2.2 Theoretical Probability

Student Text Pages
68–75

Suggested Timing
70 min

Related Resources
BLM 2-4 Section 2.2 Theoretical Probability

Link to Prerequisite Skills

Students should complete questions 1 to 6 of the Prerequisite Skills before proceeding with this section.

Warm-Up

1. Express each fraction in lowest terms.

a) $\frac{4}{52}$ **b)** $\frac{13}{52}$ **c)** $\frac{6}{36}$

2. In a standard deck of cards, how many are:

a) face cards? **b)** red? **c)** red face cards?

Warm-Up Answers

1. a) $\frac{1}{13}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{6}$

2. a) 12 **b)** 26 **c)** 6

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently, and then discuss the solutions as a class.

Section Opener

- Have students record their choice. Divide the class into two groups, and have students argue in favour of their decision. Can they think of any real-life situations that would require similar decisions? (Buying or selling stocks now or waiting for a change in price, etc.)

Investigate

- In Investigate 1, most students will know the answers to question 1 even if they do not know the definition of theoretical probability. By asking them to reflect on their thinking as they answer the questions, students may realize (or recall) how to find theoretical probability. The reference to numerator and denominator, and their knowledge of experimental probability should help them arrive at, and understand, the formula.
- Discuss the concept of "equally likely" in probability. Remind students that outcomes such as tossing a fair coin, rolling a fair die, or choosing a card from a fair deck are equally likely. Add events that are *not* equally likely. For example, when rolling 2 dice, rolling a 7 and a 5 are not equally likely events. The individual outcomes [(1, 4), (2, 3), (5, 2), etc.], are equally likely, but there are more ways to roll a 7 than there are to roll a 5.
- In Investigate 2, students are introduced to counting arrangements. Arrangements (permutations) are not in the curriculum, so having students make an organized list is all that is expected.

Investigate Answers (pages 68-70)

Investigate 1

1. a) $\frac{1}{2}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{13}$ **d)** $\frac{1}{10}$

2. The numerator is the number of outcomes that result in the given event: a) tossing heads, b) rolling a 4, c) choosing an ace, d) winning the contest. The denominator is the total number of outcomes for each situation: a) tossing heads or tails b) rolling the numbers 1 to 6, c) choosing from 52 cards, d) choosing from 100 qualified contestants.

3. Create a fraction. The numerator is the number of outcomes that result in the desired event and the denominator is the total number of outcomes.

Investigate 2

2.

Jason	Tony	Lisa
X	G	H
X	H	G
G	X	H
G	H	X
H	X	G
H	G	X

3. a) 6 ways **b)** 2 **c)** $\frac{1}{3}$

4. The desired outcome is Jason receiving Golf Legends. There are three different games Jason could possibly receive and each is equally likely, so the probability is the same as when it is calculated by examining the different combinations of games, $\frac{1}{3}$.

Examples

- In Example 1, part c), students need to be certain to count each outcome only once.
- Example 2: When rolling 2 dice, tossing 2 coins, or choosing sandwich parts (in question 9), tree diagrams provide a systematic, visual way of helping to count outcomes. The table at the bottom of page 72 provides another effective way of displaying the roll of 2 dice. It is easy to see there are 6 possible doubles, 6 ways of rolling a 7, 5 ways of rolling a 6, and so on.
- Method 2 provides a different, yet correct approach to the problem. Students should be encouraged to use logic in looking for multiple approaches to problems.

Key Concepts

- These concepts are similar to experimental probability. Note that counting outcomes is a critical part of calculating probability. Tree diagrams, tables, systematic lists, and logic can all be used to count outcomes.

Discuss the Concepts

- You may wish to assign one or more of the Discuss the Concepts questions as a journal entry.

Discuss the Concepts Suggested Answers (page 73)

D1. The number of successful outcomes can be 0; for example, drawing a red club from a deck of cards or rolling a 7 with one die.

$$\frac{0}{\text{Number of possible outcomes}} = 0$$

The number of successful outcomes can also be as large as the total number of outcomes; for example, choosing any card from a deck or rolling any number from 1 to 6 on one die.

$$\frac{\text{Number of possible outcomes}}{\text{Number of possible outcomes}} = 1$$

So, theoretical probability will be between 0 and 1.

D2. Drawing a black diamond from a standard deck of cards; diamonds are red, so it is impossible to draw a black diamond from a fair deck of cards.

Practise (A)

- Encourage students to refer to the Examples before asking for assistance.
- For **question 1**, remind students to express their answers in lowest terms. Parts b) and c) are complementary probabilities, as are parts d) and e) in **question 5**.
- Remind students that "or" means the number of successful outcomes is added, provided the outcomes do not overlap.

Apply (B)

- For **question 6**, it may be necessary to assist students who use only numbers that are given in the problem. To find the probability of not getting a chair, have students think: "How many will not get a chair?" (2 people.) "How many people are there altogether?" (10 people.)
- For **question 7**, suggest students use the strategy of using a similar-but-simpler problem. Instead of drawing a diagram with 12 charms, draw 3 or 4 charms and look at the number of places where the clasp might be. Students should then see the connection and extend it to 12 charms, without drawing a complicated diagram.
- **Question 10** is a Literacy Connect. You may wish to assign this question as a journal entry or to discuss the question as a class. If students have difficulty with the term *complement*, give examples using dice or a deck of cards.
- For **question 11**, students may think the area of the small circle is half that of the larger circle. Have students use the formula for area, $A = \pi r^2$, to calculate the area of both circles. This would be a good reminder that area varies with the square of the dimensions; while volume varies with the cube of the dimensions. You may use tiles for area and cubes for volume to demonstrate these concepts in a concrete way.
- **Question 12** links to the Chapter Problem. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up. Part c) illustrates the concept of conditional probability; by stating that it is not a bass, the number of total possible outcomes is reduced, decreasing the denominator and increasing the probability (of the fish being a carp).

Common Errors

- Some students may count outcomes twice, when given questions involving the word "or".

R_x Have students make up questions where there may or may not be overlap between events (i.e. find the probability of rolling doubles or a number greater than 7 with 2 dice).

- Some students may have difficulty counting outcomes.

R_x Have students review counting techniques such as tree diagrams, tables, organized lists, or regular diagrams.

Accommodations

Visual—encourage students to draw diagrams where appropriate and to use manipulatives as they work on the exercises

Perceptual—prepare a classroom poster showing the tree diagram and the outcome table for rolling two dice

Language—provide a deck of cards to help students recognize differences in suits and cards

Extend (C)

- Assign the Extend questions to students who are not being challenged by the questions in Apply.
- In **question 13,** parts a) and b) are in fact the same question: if Ann is not between Bob and Cathy, then Bob and Cathy are standing next to each other.
- **Question 14** connects probability to volume concepts. A cone has volume one-third that of a cylinder with the same height and radius.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	10–14
Reasoning and Proving	8, 10, 11, 14
Reflecting	10, 11, 14
Selecting Tools and Computational Strategies	1–7, 9, 11–14
Connecting	2, 12
Representing	7, 10
Communicating	8, 10, 13

Extra Practice

- You may wish to use **BLM 2-4 Section 2.2 Theoretical Probability** for remediation or extra practice.