1.4 The Cosine Law

Link to Prerequisite Skills
Students should complete Prerequisite Skills questions 1, 2, and 9 before proceeding with this section.

Warm-Up
1. Solve. Express your answers to the nearest tenth.
  a) $x^2 = 4^2 + 5^2 - 2(4)(5)(0.3456)$ **b)** $x^2 = 8^2 + 5^2 - 2(8)(5)(0.2345)$
  c) $6^2 = 9^2 + 10^2 - 2(9)(10)x$ **d)** $12^2 = 7^2 + 15^2 - 2(7)(15)x$
2. Evaluate. Round your answers to four decimal places.
  a) $\sin 60°$ **b)** $\sin 50°$ **c)** $\sin 40°$
3. Evaluate. Round your answers to four decimal places.
  a) $\cos 60°$ **b)** $\cos 50°$ **c)** $\cos 40°$

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Warm-Up Answers

1. a) 5.2   **b)** 8.4   **c)** 0.8   **d)** 0.6
2. a) 0.8660   **b)** 0.7660   **c)** 0.6428
3. a) 0.5000   **b)** 0.6428   **c)** 0.7660

Teaching Suggestions
Warm-Up
• Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class.
• The Warm-Up questions give students practice rearranging the equation to solve for x.

Section Opener
• Have students use the Internet to learn more about the construction of tunnels. When was the first tunnel built? Which tunnel is the longest? the shortest?

Investigate
• Have students work in pairs to complete the Investigate and have them discuss their results.
• Use **BLM 1-6 Section 1.3 The Sine Law Triangles** if you do not have access to *The Geometer's Sketchpad®*.

Investigate Answers (pages 34–35)

3. They are equal.
4. They are equal.
5. They are equal.
6. 0; the Pythagorean theorem.
7. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Examples
- Have students work through the Examples as a class before proceeding to the Discuss the Concepts.
- For Example 3, label the given angle and sides a, b, and c, then solve for a.
- Students could use a table to record the problems and their answers.

Key Concepts
- Have students make memory cards for the formulas with the formulas for the sides on the front and the formulas for the angles on the back.

Discuss the Concepts
- For **question D1**, it is important students know the angle is contained by the two sides.
- For **question D2**, remind students of the abbreviations to remember these cases: AAS (any two angles and a side) or SSA (two sides and an angle opposite one of the sides) for the sine law. SAS (two sides and a contained angle) or SSS (all three sides) for the cosine law. Memory aid diagrams are useful for these laws.

Discuss the Concepts Suggested Answers (page 39)

D1. To determine the measure of an unknown side, you must know the measures of the two adjacent sides and one of the opposite angles.

D2. Sine law: can be applied when you know the measures of one side and two angles, or two sides and one angle opposite a given side.
Cosine law: can be applied when you know the measures of two sides and the angle between them, or all three sides.


Practise (A)
- Encourage students to refer to the Investigate and the Examples before asking for assistance.

Apply (B)
- For **questions 3** and **4**, remind students how sides of a triangle are labelled: same lower case letters for the sides opposite the upper case letter angles.
- **Question 5** links to the Chapter Problem. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up. Supply students with this diagram if they need assistance.
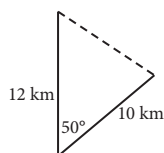
- **Question 6** is a Literacy Connect. You may wish to assign this question as a journal entry or to discuss the question as a class.
- Supply students with this diagram for **question 10** if they need assistance.
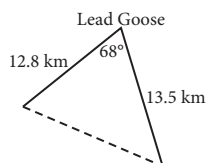
Common Errors

- Some students may make order of operation errors in Example 1, such as subtracting 640 from 656 first and then multiplying by cos 52°.

R_x Remind students that when a number and a trigonometric ratio are side by side, this means the two are multiplied. Students need to follow the order of operations: multiply first, then subtract. Use an example.

- Students may enter expressions into their calculator as shown instead of using the order of operations.

R_x Have students write out all their steps to find the value for the numerator and then divide, as in Example 2, instead of solving the entire expression at once. Or have them use brackets, $(1444 + 676 - 900) \div 1976$, if their calculators have that option.

Accommodations

Memory—feature the cosine law on a poster or on the word wall

Visual—provide diagrams on a worksheet with room to complete the solutions

Perceptual—have a partner assist with diagrams

Gifted and Enrichment—have students research an interesting problem on the Internet that is an application of the cosine law and present this problem to the class

Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- **Question 13** can be given as an in-class assignment. Allow students to discuss the question to help each other develop a solution. Then have students complete the question individually.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	11–13
Reasoning and Proving	11–13
Reflecting	6
Selecting Tools and Computational Strategies	1–5, 7–13
Connecting	6
Representing	3–5
Communicating	6, 12

Extra Practice

- You may wish to use **BLM 1-9 Section 1.4 The Cosine Law** for remediation or extra practice.