a level four solution will probably show both methods.

Level 2 Sample Response

1. Equation: $h = -(d - 25)^2 + 0.25$

2. Equation: $h = -(d - 100)^2 + 0.25$

3. (11, ?)

$$\begin{aligned} h &= -(11 - 25)^2 + 0.25 \\ &= -(-14)^2 + 0.25 \\ &= -196 + 0.25 \\ &= 195.8 \text{ cm} \end{aligned}$$

4. (?, 0.16)

$$\begin{aligned} h &= -(d - 25)^2 + 0.25 \\ 0.16 &= -(d - 25)^2 + 0.25 \\ 0 &= -(\text{d} - 25)^2 + 0.09 \\ (d - 25)^2 &= 0.09 \\ d - 25 &= 0.3 \\ d &= 25.3 \end{aligned}$$

5. $h = -(d - 9.15)^2 + 0.25$

$$\begin{aligned} 0 &= -(d - 9.15)^2 + 0.25 \\ (d - 9.15)^2 &= 0.25 \\ \text{d} - 9.15 &= 0.5 \\ d &= 9.65 \end{aligned}$$

Level 2 Notes

Look for the following:

- For questions 1 and 2, students will most likely substitute the vertex but will not be able to find the vertical compression/stretch factor *a*. They will probably make some of the substitutions correctly but will have difficulty without visual cues.
- For question 3, students will probably substitute the numbers correctly and evaluate correctly, but they may not convert the units to interpret the answer properly.
- For question 4, students will probably substitute the numbers and perform some of the algebra correctly, however they will fail to determine the correct value for *d*.
- In question 5, students will probably recognize that 9.15 m is significant but will not understand the concrete meaning in relation to the quadratic equation modeling the profile of the field. They will probably recognize that they must solve for a value using an algebraic method.

Level 1 Sample Response

1. $h = -d^2$

$h = -d^2 + 0.25$

2. $h = -d^2 + 100$ m

3. $h = -d^2 + 0.25$
$= -(11)^2 + 0.25$
$= -(121) + 0.25$

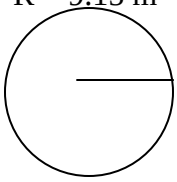
$h = 96$ m

4. $h = -d^2 + 0.25$

$h = -(0.16)^2 + 0.25$
$h = -(256) + 0.25$

h = –231 m

5. R = 9.15 m ?



Level 1 Notes:

Look for the following:

- Students will probably be confused by the questions and just make stabs at writing quadratic equations but show no understanding of the situation.
- For question 5, they will most likely draw a circle and label the radius correctly.